UNIT 1

Physics for Class XI

SCALARS AND VECTORS

F- 15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

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PREFACE

This unit is third in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project. At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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30 May 1982

INTRODUCTION

In secondary classes you have studied some basic concepts of vectors and scalars The rules in vector algebra are much different from the rules followed by scalars. You know that a mass of 2 kilograms added to another mass of 2 kilograms always results in 4 kilograms. But two vectors of magnitude 2 units each may give a sum varying from zero to four units, depending upon the directions of the two vectors. The rules for multiplication of vectors are also much different from the rules of multiplication of numbers. Vectors are extensively used in all branches of physics In this unit you will learn more about some introductory concepts and language of vectors and scalars.

OBJECTIVES

To be able to

1. Add, subtract and multiply vectors.
2. Determine the components of a vector along two mutually perpendicular directions.
3. Apply the rules for addition, subtraction and multiplication of vectors to solve simple problems

SUGGESTED READING MATERIAL

1. Vector representation

Suppose you use your finger to apply a force on a block lying on a table If you push the block vertically downwards, it will not move no matter how 'hard' you push On the other hand the block will move if you push it in a horizontal direction, and the harder you push the faster the block will move Therefore, if you want to study the motion of the block you must know how 'hard' (*magnitude*) you are pushing it and in which *direction*. A physical quantity which requires a magnitude and a direction for its complete specification is called a vector quantity. Other examples of vectors are displacement, velocity, momentum, etc. Vectors are represented by an arrow. The length of the arrow (Fig. 1), to some chosen scale, gives its magnitude and the arrow head points to its direction. As an illustration, let a force of magnitude 10 N be applied to a block, using a string making an angle of 45 degrees with the horizontal [Fig. 1(a)]. This force vector can be represented by an arrow of length 2 cm (for convenience a scale representing 5 N by 1 cm length is chosen) making an angle of 45 degrees with the horizontal as in Fig 1(b). Analytically a vector may be represented by bold face print letter such as **A** and its magnitude by light face print letter *A*. In your notes you may represent this vector using a single letter with an arrow at the top, such as $\vec{A}$, whose magnitude may be represented as $|\vec{A}|$. The scalar quantities, such as mass, time, etc. on the other hand have only a magnitude but do not require a direction for their specification. Scalars are added as ordinary numbers and represented by light faced letters.

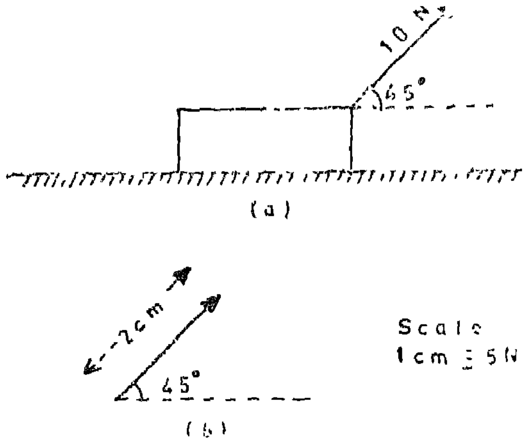

Fig. 1. Representation of force vector by an arrow.

2. Equal and unequal vectors

Two vectors of the same physical quantity are said to be equal if and only if they possess equal magnitudes and same direction. Vectors ***A*** and ***B*** shown in Fig 2(a) are equal because they have equal length and same direction. Vectors ***C*** and ***D*** are not equal and so are vectors ***E*** and ***F***, why?

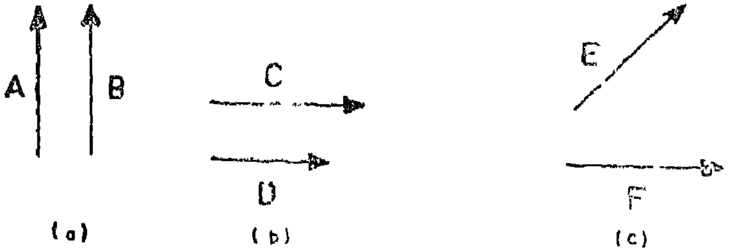

Fig. 2. Equal and unequal vectors.

3. Composition of vectors

As it has been pointed out in the introduction, unlike scalars, vectors posses magnitudes and directions. That is why their addition, etc. are *little* complicated In the present article we shall familiarise you with the relevant rules for composition of vectors. By the term *composition of vectors* we essentially mean, given two or more vectors, how to reduce them into a single equivalent vector called the resultant

3.1. *Addition of vectors* · Let a boat row with a speed of 4 km/h in a direction at right angles to the flow of a stream where the velocity of the stream is 3 km/h. From your intuition, you can say that the flow of the stream will not let the boat move with its orginal velocity of 4 km/h and even the direction of motion of the boat will not remain the same as desired. But in order to determine the resultant velocity of the boat, simple arithmetic of 4 km/h plus 3 km/h equal to 7 km/h will not work, simply because directions of velocities are different. As the boat is propelled towards the opposite bank, the river water drags it along the stream as in Fig 3 (a).

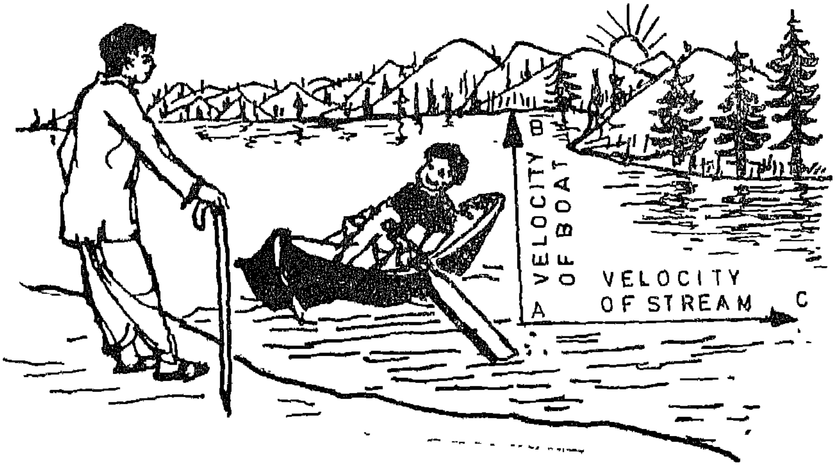

Fig. 3(a). A boat crossing a river.

We find that the resultant velocity of 5 km/h is given by the diagonal of the rectangle, as shown in Fig. 3(b), whose adjacent sides represent the original velocities The proper diagonal for the velocity is the concurrent (starting from the vertex where tails of these two vectors meet) diagonal.

This is a special case of the *law of parallelogram of vector addition* stated below.

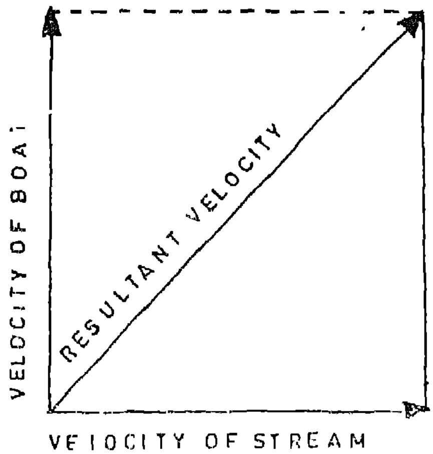

Fig. 3(b). Addition of velocity vectors.

"If two vectors acting at a point are represented in magnitude and direction by adjacent sides of a parallelogram, then the concurrent diagonal of the parallelogram will represent their resultant in magnitude and in direction."

Let A and B be two vectors acting along OA and OB respectively as shown in Fig. 4. The diagonal of the parallelogram OACB is OC, which represents the resultant R of A and B.

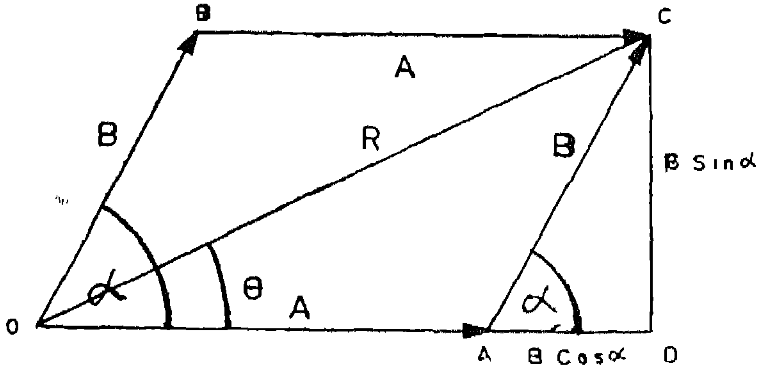

Fig. 4. Resultant of two vectors acting at a point.

In vector from one can write.

$$R = A + B \qquad \ldots(1)$$

The magnitude and direction of $\boldsymbol{R}$ are given by

$$R=\sqrt{A^2+B^2+2AB\cos\alpha} \qquad \ldots (2)$$

and

$$\tan\theta=\frac{B\sin\alpha}{A+B\cos\alpha} \qquad \ldots\ldots(3)$$

Special case

If two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ in Fig. 5 act at right angles to each other, then from Eq.(2)

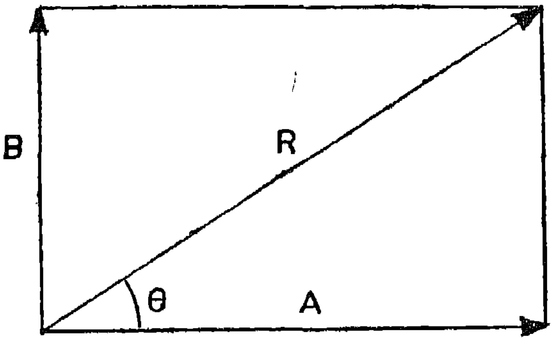

Fig. 5. Resultant of two vectors at right angle.

$$R=\sqrt{A^2+B^2} \qquad \text{because } \cos 90^\circ=0 \qquad \ldots\ldots(4)$$

which gives the magnitude of the resultant.
Similarly, from Eq.(3), the direction of $\boldsymbol{R}$ is given by

$$\tan\theta=\frac{B}{A} \qquad \ldots\ldots(5)$$

The parallelogram law of addition of vectors can be translated to slightly different form called the *triangle law of vector addition.*

3.2. *Triangle law of vectors* · Here we move vector $\boldsymbol{B}$ parallel to itself until the tail of $\boldsymbol{B}$ is over the head of vector $\boldsymbol{A}$. Then the side closing the triangle gives the sum $\boldsymbol{R}$, both in magnitude and direction as shown in Fig. 6.

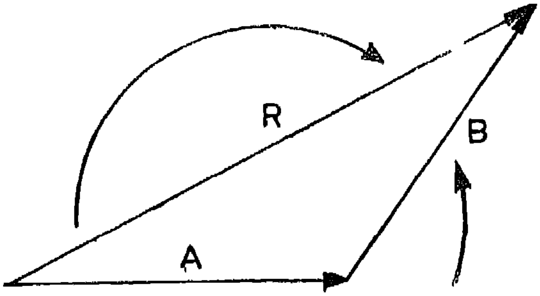

Fig 6. Addition of vectors by triangle method.

Question

Do you change a vector when you move it parallel to itself?

Ans. No.

If we wish to add several vectors A, B, C, D, ... we could add them by applying parallelogram or triangle constructions again and again getting the resultant of A and B and adding it to C, adding the new resultant to D and so on. Equivalently we add A and B by head to tail, then place the tail of C over the head of B, then place the tail of D on the head of C and so on. We can omit the intermediate resultant R by joining tail of the first vector to the head of the last vector, as shown in Fig 7. This is what is known as *polygon law of vector addition*

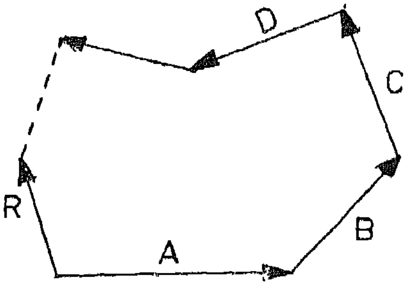

Fig 7. Addition of vectors by the head to tale polygon method.

Example 1

A ship surrounded by fog is pointed due north and sailing, as the navigator thinks, at 50 km/h due north in still water. Actually it is in a current moving 1.5 m/s due east. If the fog disappears and the navigator can observe nearby islands, in what direction will he be really moving? How fast?

Solution

The velocity of the ship,

$$U = 50 \text{ km/h}$$

$$= (125/9) \text{ m/s}$$

The velocity of the stream,

$$V = 1.5 \text{ m/s}$$

Let θ be the angle between the direction of resultant velocity W, and velocity of the stream V. Then from parallelogram law of vectors, resultant velocity of the ship is (see Fig. 8).

$$W = \sqrt{U^2 + V^2}$$

$$= \sqrt{\left(\frac{125}{9}\right)^2 + \left(\frac{3}{2}\right)^2} \text{ m/s}$$

$$W = 14 \text{ m/s}$$

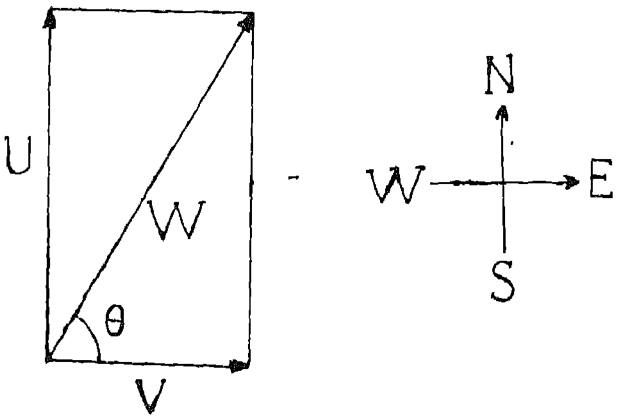

Fig. 8. Vector addition of two velocities.

And $\tan\theta = \frac{U}{V} = \frac{125/9}{3/2} = 9.26$

or $\theta = 83^\circ\ 50'9''$

Example 2

Two forces F_1 and F_2 of 30 N add 40 N respectively act on a particle P. If the angle between F_1 and F_2 is 60°, determine the force that is required to prevent P from moving.

Solution

From the parallelogram law of vectors, the magnitude of the resultant vector R will be given by

$$R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos 60^\circ}$$
$$= \sqrt{(30)^2+(40)^2+2(30)(40)\tfrac{1}{2}}\ N$$
$$=61\ \text{N}$$

The angle between F_1 and R is given by

$$\tan\theta = \frac{F_2\sin 60^\circ}{F_1+F_2\cos 60^\circ} = \frac{40\times 0.866}{30+40\times 0.5} = 0.693$$

or $\theta = 35^\circ$

In order to prevent particle P from moving, a force of 61 N has to be applied to P in the direction opposite to the direction of R.

Questions

1. If two vectors A and B are parallel to each other and act along the same line, what will be their sum?

Ans. $R = A + B$

2 If two vectors $\mathbf{A}$ and $\mathbf{B}$ are directed in opposite directions but along the same line, what will be their sum ?

Ans. $\mathbf{R} = \mathbf{A} - \mathbf{B}$

3.3. Subtraction of vectors

The difference of two vectors $\mathbf{A}$ and $\mathbf{B}$ is

$$\mathbf{C} = \mathbf{A} - \mathbf{B}$$

which may be computed from ...(6)

$$\mathbf{C} = \mathbf{A} + (-\mathbf{B})$$

where $-\mathbf{B}$ has same magnitude as $\mathbf{B}$ but opposite direction. Vectors $\mathbf{A}$ and $\mathbf{B}$ are shown in Fig. 9(a). Fig 9(b) shows the addition of $\mathbf{A}$ and $-\mathbf{B}$ by vector triangle method. Therefore, rules for the subtraction of vectors is same as the addition of vectors. How will you find $\mathbf{A}-\mathbf{B}$ by vector parallelogram method? ... (6)

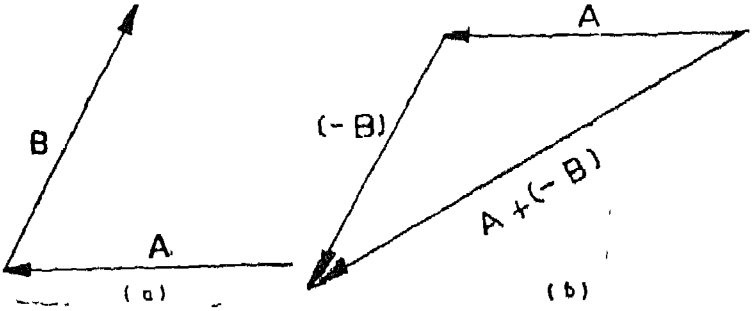

Fig. 9. Subtraction of two vectors.

Example 3

Given two vectors $\mathbf{A}$ and $\mathbf{B}$ such that

$$|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$$

show that $\mathbf{A}$ and $\mathbf{B}$ are at right angles to each other (where $\mathbf{A}$ and $\mathbf{B}$ are non-zero vectors and vertical bars denote the magnitudes).

Solution

If α is the angle between $\mathbf{A}$ and $\mathbf{B}$, then from parallelogram law

$$|\mathbf{A}+\mathbf{B}| = \sqrt{A^2+B^2+2\,AB\cos\alpha}$$

$$|\mathbf{A}-\mathbf{B}| = \sqrt{A^2+B^2+2\,AB\cos(180-\alpha)}$$

Since $|\boldsymbol{A}+\boldsymbol{B}| = |\boldsymbol{A}-\boldsymbol{B}|$

$$\sqrt{A^2+B^2+2\,AB\cos\alpha} = \sqrt{A^2+B^2-2\,AB\cos\alpha}$$

which on squaring both sides gives

$4AB\cos\alpha=0$

Since $A\neq0$, $B\neq0$, $\cos\alpha=0$

Therefore $\alpha=90^\circ$

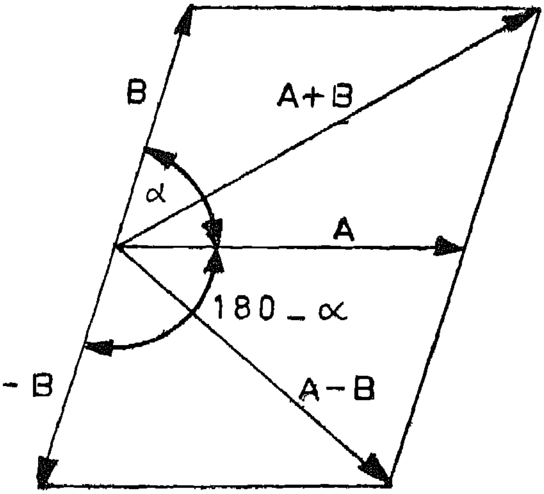

Fig 10. Addition and subtraction of A and B.

Questions

1. When can you add vectors like scalars?

2. Time moves from past to future through present. Would you consider time as a vector or a scalar? Justify your answer.

3. Can two vectors of different magnitudes be combined to give a zero resultant? Can three vectors?

4 Does it make any sense to call a quantity vector when its magnitude is zero ?

5. Given two vectors of magnitudes 3 and 4 units, how will you combine them to get resultant vectors of magnitude 1 unit, $\sqrt{13}$ units, 5 units, $\sqrt{37}$ units and 7 units ?

4. Resolution and addition of vectors, analytic method

In article 3 we discussed composition of vectors. In the present article we will discuss exactly the reverse, i.e. given a single vector acting at a point, how to break it up into two vectors at right angles to each other. Hence the process is called resolution of vectors.

Given a vector A, we want to resolve it into two vectors A_x and A_y along mutually perpendicular X and Y axes. Let the angle between direction of X axis and A be θ as shown in Fig. 11

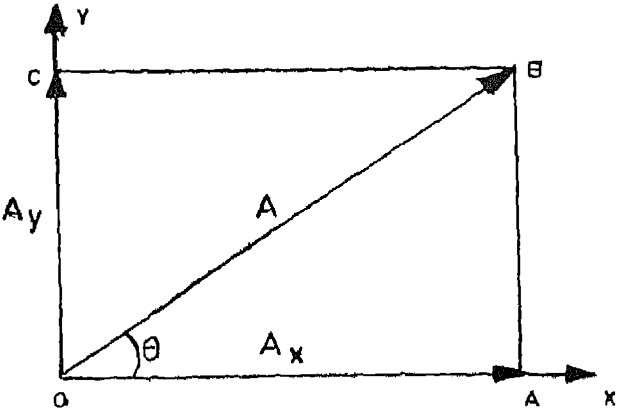

Fig. 11. Resolution of a vector along two mutually perpendicular X and Y axes. The projection of A along X axis represents its X component A_x while its projection along Y axis represents its Y component A_y.

$$\frac{A_x}{A} = \cos\theta$$

$$A_x = A\cos\theta \qquad \cdots (7)$$

Similarly, since AB is equal and parallel to OC, one gets

$$\frac{A_y}{A} = \sin\theta$$

$$A_y = A\sin\theta \qquad \cdots (8)$$

Thus the effect of A is same as the combined effect of A_x and A_y which are called the vector components of A. A lawn-mower (Fig. 12) is one example where motion can be understood by resolution of a vector. A lawn-mower is normally pushed (otherwise it won't cut grass) and not pulled. But it is always easier to pull it because in this case 'pull' can be resolved into two mutually perpendicular components as in Fig. 13(a). The horizontal component will overcome friction whereas the vertical com-

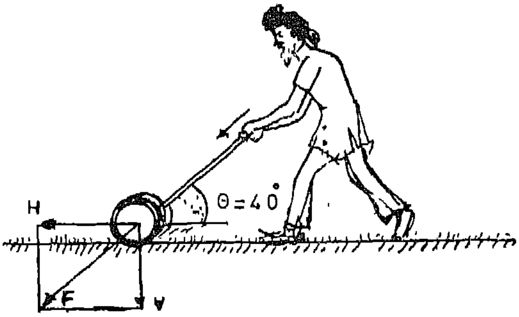

Fig. 12. A lawn-mower being pushed. The applied force F is resolved into horizontal component H and vertical component V.

ponent will counter balance the weight of the mower and the man to some extent. On the other hand in case of 'push' although horizontal component will still counter balance the force of friction, vertical component acting in the downward direction [as shown in Fig 13 (b)] will make the effective weight increase. Thus it is easier to pull a lawn-mower than to push it.

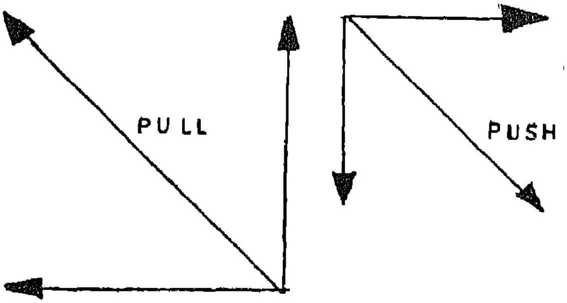

Fig. 13. 'Pull and Push' resolved into vertical and horizontal components.

Example 4

A vector, inclined to X axis at angle of 30°, has a component of $8\sqrt{3}$ units along X axis. Determine the magnitude of the vector and its component along Y axis.

Solution

Referring to Fig. 14

$$A = \frac{A_x}{\cos 30^\circ}$$

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$$A = \frac{8\sqrt{3}}{\sqrt{3}/2} \text{ units} = 16 \text{ units}$$

Similarly $A_y = A \sin 30°$

$$= 16 \times \tfrac{1}{2} \text{ units} = 8 \text{ units}$$

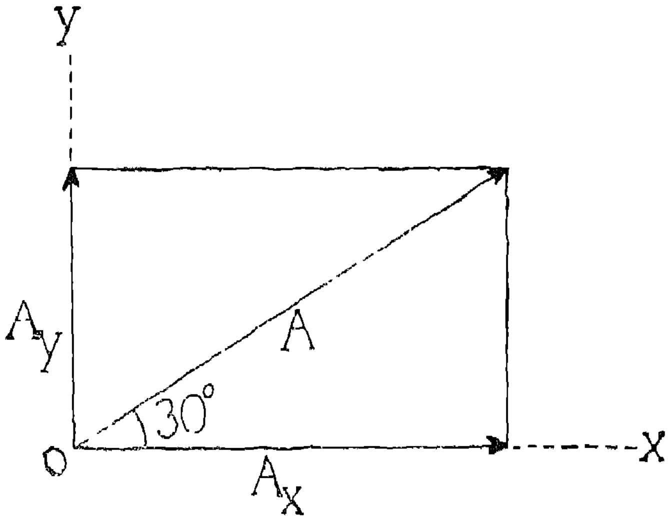

Fig 14. Vector A resolved into components along the X and Y axes

The description of a vector A in terms of A and θ is equivalent to its description in terms of components A_x and A_y and can be used interchangeably. Equations (7) and (8) give A and θ from A_x and A_y.

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = A_y/A_x$$

In the analytic method vectors are resolved into their components. The choice of axis is arbitrary and any two mutually perpendicular axes may be chosen for convenience.

Questions

1. It is much easier for making calculations if axes are so chosen that most of the vectors lie on them. Why?

2. In solving physics problems two axes perpendicular to each other are chosen. What is the advantage of this choice over a pair of axes not at right angles to each other.

Referring to Fig. 4 and Eq. (1) if X axis is chosen along OD and Y axis perpendicular to it, it can be seen that in any coordinate system two vectors such as R and $A+B$ can only be equal if corresponding components are equal, or

$$R_x = A_x + B_x \qquad 9(a)$$

$$R_y = A_y + B_y \qquad 9(b)$$

Taken together, these two algebraic equations are equivalent to vector relation Eq. (1). These two give R and θ by

$$R = \sqrt{R_x^2+R_y^2} \qquad \ldots\ldots 10(a)$$

$$\tan\theta = R_y/R_x \qquad \ldots 10(b)$$

Thus in the analytic method all the vectors are resolved into their X and Y components. The algebraic sum of all the X components gives R_x and the algebraic sum of all the Y components gives R_y. Equations 10 (a) and 10 (b) then give the resultant R.

Example 5

Two vectors A and B of magnitude 20 and 10 units respectively act on point O as shown in Fig. 15. Find the sum of A and B by analytic method.

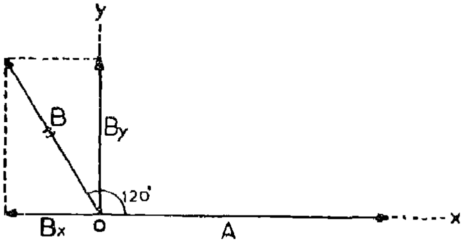

Fig. 15. Finding the sum of A and B by analytic method

Solution

For convenience choose the X axis along vector A as shown in Fig. 15

$$R_x = A_x + B_x$$

$$= A - B\cos 60^\circ$$

$$= (20 - 10\times\tfrac{1}{2})\text{ units}$$

$$= 15\text{ units}$$

$$R_y = A_y + B_y$$

$$= 0 + B\sin 60^\circ$$

$$= (0 + 10 \times 86) \text{ units}$$

$$= 8.6 \text{ units}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(15)^2 + (8.6)^2} \text{ units}$$

$$= 17.3 \text{ units}$$

$$\tan\theta = \frac{R_y}{R_x}$$

$$= \frac{8.6}{15}$$

$$= 0.57$$

$$\text{or } \theta = 30^\circ$$

5. Multiplication of scalars and vectors with vectors

Let a force of 8 N acting upon a body towards east be doubled (i e. multiplied by a scalar 2). We get a force of 16 N acting in the same direction Hence, in general, if a vector V acting at a point P in a certain direction is multiplied by a scalar k (a pure number), it gives us a vector kV whose magnitude is kV and direction is the same as of V as in Fig 16.

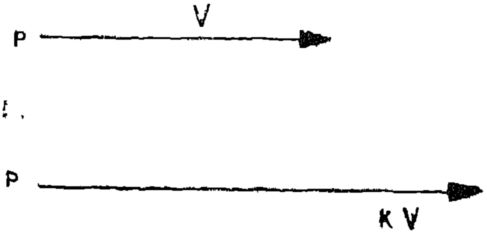

Fig 16. Vector multiplied by a scalar.

The multiplication of two vectors is, for convenience, defined in two different ways to give us products as scalar or vector The former type of multiplication is called scalar (or dot) product and the latter type is known as vector (or cross) product.

5.1. *Scalar (or dot) product*

The scalar product of two vectors A and B inclined to each other at an angle θ is given by

$$A\ B = AB\cos\theta \qquad \ldots\ldots(11)$$

L.H S. of Eq. (11) is read as vector A dot vector B or simply A dot B. It is clear from Fig 17, that $A.B$ is the product of A and projection of vector $\boldsymbol{B}$ on vector A, i.e. OC

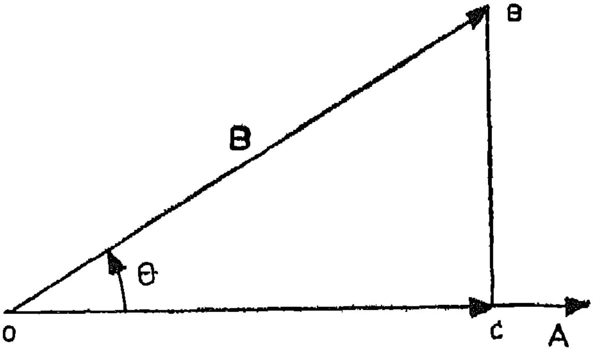

Fig 17. Scalar product of two vectors.

Thus $A\ B=(\text{OA})\ (\text{OC})$

(Since $\text{OC}=\text{OB}\cos\theta$)

In Eq. (11), if instead of considering $A\ \boldsymbol{B}$, we consider $\boldsymbol{B}.A$ we find

$\boldsymbol{B}.A=B\ A\cos(-\theta)$

But $\cos(-\theta)=\cos\theta$

hence $\boldsymbol{B}.A=BA\cos\theta$

$=A.B$

Thus the scalar (or dot) product is commutative.

Since by definition $A.B$ is scalar, it does not have any direction.

5 2 *Vector (or cross) product of two vectors*

Given two vectors A and B inclined to each other at an angle θ, the magnitude of their vector product (read as A cross B) will be (see Fig. 18).

$|A\times B|=BA\sin\theta$ ...(12)

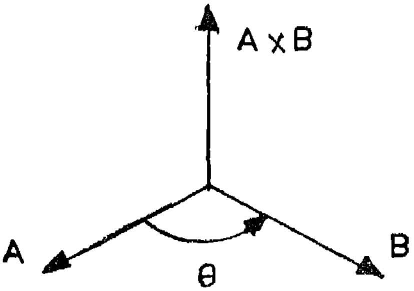

Fig. 18. Cross product of two vectors

$A \times B$ is a vector, whose direction is given by what is known as the *right hand thread rule or the right screw rule. The right screw rule* states that in order to determine the direction of $A \times B$, right screw with its head in the plane determined by A and B, is to be rotated in the direction in which A becomes coincident with B through the smaller angle, then the direction pointed out by the advance of screw head will indicate the direction of $A \times B$ as shown in Fig 19(a).

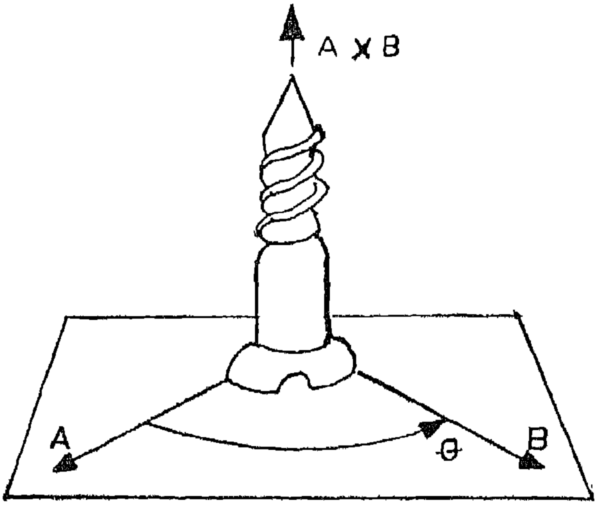

Fig 19 (a). Right screw rule.

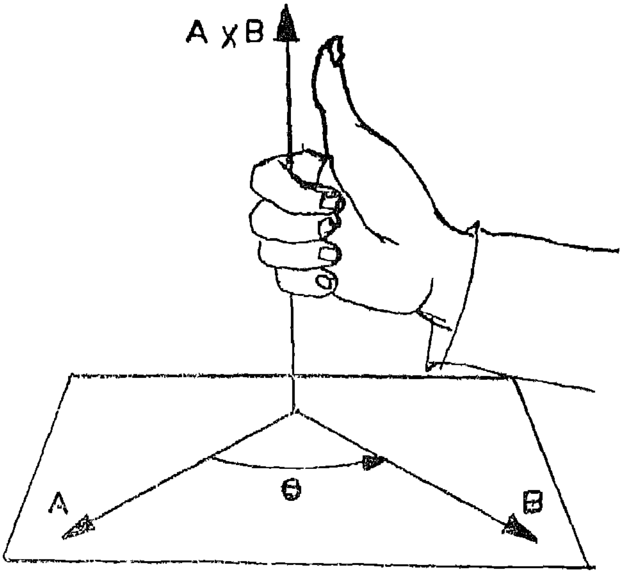

Fig. 19 (b) Right hand thread rule.

Right hand thread rule states that in order to find the direction of $\boldsymbol{A}\times\boldsymbol{B}$, four fingers (excluding the thumb) of the right hand are to be curled from nuckles to nails in the direction in which $\boldsymbol{A}$ is to be rotated to bring it into coincidence with the direction of $\boldsymbol{B}$, then the thumb will point in the direction of $\boldsymbol{A}\times\boldsymbol{B}$ as in Fig. 19(b).

It is clear from the foregoing discussions that

$$|\boldsymbol{A}\times\boldsymbol{B}| = AB\sin\theta$$

But referring to Fig. 20 one finds that $\boldsymbol{B}\times\boldsymbol{A}$ is in the direction opposite to $\boldsymbol{A}\times\boldsymbol{B}$, hence one has to write

$$\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A} \qquad \text{.....(13)}$$

Thus the cross product is not commutative.

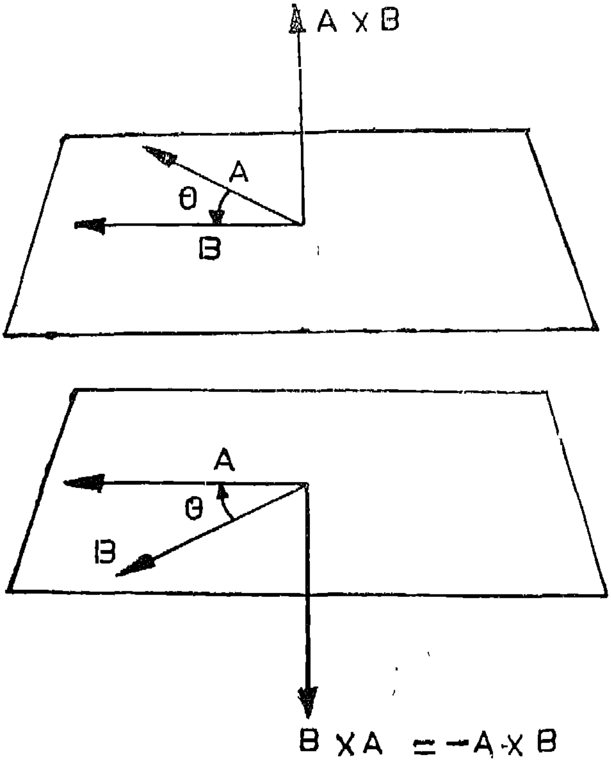

Fig. 20 Vector product $B \times A$ is opposit to $A \times B$.

Geometrically the magnitude of the cross product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ means the area of the parallelogram determined by considering $\boldsymbol{A}$ and $\boldsymbol{B}$

to be its adjacent sides, because area of the parallelogram OACB (Fig. 21) is

$=(OA)\ (CD)$
$=AB \sin\theta$ where $CD=B\sin\theta$
$=|A\times B|$

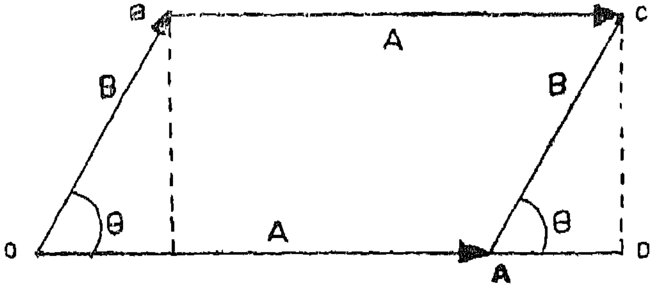

Fig. 21. Geometrical meaning of the cross product.

Example 6

Two vectors act at O in the directions as shown in Fig. 22, where $A=3$ units, $B=4$ units and × denotes going perpendicularly into the plane of the paper

Determine $A\times B$ and $A.B$

Solution

Since angle between A and B is 90°
$A.B=AB\cos 90^\circ=0$
$|A\times B|=AB\sin 90^\circ=(3\times 4\times 1)$ units $=12$ units

The direction of $A\times B$, by right hand thread rule, will be in the plane of the paper in the direction as shown in Fig. 22.

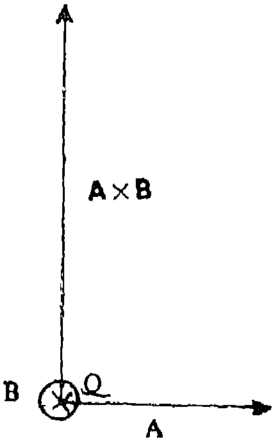

Fig. 22. Vectors acting at point O.

ASSIGNMENT

1. Vectors ***V*** and ***B*** act at a point as shown in Fig. 23. Determine ***V.B***, ***V***×***B*** and ***B***×***V*** where =3 *V*units, *B*=4 units

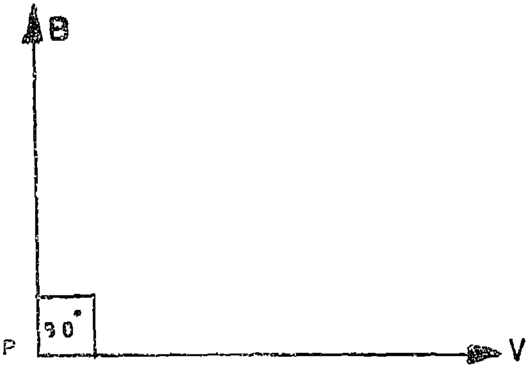

Fig. 23. Two vectors acting at point P.

Ans. 0, 12 units (perpendicular to the plane of the paper and towards you),—12 units going perpendicularly into the paper

2. Two vectors F_1 and F_2 act at P. F_1 is in the plane of the paper and F_2 is coming out of the paper towards the reader. Determine

$F_1 \times F_2$ and $F_1 . F_2$

where $F_1=6$ units and $F_2=7$ units

Ans. 42 units, in the plane of the paper and downwards, 0

3. Let *A* and *B* be two vectors acting along the edges of a cube of side *l* as shown in Fig. 24.

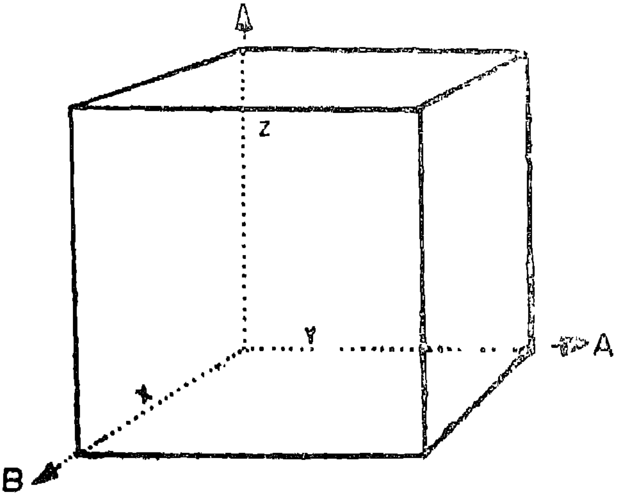

Fig. 24. Two vectors acting along edges of a cube.

(a) Determine $C = A \times B$

(b) Find the value of $(C.A)$ and $(C.B)$

Ans. l^2 (along Z direction), 0, 0

4. If a vector inclined to X axis at an angle of 60°, has a component of 8 units along X axis, determine the magnitude of the vector and its component along Y axis.

Ans. 16 units, $8\sqrt{3}$ units

5. A river flows at 3 km/h. A swimmer whose velocity in still water is 4 km/h requires 15 minutes to reach the opposite bank. What total distance will he swim ?

Ans. 0 461 km

6 A car is driven 50 km eastwards then 30 km northwards and then 25 km westwards. Draw a vector diagram and determine the total displacement of the car from its starting point

Ans. 39 km, 50° 11′ 39″ east of north

7 A vector A has a magnitude 2 5 metres and points due north. What are the magnitudes and directions of the following vectors ?

(a) A (b) $A/2$ (c) $-2.5\,A$ and (d) $4A$

Ans. 2.5 m (due north), 1.25 m (due north), 6.25m (due north)
10m (due north)

8. Show for any vector A, that $A\,A = A$ and that $A \times A = 0$.

9. A force of 10 N is represented by vector A as shown in Fig. 25. Determine its components along X and Y axes.

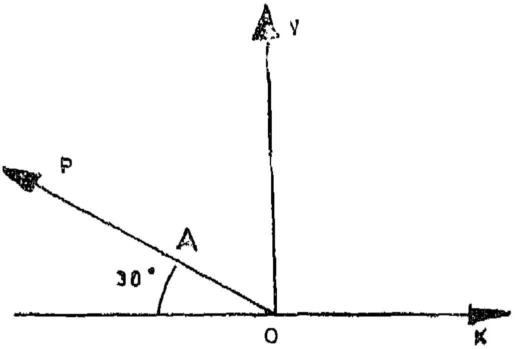

Fig. 25. A force acting at point o.

Ans. X component $=-5\sqrt{3}$ N, Y component $=5$ N

10. Show that vector addition is associative.

11. Establish Eq. (2) and Eq. (3) of the text by taking components along two perpendicular directions.

SELF ASSESSMENT

1. A particle undergoes three successive displacements as follows, $3\sqrt{3}$ metres south, 5.0 metres east, 6 0 metres in a direction 60° north of east. Find the magnitude of the resultant displacement
2. An aeroplane takes off at an angle of 45° to the horizontal. If the vertical component of its velocity is 300 km/h, calculate its actual velocity and the horizontal component of velocity.
3. A force A of magnitude 10 N and another force B of magnitude 6N point in the directions differing by 60° Find their vector and scalar products.
4. Two forces F_1 and F_2 act upon a body in such a manner that the resultant force R has a magnitude equal to F_1 and makes an angle of 90° with F_1. Let $F_1=R=10$ N. Find the magnitude and the direction (relative to F_1) of the second force F_2.
5. Use the standard (right hand) X,Y,Z system of coordinates (Fig. 26). Given vector A in the + X direction, vector B in the +Y direction, and the scalar quantity d.
   - (a) What is the direction of $A \times B$?
   - (b) What is the direction of $B \times A$?
   - (c) What is the direction of B/d?
   - (d) What is the magnitude of $A\ B$?

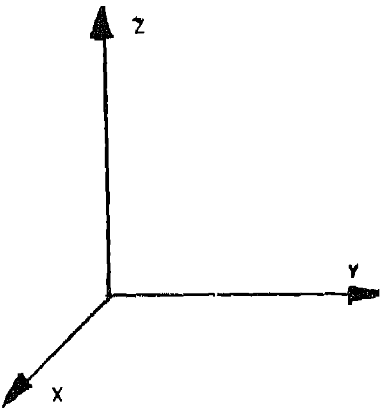

Fig. 26. The right hand system of coordinates.

UNIT 2

Physics for Class XI

MEASUREMENT

F-15/87

विद्याऽमृतमश्नुते

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राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is second in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI Physics course of the Central Board of Secondary Education A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project. I am also thankful to Dr. S. C. Abbi, Physics Department, I.I.T., Delhi for his help in editing this unit and for helping me in many other ways on volunteer basis. At this stage this unit is an experimental edition. I shall welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
N.C.E.R.T.

New Delhi
30 May 1982

INTRODUCTION

The physical quantities are the building blocks of physics. The laws of physics are expressed in terms of physical quantities. Testing a theory or verifying a law involves measuring these physical quantities For this purpose we have chosen certain quantities in terms of which other physical quantities can be expressed. Such quantities are called fundamental quantities. For the sake of precision, there must be standards for these fundamental quantities The criterion for selecting these standards should be that they do not vary over a period of time due to repeated use and are easily accessible or can be imitated easily. In your previous classes, you have studied the need for adopting the standards and also about units in which they are expressed You will now learn how these units are chosen and the most recently accepted units for these quantities. You will also study more about the units of fundamental quantities and how other physical quantities can be expressed in terms of these fundamental quantities. You will learn, how very large or small distances or quantities can be expressed more conveniently, and how large distances such as the distances of stars and planets from the earth, and very small distances such as diameters of atoms, etc can be measured. This unit may not be very interesting, yet you should know various types of units to understand the physics for your course.

OBJECTIVES

To be able to

1. Define the base units in S I. units (The International System of Units)
2. State the order of magnitudes of large and small quantities.
3 Describe indirect methods such as triangulation or parallax method, and method based on inverse square law of distance in respect of intensity of light to determine astronomical distances.
4. Describe indirect method for the determination of the size of an atom using Avogadro's hypothesis.
5. Deduce dimensions of various physical quantities.
6. Use dimensional method for
   (a) testing the accuracy of expressions deduced,
   (b) deducing certain formulae,

SUGGESTED READING MATERIAL

1. Fundamental and derived units

Suppose we want to measure the length of a certain plot. First of all we have to specify a unit for measurement (say, the length of a stick). Using this we find out how many times the plot is as long as the given stick. If we call the length of the stick as one 'a' and we find the length of the plot to be twenty times the length of the stick, then we can say that the length of plot is 20a.

But if every one chooses different units the result will be a chaos. Hence international agreement is necessary concerning the specification of units for the measurement of various quantities. From this example it is clear that procedure of measurement of a physical quantity, in general, involves two steps, viz. the choice of a unit and the establishment of methods of comparing the unit to the quantity to be measured so that a number and a unit are determined as the measure of that quantity.

Physical quantities are divided into two categories, viz , fundamental quantities and derived quantities. *Fundamental quantities are those physical quantities whose defining operations (i. e. procedure of measurement) do not involve any other physical quantity and derived quantities are defined on the basis of fundamental quantities.* Examples of quantities usually viewed as conventionally fundamental are mass, length, time, etc. Examples of derived quantities are velocity, acceleration, volume, etc.

We have four systems of measurements namely, F.P.S. (Foot, Pound, Second), C.G.S. (Centimeter, Gram, Second), M.K.S. (Meter, Kilogram, Second), and S.I. units. The reason for the tremendous popularity of S.I. units is that they are easiest for remembering and practical work. *The important features of SI units can be summerised in three terms . rationality, coherence and comprehensibility* The terms need a little explanation. *The SI units is rational because it absorbs the rationalised MKS system and is made more efficient by logically reducing or eliminating unnecessary complexities. It is coherent because it is based on a certain set of 'basic units' from which all derived units are obtained by multiplication or division without introducing numerical factors. Its comprehensibility is based upon the fact that it uses only seven base units and two dimensionless units to cover all the fields of science and engineering.*

Given below are the definitions of the base and dimensionless units.

Base units

Kilogram (kg) : The kilogram is the unit of mass equal to the mass of the international prototype of the cylinder at Sevres, France.

Meter (m) : The meter is the length equal to 1650763.73 wave lengths in vacuum of the radiation corresponding to the transition between the levels 2 P_{10} and 5 d_5 of the krypton—86 atom.

Second (s) The second is the duration of 9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of caesium—133.

Kelvin (K) : The kelvin, unit of thermodynamic temperature, is the fraction 1/273.18 of the thermodynamic temperature of the triple point of water.

Ampere (A) : The ampere is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 1×10^{-7} newton per meter length.

Candela (cd) · The candela is the luminous intensity, in the perpendicular direction, of a surface of 1/600000 square meter of a black body at the temperature of freezing platinum under a pressure of 101325 newton per square meter.

Mole (Mol) · The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12.

Dimensionless units

Radian (rad) . The radian is the plane angle between two radii of a circle which cuts off on the circumference an arc equal in length to the radius.

Steradian (sr) : The steradian is the solid angle which, having its vertex at the centre of a sphere, cuts off an area of the surface of the sphere equal to that of a square having sides of length equal to the radius of the sphere.

Derived units

The derived units can be obtained by using the coherent features. For example, the velocity is defined as displacement per unit time. The dimension of velocity, written as, | velocity | , can be written as

$$|\text{velocity}| = \frac{|\text{displacement}|}{|\text{time}|}$$

In SI units

$|\text{velocity}|_{SI} = \text{m/s}$

Similarly,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$|\text{acceleration}|_{SI} = \frac{\text{meter/second}}{\text{second}}$$

$$= \text{m/s}^2$$

force = mass × acceleration

$|\text{force}|_{SI} = \text{kg} \times \text{m/s}^2 = \text{kg m/s}^2$

Derived units will be discussed in more details in the article 4.1.

2. Order of magnitude of very large and very small quantities

In our every day life we encounter objects of widely differing sizes. Some of them are as big as mountains and others are as small as a pinhead. When we go beyond these limits, either in the direction of much larger objects or in the direction of much smaller ones, it becomes increasingly difficult to grasp their actual sizes.

For example the diameter of the sun is 1,390,000,000 meters and the diameter of the hydrogen atom is only 0.000000000106 meters. Scientists customarily express such numbers in scientific notation

In scientific notation we write the diameter of the sun and the hydrogen atom as 1.39×10^9m and 1.06×10^{-10} m respectively.

The exponent is called the order of magnitude of the physical quantities

Questions

1. Determine the order of magnitude of the numbers 1.2×10^9 and 1.1×10^{-23} Ans. 9,—23

2. One light year is defined as the distance travelled by light in one year. Express one light year in meters. Ans. 9.43×10^{15}m

The calculations becomes relatively simpler after the numbers are expressed in scientific notation.

Table 1 gives the order of magnitude of typical lengths.

TABLE 1

The order of magnitudes of lengths from astronomical to sub-atomic scale

Distance	Order of magnitude (in m)
Farthest galaxy (Quasars) photographed	10^{26}
Distance to the nearest galaxy	10^{22}
Diameter of our galaxy	10^{20}
Distance to the north star	10^{19}
Distance to the nearest star (Alpha Centauri)	10^{17}
Mean distance of the earth from the sun†	10^{11}
Radius of the sun	10^{9}
Average distance of the moon from the earth	10^{8}
Radius of the earth	10^{7}
Length of a football field	10^{2}
Width of your hand	10^{-1}
Thickness of a pencil	10^{-2}
Thickness of a piece of paper	10^{-4}
Size of a biological cell*	10^{-6}
Size of a virus	10^{-8}
Diameter of a hydrogen atom**	10^{-10}
Size of a large nucleus***	10^{-14}
Diameter of a proton	10^{-15}

---

†Also defined as one astronomical unit and equals 1.498×10^{11}m

*For expressing such length a unit equal to 10^{-6}m called a micron (μ) is used.

**This is approximately equal to one Angstrom (1A°) unit which equals 10^{-10}m.

***Nuclear size is usually expressed in fermi, 1 fermi$=10^{-15}$m.

As you have seen 'time' too, is a fundamental quantity. To measure it roughly, we have pendulum clocks and watches which produce ticks at intervals of one second. Many other repetitive phenomena such as heart beat, motion of planets round the sun, etc. can be adopted as the time standard. But as you know scientists always strive for greater accuracy. Hence they adopted 'second' based on caesium (Cs) clock as international time standard in 1964. Using caesium the value of a second can be measured with an accuracy of 1 part in 10^{11}. And quartz crystal clocks, based on the electrically sustained natural periodic vibrations of a quartz wafer, serve as the secondary time standard.

To measure very short time intervals we have electronic devices and still shorter time intervals require indirect means. On the other hand, to measure long intervals of time, we have methods such as the ones based upon radioactive disintegration, Doppler shift, etc. In Table 2 given below we have listed order of magnitude of time for very long as well as very short time intervals.

TABLE 2

Event	Time interval (s)	Event	Time interval (s)
Age of the universe	10^{18}	Time for light to cross a room	10^{-8}
Life of earth	10^{16}	Time for electron to revolve round the hydrogen nucleus	10^{-15}
Human race	10^{12}		
A year	10^{8}	Time for inner most electron to revolve around nucleus in a heavy atom	10^{-20}
An hour	10^{4}		
A wink	1		
Time period of sound	10^{-2}—10^{-4}	Time for proton to revolve once in nucleus	10^{-22}

Questions

1. Name several repetitive phenomena occuring in nature which could serve as reasonable time standards.
2. Do you think that a definition of a physical quantity for which no method of measurement is known or given has any meaning ?
3. What criteria should a good clock satisfy ?

4. Can you think of a way to define length standard in terms of time standard and vice versa ? If so, can length and time be regarded as fundamental quantities ?

   Hint . Time period of a pendulum is related to length of the pendulum.

3.1. Indirect methods to measure large distances

If you are to measure astronomical distances, direct methods such as using your meter stick will be of no use. That is why, we have indirect methods, e.g. triangulation (parallax) method by which distances to nearby stars can be determined.

To illustrate the principle, let us consider the simple case shown in Fig. 1. Let N be a nearby star whose distance is to be determined. We choose a far away star (say F) whose direction may be taken practically to be the same at all positions of the earth in its orbital motion.

Suppose at some position of the earth, A, one measures the angle between the direction of F and the direction of N, i.e. the angle FAN (θ). Then the observer waits for six months so that the earth is at B, a position diametrically opposite to the position A of the earth in its orbit around the sun S. At B, again he measures the angle NBF (ϕ) Now knowing

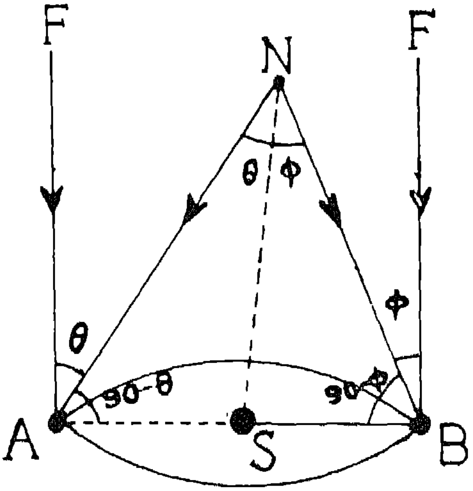

Fig 1. Lines of sight of a distant star F and a nearby star N taken from two diametrically opposite points A and B of earth's orbit around the sun S. Measuring angles θ and ϕ and knowing the distance between the earth and the sun, *d*, the distance of N from earth can be determined

the distance d between the earth and the sun, the distance of N can be determined using the following equation.

$$\frac{2d}{\sin(\theta+\phi)} = \frac{AN}{\sin(90-\phi)} = \frac{BN}{\sin(90-\theta)} \quad ..(1)$$

By this method, distances of stars which are relatively near to the earth can be determined and they are only a few hundreds in numbers In case of far away stars, other indirect methods such as methods based on inverse square law of distance, Doppler shift, etc. are used. In the former method intensity of faint image of a very distant star on a photographic plate is compared with the intensity of a nearby star.

If I_1 = Intensity of a nearby star

d_1 = Distance of the nearby star from the earth (This distance can be determined by triangulation method)

I_2 = Intensity of a distant star

d_2 = Distance of the distant star from the earth (This is to be determined)

Then from inverse square law of distance

$$I_1/d_1^2 = I_2/d_2^2$$

$$\text{or } d_2 = \sqrt{I_2 d_1^2/I_1} \quad . (2)$$

In order to determine the distance of a far away galaxy by Doppler shift (Questions on this may be asked in Class XII) one has to determine the shift in apparent wavelength of a spectral line ($\Delta\lambda$) of the distant galaxy compared to the same line photographed from a stationary laboratory source. Then relative velocity of the galaxy with respect to the earth is given by

$$V = \frac{c\,\Delta\lambda}{\lambda} \quad ...(3)$$

Where λ = Wavelength of the spectral line concerned

c = Velocity of light

Now from Hubble's law we know, that the velocity of recession of a galaxy is given by the formula

$$V_{rec} = Hr \quad ..(4)$$

where r = Distance of the galaxy from the earth

H = Hubble's constant

= 100 m/s for every million light years ...(5)

With the help of Eq. (3) and Eq. (4) one can estimate the distance of the far away galaxies.

3.2. Indirect methods to measure very small distances

For the measurement of very small distances such as those of the molecules and atoms, only indirect methods can be adopted. Determination of the size of an atom by Avogadro's hypothesis and the determination of the size of a gold nucleus by alpha-particle scattering experiment are two typical examples.

In order to understand the principle and the method of determination of size of an atom by Avogadro's hypothesis, let us consider a monatomic element such as copper.

Let m = Mass of the given body made of copper

M = Molecular mass of copper

V = Volume of the given body

We know M grams of copper will contain $N (=6.023 \times 10^{23})$ atoms (since Cu is monatomic) of copper where N is Avogardo's number. Hence m grams of specimen will contain $\frac{Nm}{M}$ atoms. If r is the radius of the copper atom, assuming atoms to be spherical, one can write

$$\frac{4}{3}\pi r^3 \frac{Nm}{M} = \text{Total volume occupied by the atoms} \quad . (6)$$

Assuming that atoms occupy two third of the volume of the specimen one gets

$$\frac{4}{3}\pi r^3 \frac{Nm}{M} = \frac{2}{3} V \quad ...(7)$$

or

$$r = \left(\frac{MV}{2\pi\, Nm}\right)^{1/3} \quad .. (8)$$

To estimate the radius of the nucleus (questions on this method will not be asked) let us consider an α particle of energy E joules incident at the centre of the nucleus as shown Fig. 2. As you must have studied in earlier class, α particle will retrace its path after reaching a minimum distance r_0 from the centre of the nucleus because K.E. of the α particle is getting transformed into P.E. due to Coulomb's repulsion between α particle and nucleus and at r_0. The kinetic energy of the α particle will become exactly equal to the potential energy of α particle nucleus system, which is given by $k\, z_1 z_2 e^2/r_0$ where z_1 and z_2 are atomic numbers of α particle and nucleus respectively

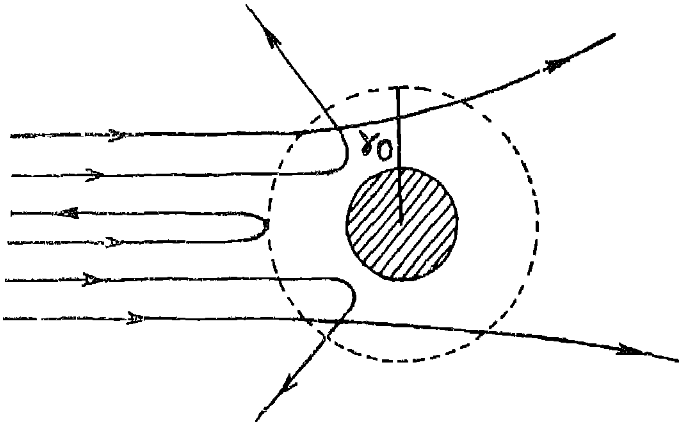

Fig. 2. Large angle scattering of alpha particles from a nucleus.

($k = 9 \times 10^9$ Nm^2/C^2, $e = 1.6 \times 10^{-19}$ coulombs)

Radius of the nucleus must be smaller than r_0 (why) ?

Example 1

Estimate the size of Ag atom, given density of Ag = 10 g/cm^3 and atomic mass of Ag = 108

Solution

From equation (8)

$$r = \left(\frac{M}{2\pi N d}\right)^{1/3}$$

where $$d = \frac{m}{V}$$

$$= 10 \text{ g/cm}^3$$

$$N = 6.023 \times 10^{23}$$

$M = 108$, r=Radius of Ag atom.

Substituting these values in the above formula, one finds

$$r = \left(\frac{108 \times 1000}{2 \times \frac{22}{7} \times 6.023 \times 10^{23} \times 10}\right)^{1/3} \text{cm}$$

$$r = 1.4 \times 10^{-8} \text{ cm}$$

Questions

1. How will you measure the distance between the moon and the earth ?

2. How will you show that the nucleus occupies a very small fraction of the total volume of an atom ?

4.1. Dimensional analysis

It has been noted in article 1 that derived quantities can be expressed in terms of fundamental quantities like mass, length, time, current, etc. Let us suppose M, L, T, and I are chosen to be the dimensions of mass, length, time and current respectively. Hence

$$|\text{ velocity }| = |\text{ displacement/time }|$$

$$= \mathrm{L/T} = \mathrm{LT^{-1}}$$

$$|\text{ density }| = |\text{ mass/volume }|$$

$$= \mathrm{M/L^3}$$

$$= \mathrm{ML^{-3}}$$

$$|\text{ pressure }| = |\text{ force/area }| = \mathrm{MLT^{-2}/L^2}$$

$$= \mathrm{ML^{-1}\,T^{-2}}$$

$$|\text{ angle in radian }| = |\text{ arc length/radius }|$$

$$= \mathrm{L/L} = 1$$

(Hence we find angle to be a pure number)

$$|\text{ charge }| = |\text{ current} \times \text{time }|$$

$$= \mathrm{IT}$$

From the examples discussed above it is clear that the dimensions of a derived quantity (say, A) will be dependent on some powers of mass, length, time, current and so on. To put it mathematically

$$|A| \propto \mathrm{M}^x$$

$$\propto \mathrm{L}^y$$

$$\propto \mathrm{T}^z$$

$$\propto \mathrm{I}^w$$

$$\text{or } |A| = k\mathrm{M}^x\ \mathrm{L}^y\ \mathrm{T}^z\ \mathrm{I}^w$$

where k is a constant of proportionality.

Eq. (9) is called dimensional equation.

Hence one can define the powers to which the fundamental units of mass, length, time, etc. be raised to denote the dimensions of the physical quantity.

One most important feature of a dimensional equation is what is known as dimensional homogenity, i.e. powers of M, L, T, I, etc. on both sides of a dimensional equation must be same.

A glance at Table 3 will give an idea of how different derived quantities can be expressed dimensionally. Fill in the blanks and check your answers from the key on the next page (Table 4).

TABLE 3

SI Units for common physical quantities

Physical Quantity	Definition	Dimension	Unit	Unit symbol
velocity	$\frac{\text{displacement}}{\text{time}}$	LT^{-1}	meter/second	m/s
acceleration	$\frac{\text{change in velocity}}{\text{time}}$	—	—	—
density	mass/volume	ML^{-3}	kilogram/meter^3	kg/m^3
force	mass × acceleration	—	newton	N(—)
work/energy/heat	force × displacement	—	joule	J(—)
power	work done/time	—	watt	W(—)
charge	ampere × time	IT	coulomb	C(=As)
potential	work/charge	$LM^2T^{-3}\ I^{-1}$	volt	V(=J/C)
resistance	potential/current	$ML^2T^{-3}\ I^{-2}$	ohm	Ω(=V/A)
specific resistance	$\frac{\text{resistance} \times \text{area}}{\text{length}}$	—	—	—

TABLE 4

Key

Physical quantity	Definition	Dimension	Unit	Unit symbol
velocity	$\frac{\text{displacement}}{\text{time}}$	LT^{-1}	meter/second	m/s
acceleration	$\frac{\text{change in velocity}}{\text{time}}$	LT^{-2}	$\frac{\text{meter}}{\text{second}^2}$	m/s^2
density	mass/volume	ML^{-3}	kilogram/meter^3	kg/m^3
force	mass × acceleration	MLT^{-2}	kilogram meter/ second^2 or newton	$N(= kg\ m/s^2)$
work/energy/ heat	force × displacement	$ML^2\ T^{-2}$	$\frac{\text{kilogram meter}^2}{\text{second}^2}$ or joule	$J(= kg\ m^2/s^2)$
power	work done/time	$ML^2\ T^{-3}$	$\frac{\text{kilogram meter}^2}{\text{second}^3}$ or watt	$W(= kg\ m^2/s^3)$
charge	ampere × time	IT	coulomb	C(= As)
potential	work/charge	$ML^2T^{-3}\ I^{-1}$	volt	V(= J/C)
resistance	potential/current	$ML^2T^{-3}\ I^{-2}$	ohm	Ω (= V/A)
specific resistance	$\frac{\text{resistance} \times \text{area}}{\text{length}}$	$ML^3\ T^{-3}\ I^{-2}$	ohm × meter	Ω m

4.2. Application of dimensional analysis

Dimensional analysis can be used to deduce relationships and to check the dimensional accuracy of a deduced expression.

(*i*) *To check the accuracy of an expression*

To illustrate this statement let us consider expression for time period T, of a simple pendulum given by

$$T = 2\pi \sqrt{\frac{l}{g}} \qquad \ldots(10)$$

where l is the length of the pendulum and g is the acceleration due to gravity.

Dimensions of L.H S. $= M^0 L^0 T^1 I^0$

Dimensions of R.H S. $= \sqrt{\dfrac{L}{LT^{-2}}}$

$$= \sqrt{T^2}$$

$$= M^0 L^0 T^1 I^0$$

We see that the dimensions of LHS and RHS are same hence Eq. (10) is dimensionally homogeneous.

(*ii*) *To deduce certain formula*

To illustrate this we discuss the following example .

Example 2

Obtain by the method of dimensional analysis expression for the surface tensions σ of a liquid rising in a capillary tube.

Solution

Assume that surface tension depends on the mass M, pressure P and radius r of the capillary tube and the constant $k = \frac{1}{2}$.

We can write $\sigma = \frac{1}{2} M^x P^y r^z$ where x, y, z are to be determined.

| surface tension | = | force per unit length |

$$= | F/l |$$

$$= MLT^{-2}/L = MT^{-2}$$

| pressure | $= ML^{-1} T^{-2}$

Thus dimensional relation for surface tension from the relation given above is $MT^{-3} = (M)^x (ML^{-1} T^{-2})^y (L)^z$

or $$M^1 \; L^0 \; T^{-2} = M^{x+y} \; L^{-y+z} \; T^{-2y}$$

Dimensional homogeneity demands that powers of M, L and T on both sides of the equation must be same. Hence

$$x+y = 1, \; -y+z = 0, \; -2y = -2$$

which gives $x = 0, y = 1, z = 1$

Using these values in above equation, surface tension can be given by

$$\sigma = \frac{1}{2} Pr$$

ASSIGNMENT

1. If in specifying the standard length, the temperature has to be taken into account, will the length be still called the fundamental quantity for measurement ?

2. The size of a nucleus is of the order of 10^{-15} m, and if it is considered to be size of an orange, what will be the size of atom on this scale, given that order of magnitude of an atom is 10^{-10}m.
Ans. A sphere of 10^3m diameter

3 Outline a method to measure the height of a tall building or a tree.

4. Which one of the following is used as the standard for time now and why ?
   (a) Vibration of molecules
   (b) Vibration of atoms
   (c) Vibration of the pendulum
   (d) Motion of earth round its axis

5. Estimate the radius of a gold nucleus when an α particle of energy 6 Mev (1 Mev $= 1.6 \times 10^{-13}$ joules) is incident on its centre and gets scattered through 180°. (Atomic number of Au $= 79$, $k = 9 \times 10^9$ Nm^2/C^2, $e = 1.6 \times 10^{-19}$C.) Ans. 6.1×10^{-14} m

6. A stellar spectrum is photographed on the same plate and with the same spectroscopic equipment as the spectrum of a certain comparison element in the laboratory. One hydrogen line has a wavelength of 6562.8A° and if the corresponding Fraunhofer line in the stellar spectrum falls at a location corresponding to 6564.2A°, determine the velocity of recession of the star. (Given $c = 3 \times 10^8$m/s)
Ans. 64 km/s

7. The velocity of sound waves through a medium may be assumed to depend on (1) the density of the medium d (2) modulus of elasticity E. (Modulus of elasticity is a ratio of stress to strain where stress is force per unit area and strain is the ratio of elongation to original length). Deduce by the method of dimensions the formula for the velocity of sound. Ans. $U = k\sqrt{E/d}$

8. Check the accuracy of following equations by the dimensional method, where the symbols have their usual meaning.
   (a) $v = u + at$
   (b) $v^2 = u^2 + 2as$
   (c) $s = ut + \frac{1}{2}at^2$

9. Force of attraction F between two masses m_1 and m_2 separated by a distance r is given by the formula

$$F = G\frac{m_1 m_2}{r^2}$$

where G is universal gravitational constant. Determine the dimensions of G. Ans. $M^{-1}\ L^3\ T^{-2}$

SELF ASSESSMENT

1. Check the dimensional accuracy of the following equations :

   (a) $eV = \frac{1}{2}mv^2$;

   where e = electronic charge

   V = voltage, m = mass of the electron, v = velocity of the electron

   (b) $F\,S = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$, where F = force, S = distance

2. Deduce an expression for the frequency n of the transverse vibration of a stretched string assuming n depends on (i) length of the vibrating segment L (ii) mass per unit length m of the string (iii) tension T

3. Name distances corresponding to each of the following

   (i) 10^{-13}m (ii) 10^{-6}m (iii) 10^{-2}m (iv) 10^{3}m (v) 10^{11}m

4. Estimate the size of copper atom, given the density of copper as 8890 kg/m^3 and its atomic mass 63.

5. Describe one method to determine the distance of a near star from the earth.

UNIT 3

Physics for Class XI

APPLIED MATHEMATICS AND KINEMATICS

F-15/87

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is fifth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this units are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful

In preparing the draft of this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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New Delhi
30 May 1982

INTRODUCTION

Applied Mathematics plays a key role in verifying physical facts and provides us mathematical form of a physical concept In the present unit we intend to give you working knowledge of calculus (a branch of mathematics developed by Lebnitz, Newton, etc.), whose application has become almost inevitable in studying lower as well as higher physics. You will find the various concepts discussed here very useful in understanding the contents of subsequent units. You will also study and solve problems based on equations of motion which coincide very nearly with the problems faced in practical life. The problems in kinematics are directly related to the concept of vectors because these problems involve velocity, acceleration, etc. This unit is probably not very interesting, but don't ignore it as this is the foundation.

OBJECTIVES

To be able to

1. Differentiate and integrate simple algebraic functions.
2. Derive equations of motion in straight line with the help of integration.
3. Solve problems involving calculus and equations of motion.

SUGGESTED READING MATERIAL

1. Differential calculus

In mathematics we come across various quantities which can be divided into two classes, viz. constants (1,2,3, etc.) and variables (x,y,z, etc.). Variables are of two types, viz. independent variables (represented by x) and dependent variables (represented by y) In a simple experiment of plotting temperature variation against time, one notices that as the time changes so does the temperature and not vice versa. Hence time is an independent variable.

A mathematical relationship between independent variable x and dependent variable y is called a function and is symbolically represented as

$$y=f(x)$$

i.e. y is a function of x, e.g.

$y=x^2+bx+c$: Algebraic relation

$y=\sin x$: Trignometric relation

$y=\log_e x$ · Logarithmic relation

and so on.

By the limiting value of a variable, e.g. limit $x\rightarrow 1$ (x is tending towards one) we mean either x is very slightly greater than one or x is very slightly less than one but it can never be exactly equal to one. So if, limiting value of function

$$\lim_{x\rightarrow 2}\left[\frac{x^2-4}{x-2}\right]$$

is being asked to calculate, obviously we are not interested in exact value. But here is a minor hitch because if you put $x=2$, to evaluate the limiting value of the function $(x^2-4)/(x-2)$ as x is tending towards 2, we get division by zero (which is never allowed). Hence to get rid of this difficulty we proceed in the following manner.

$$\lim_{x\rightarrow 2}\left(\frac{x^2-4}{x-2}\right)=\lim_{x\rightarrow 2}\left[\frac{(x+2)\,(x-2)}{(x-2)}\right]=\lim_{x\rightarrow 2}(x+2)=2+2=4$$

Problems

Evaluate the following limits

(i) $\lim\limits_{x\rightarrow \infty}\left[\dfrac{ax^2+bx+c}{dx^2+ex+f}\right]$ $\left(\text{Hint: Put } x=\dfrac{1}{y},\text{ Hence as } x\rightarrow\infty,\ y\rightarrow 0\right)$

(ii) $\lim\limits_{x\rightarrow a}\left[\dfrac{x^2-a^2}{x-a}\right]$

Consider a particle moving along the X axis as shown in Fig. 1. The curve in Fig. 2 is a graph of its co-ordinate x as a function of time t. At a time t_1 the particle is at point A in Fig 1, where its coordinate is x_1 and at a later time t_2 it is at point B, whose co-ordinate is x_2. The corresponding points on the coordinate-time graph in Fig. 2 are lettered a and b.

The displacement of a particle as it moves from one point of its path to another is defined as the vector $\triangle x$ as in Fig. 1 The vector AB of magnitude $x_2-x_1=\triangle x$, is the displacement. The average velocity of the

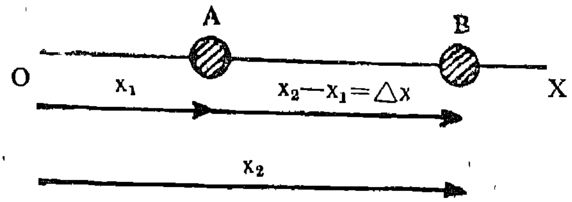

Fig. 1. Particle moving on the X axis.

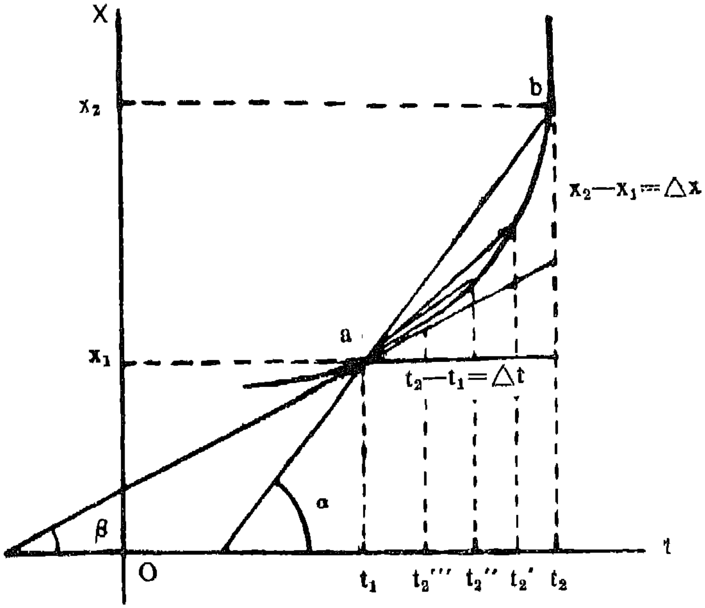

Fig. 2. Coordinate-time graph. Slope of the chord ab (tan α) equals average velocity between t_1 and t_2. The instantaneous velocity at point a is equal to the slope of the tangent at point a (tan β).

particle is defined as the ratio of the displacement to the time interval $t_2 - t_1 = \Delta t$. We shall represent average velocity by $\bar{v}$ (read as v bar) as

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \ldots (1)$$

In Fig. 2, the average velocity is represented by the slope of the chord ab (slope of the line ab is tangent of the angle that line ab makes with positive direction of t axis, which is tan α)

The average velocity as defined above, depending only on the displacement and time interval between A and B, does not tell anything at all about the motion between A and B. The actual path between A and B could be a straight line or a curve. The motion could have been steady or eratic. For example, if a man starts from his house, goes around the world and returns to his house, his average velocity will be zero because his displacement is zero for the time he takes for the trip. The motion can be described in greater details if we measure velocity at several points between A and B. If the average velocity turned out to be same between any two points on the path, you may conclude that the particle moves with constant velocity, that is, along straight line at a uniform rate. If the path of the particle between A and B is a curve then, in general, the average velocity measured at different time intervals will be different. The velocities for shorter time intervals will then describe the motion in greater detail and ultimately we may want to know the velocity at each instant, called the *instantaneous velocity*. For the motion shown in Fig 2 we determine $\Delta x/\Delta t$ for time interval $(t_2'-t_1)$, $(t_2''-t_1)$, $(t_2'''-t_1)$, etc and for the time interval approaching zero in the limiting case The instantaneous velocity at A is defined as

$$v=\lim_{\Delta t\to 0}\left[\frac{\Delta x}{\Delta t}\right]=\frac{dx}{dt} \qquad ..\ (2)$$

which gives the ratio $\Delta x/\Delta t$ when B approaches A or when $\Delta t\to 0$. In the notation of calculus it is written as dx/dt and is called the derivative of x with respect to t.

The magnitude of instantaneous velocity is $|\,dx/dt\,|$ which is called the speed. You can see that as b approaches a, in the limiting case, when $\Delta t\to 0$, chord ab becomes tangent at a and gives the direction of the instantaneous velocity. *Thus the slope of the tangent* (*tan* β) *at any point* (*or derivative*) *of displacement-time graph gives the instantaneous velocity at that point.*

In certain special cases, the velocity of a moving body changes continuously as the motion proceeds. When this is the case the body is said to move with accelerated motion, or to have an acceleration.

Let v_2 and v_1 be the instantaneous velocities at points B and A having time co-ordinates t_2 and t_1 respectively Then the average acceleration between A and B is the ratio of change in velocity, $\Delta v=v_2-v_1$ and time interval $\Delta t=t_2-t_1$

$$a=\frac{\Delta v}{\Delta t} \qquad ..\ (3\text{ a})$$

The instantaneous acceleration of the body, that is, its acceleration at some instant of time is defined in a similar way as instantaneous velocity. The acceleration vector is defined as

$$a = \lim_{\Delta t \to 0} \left[\frac{\Delta v}{\Delta t}\right] = \frac{dv}{dt} \qquad \text{...(3b)}$$

In the language of calculus instantaneous acceleration is the first derivative of velocity with respect to time The direction of the instantaneous acceleration is the limiting direction of the vector change in velocity, Δv. Taking cue from discussion in the case of instantaneous velocity, one can consider *the instantaneous acceleration at any point of the velocity-time graph equal to the slope of the tangent to the graph at that point.*

Now we are in a position to generalise the concept of differentiation Let us suppose we have two variables x and y where y is a function of x which is written as

$$y = f(x)$$

Let us give small increment Δx to x and corresponding increment in y is Δy

or $\qquad y + \Delta y = f(x + \Delta x)$

then $$\lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right] = \lim_{\Delta x \to 0} \left[\frac{\Delta y}{\Delta x}\right] \qquad \text{...(4)}$$

The above $\lim_{\Delta x \to 0} \left[\frac{\Delta y}{\Delta x}\right] = \frac{dy}{dx}$ is first differential coefficient (or derivative) of y with respect to x, and it gives the rate at which y varies with respect to x at any point (x, y) in the graph $y = f(x)$ Hence we come to the conclusion that the derivative is a rate measurer and Eq (4) gives the definition of first derivative of y with respect to x.

Let us now find out differential coefficient of a function $y = f(x) = x^n$ (5)

where n is a positive integer. The method that will be adopted is known as differentiation from first principle (ab initio). Let us give small increment Δx in x and corresponding increment in y is Δy

$$y + \Delta y = (x + \Delta x)^n \qquad \text{.. (6)}$$

Subtracting Eq. (5) from Eq (6)

$$\Delta y = (x + \Delta x)^n - x^n \qquad \text{. (7)}$$

Expanding $(x+\Delta x)^n$ by Binomial theorem† for positive integral index one gets Eq. (7) as

$$\Delta y=[x^n+{}^nC_1x^{n-1}\Delta x+{}^nC_2x^{n-2}(\Delta x)^2+\ldots+{}^nC_rx^{n-r}(\Delta x)^r+\ldots(\Delta x)^n]-x^n \qquad \ldots(8)$$

where ${}^nC_r$ is combination of n things taken r at a time

$$\text{or} \quad {}^nC_r=\frac{n!}{r!\,(n-r)!} \quad \text{here } n!=1.2.3\ldots\ldots(n-1)\,n \qquad (9)$$

Dividing both sides of Eq. (8) by Δx, one gets

$$\frac{\Delta y}{\Delta x}=\left\{{}^nC_1x^{n-1}+{}^nC_2x^{n-2}\Delta x+\ldots+(\Delta x)^{n-1}\right\} \qquad \ldots(10)$$

Taking limit of Eq. (10) as $\Delta x\rightarrow 0$ we get

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{dy}{dx}={}^nC_1x^{n-1} \qquad \ldots(11)$$

But ${}^nC_1=n$ [Refer to Eq. (9)]

hence $$\frac{d}{dx}(x^n)=nx^{n-1} \qquad \ldots(12)$$

Similarly one can show that even if n happens to be any rational number the result, Eq. (12) will be valid.

Other derivatives are found in a similar way. Some important results are given below for your use.

(1) If c is a constant

$$\frac{dc}{dx}=0$$

---

† Binomial theorem for +ve integral index states that $(a+x)^n$, where n is positive integer, can be expanded as $(a+x)^n=a^n+{}^nC_1\,a^{n-1}\,x+\ldots+{}^nC_r\,a^{n-r}\,x^r\ldots+x^n$. You will learn about this theorem in your mathematics class For this unit you may assume the expansion It is used to give you a feel for derivative but you will not be tested on this in this unit.

(2) $\frac{dx}{dx}=1$

(3) If u and v are two functions of x, and c is a constant, then

(a) $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$

(b) $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

(c) $\frac{d}{dx}(c\,u)=c\frac{du}{dx}$

(4) $\frac{d}{dx}(\sin x)=\cos x$

(5) $\frac{d}{dx}(\cos x)=-\sin x$

(6) $\frac{d}{dx}(\log x)=\frac{1}{x}$

Question

Calculate $\frac{d}{dx}(x^2)$

Ans. $2x$

Example 1

A particle moves in such a way that the distance s, in meters, travelled by it varies with time t, in seconds, according to the relation $s=at+bt^2$ where $a=20$ m/s and $b=5$ m/s². Determine the velocity of the body at time $t=4$ seconds and show that in this case acceleration is independent of time.

Solution

Velocity of the body v is given by

$$v=\frac{ds}{dt}=\frac{d}{dt}(at+bt^2)$$

$$=\frac{d}{dt}(at)+\frac{d}{dt}(bt^2)$$

$$=\frac{adt}{dt}+b\frac{d}{dt}(t^2)$$

$$=a+2bt \quad \left\{\frac{dt}{dt}=1 \text{ and } \frac{d}{dt}(t^2)=2\,t^{2-1}=2t\right\}$$

Hence velocity after a lapse of 4 seconds is

$$V\Big|_{t=4s} = a+2b\times4$$

$$= 60 \text{ m/s}$$

and acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(a+2b\ t) = \frac{d}{dt}(a) + \frac{d}{dt}(2b\ t)$

$$= 0 + 2b\frac{dt}{dt} = 2b\times1$$

$$= 10 \text{ m/s}^2$$

Hence acceleration is independent of time and the motion is uniform.

Problem

If the velocity of a particle after t seconds is 30 m/s, calculate t if the distance s is given by

$$s = at^3 + bt^2 + ct + d.$$

where $a = 3$ m/s³, $b = 4$ m/s², $c = 5$ m/s and $d = 6$m

Also calculate the acceleration at time $t=3$ seconds.

2. Integral calculus

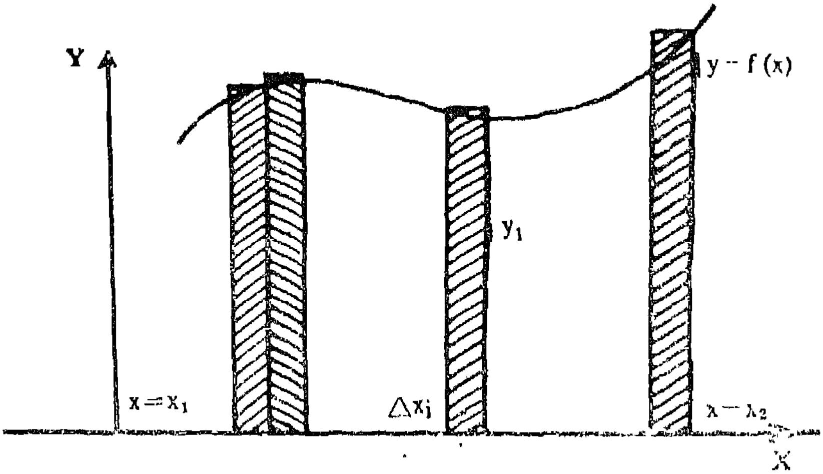

Fig. The area under the curve represented by $y=f(x)$ from $x=x_1$ to $x=x_2$ is equal to the sum of areas of strips $y_i \Delta x_i$ from $x=x_1$ to $x=x_2$. When Δx_i becomes infinitesimal $=dx_i$ the total area becomes equal to $\int_{x=x_1}^{x=x_2} y\,dx$.

Suppose we want to determine area A under the curve $y=f(x)$ from $x=x_2$ to $x=x_1$.

Let us take a small segment of thickness $\triangle x_i$, corresponding ordinate is y_i as shown in Fig 3. The strip thus obtained is a rectangle whose (one of the) shorter sides will coincide very nearly with the curve and its area will be thus, $A_i=y_i \triangle x_i$

Let us imagine the whole area to be made up of many (say n) such strips.

The total area A will be approximately given by sum of areas of these strips.

$$A=y_1\triangle x_1+y_2 \triangle x_2+\cdots y_i \triangle x_i+\cdots y_n \triangle x_n$$

or
$$A=\sum_{i=1}^{n} y_i \triangle x_i$$

Where index i runs over 1 to n and Σ stands for summation.

It is an approximation because one of the shorter sides of the strips does not exactly coincide with the curve $y=f(x)$.

In order to have better approximation we make $\triangle x_i$ smaller until it becomes infinitesimal, i.e $\triangle x_i \rightarrow dx_i$. The area of ith strip is $y_i\, dx_i$ and the number of strips becomes very large. This sum over all these very large number of strips is called integral and is represented by the symbol $\int$.

Thus the area becomes

$$A=\int_{x=x_1}^{x=x_2} y\, dx$$

The expression $\int_{x=x_1}^{x=x_2} y\, dx$, often written as $\int_{x_1}^{x_2} y\, dx$ simply means that the areas of all the strips $y_i\, dx_i$ (or $y\, dx$) between $x=x_1$ and $x=x_2$ (or between x_1 and x_2) have been added to give total area. Thus

$$A=\int_{x_1}^{x_2} y\, dx \qquad \text{. .12(a)}$$

This process of addition is called integration.

One most essential condition for the Eq. (12a) to be valid is that the curve $y=f(x)$ must be a continuous curve (i.e with no jumping or discontinuity, at any point as shown in Fig. 2). Hence integration stands for summation of a continuous variable Yet another interpretation of integration is that *it is the reverse process of differentiation.* To illustrate this let us consider the motion of a particle under constant acceleration a having velocities v_1 and v_2 at time t_1 and t_2 respectively as in Fig 4. The area under the curve from t_1 to t_2 is same as the area of a rectangle of height a and base (t_2-t_1) which is $a\,(t_2-t_1)$. This has been defined as

$$\int_{t_1}^{t_2} a\,dt$$

which from Eq. (3b) gives change in velocity. Thus by integration we have found change in velocity knowing acceleration as a function of time. This is reverse of differentiation where we find acceleration knowing velocity as a function of time.

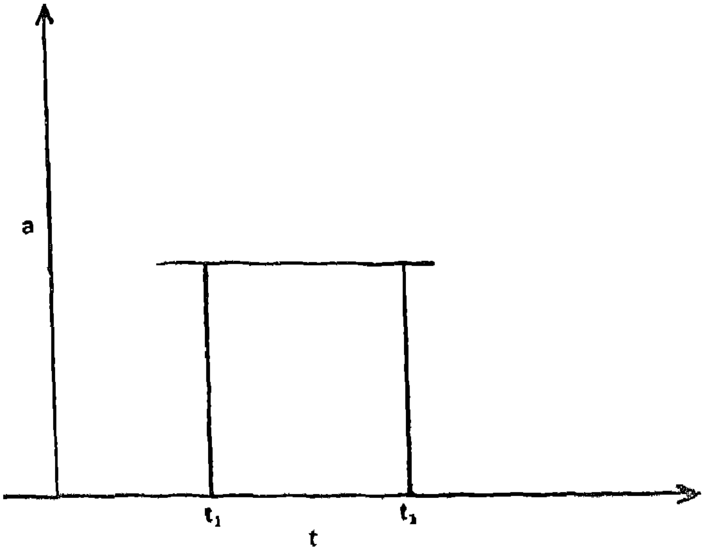

Fig. 4. Acceleration-time graph of a particle moving with constant acceleration. The area under this curve from $t=t_2$ to $t=t_1$ gives changes in velocity during this time interval.

In general starting from dy/dx a processs of finding $y=f(x)$, is called integration. Putting it mathematically

$$\text{if } \frac{dy}{dx} = \phi(x)$$

$$\text{[then } y = \int \phi(x)\ dx + \text{Constant}$$

In order to show how integrations are to be performed with elementary functions, let us take a specific example. We know that

$$\frac{d}{dx}(x^{n+1}) = (n+1)\ x^n \qquad \text{.. (12b)}$$

or $$d(x^{n+1}) = (n+1)x^n\ dx$$

or $$\frac{d(x^{n+1})}{n+1} = x^n\ dx \qquad \text{. (13)}$$

Integrating both sides of the above equation, we get

$$\int x^n dx = \int \frac{d(x^{n+1})}{n+1} + K \qquad \text{...(14)}$$

Since integration is the reverse process of differentiation, ∫ (integration) and d (differentiation) will cancel each other like (log) and (anti log) cancel each other. Hence we get Eq. (14) as

$$\int x^n dx = \frac{x^{n+1}}{n+1} + K \qquad \text{...(15)}$$

where an integration constant K has been added because in Eq. (12b) had it been $\frac{d}{dx}(x^{n+1} + K)$ instead of $\frac{d}{dx}(x^{n+1})$, even then we would have got same R H.S. as in Eq. (13). The value of K can be determined from the initial conditions.

Eq. (15) is called indefinite integral.

If in Eq. (15) we specify the limits of integration say from $x=x_1$ to $x=x_2$ as we have done in writing the expression for area A in Eq. (12a) then the integral becomes

$$\int_{x=x_1}^{x=x_2} x^n\ dx$$

This is a definite integral because the limits of integration are known

$$\int_{x=x_1}^{x=x_2} x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_{x_1}^{x_2} \qquad \ldots(16)$$

Note that no integration constant appears over here. Why?

R.H.S. of Eq. (16) can be evaluated by substituting x_2 in place of x followed by substitution of x_1 in place of x with a minus sign in between.

$$\text{Thus} \int_{x=x_1}^{x=x_2} x^n\,dx = \left|\frac{x^{n+1}}{n+1}\right|_{x=x_2} - \left|\frac{x^{n+1}}{n+1}\right|_{x=x_1} \qquad \ldots(16a)$$

$$= \left[\frac{x_2^{\,n+1}}{n+1} - \frac{x_1^{\,n+1}}{n+1}\right]$$

An example, perhaps will make the idea clear.

Example 1

$$\text{Evaluate} \quad \int_1^2 x^2\,dx$$

$$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \left[\frac{2^3}{3} - \frac{1^3}{3}\right]$$

$$= \left[\frac{8}{3} - \frac{1}{3}\right] = \frac{7}{3}$$

Let us evaluate more integrals. You know that

$$\frac{d}{dx}(\sin x) = \cos x$$

hence $\cos x\,dx = d(\sin x)$

or $\int \cos x\,dx = \int d(\sin x) + K$

or $\int \cos x\,dx = \sin x + K$ ...(16b)

Similarly $\frac{d}{dx}(\cos x) = -\sin x$

$\sin x \, dx = -d(\cos x)$

$\int \sin x \, dx = -\int d(\cos x) + K$

$\int \sin x \, dx = -\cos x + K$ ...(16c)

We have also given that

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\frac{dx}{x} = d(\log x)$$

$$\int \frac{dx}{x} = \int d\,(\log\ x) +$$

$$\int \frac{dx}{x} = \log x + K$$

Example 2

In calculating moment of inertia of a circular disc of radius r, we come across the integral

$$I = \int_0^r 2\pi m\ x^3 dx \qquad \text{(where } 2\pi\ m \text{ is a constant)}$$

Evaluate the above integral.

Solution

Integral

$$I = \int_0^r 2\pi m\ x^3 dx$$

Since $2\pi\ m$ is a constant, it can be put outside the integral

$$I = 2\pi m \int_0^r x^3\ dx$$

$$= 2\pi\ m \Big[\ x^4/4\ \Big]_0^r$$

In order to evaluate $\Big[x^4/4\ \Big]_0^r$, put

first r in place of x and then 0 in place of x and subtract the two.

$$I = 2\pi m \left[r^4/4 - 0^4/4 \right]$$

$$= 2\pi m r^4/4 = \tfrac{1}{2}\pi\, m r^4.$$

Example 3

In calculating escape velocity of a projectile we come across the integral

$$I = \int_r^\infty \frac{k\,dx}{x^2}\text{; where } k \text{ is a constant}$$

Evaluate the above integral.

Solution

$$I = \int_r^\infty \frac{k\,dx}{x^2}$$

$$= k\int_r^\infty \frac{dx}{x^2}$$

$$= k\left[\frac{x^{-2+1}}{-2+1}\right]_r^\infty$$

hence $I = (-k)\left[\frac{1}{x}\right]_r^\infty$

$$I = (-k)\left[\frac{1}{\infty} - \frac{1}{r}\right]$$

$$I = (-k)(-1/r)$$

hence $I = k/r$

Problems

Evaluate the following integrals :

(i) $\int \frac{2}{x^3}\,dx$ (ii) $\int kx^2\,dx$

(iii) $\int_0^a kx\,dx$ (iv) $\int_0^1 2\sqrt{x}\,dx$

3. Derivation of equations of motion in a straight line

You are already familiar with the equations of motion in a straight line, viz.

$$v = u + at$$
$$v^2 = u^2 + 2\,as \qquad \text{...(16d)}$$
$$s = ut + \tfrac{1}{2}at^2$$

where u = Initial velocity at $t = 0$

v = Final velocity at $t = t$

a = Acceleration (constant)

s = Distance travelled

Let us now deduce them again with newly acquired knowledge of calculus.

(i) Prove $v = u + at$

You know that the rate of change of velocity equals acceleration, i.e.

$$\frac{dv}{dt} = a$$

or

$$\int dv = a \int dt + K$$
$$v = a \int dt + K$$
$$v = at + K \qquad \text{...(17)}$$

To determine K let us use the initial condition that the motion starts at $t = 0$ when $v = u$. Substituting this in Eq. (17) we get

$$u = a \times 0 + K \text{ or } K = u$$

Substituting the value of K in Eq. (17) we get

$$v = u + at \qquad \text{...(18)}$$

(ii) Prove $s = ut + \frac{1}{2}\,at^2$

Using $v = \dfrac{ds}{dt}$ in Eq. (18) we get

$$\frac{ds}{dt} = u + at$$

$$\int ds = \int (u + at)\,dt$$

$$s = \int u\, dt + \int at\, dt$$

$$s = u \int dt + a \int t\, dt$$

$$s = ut + \tfrac{1}{2} at^2 + C \qquad \ldots(19)$$

Using the initial condition that at $t = 0$, $s = 0$, i.e. particle is at origin at $t = 0$, in Eq. (19) we get

$$0 = u \times 0 + \tfrac{1}{2} a\, 0^2 + C.$$

hence $C = 0$

Substituting the value of C in Eq. (19), we get

$$s = ut + \tfrac{1}{2} at^2 \qquad .(20)$$

(iii) Prove $v^2 = u^2 + 2\, as$

We know

$$a = \frac{dv}{dt}$$

A little manipulation will yield

$$a = \frac{dv}{ds} \frac{ds}{dt} \text{ (How ?)}$$

$$a = \frac{dv}{ds}\, v \quad \left(\text{Since } \frac{ds}{dt} = v\right)$$

$$\int_u^v v\, dv = \int_o^s a\, ds$$

$$\text{or} \quad \left|\frac{v^2}{2}\right|_u^v = \left| as \right|_o^s \qquad \ldots(21)$$

$$\text{or} \quad \frac{v^2}{2} - \frac{u^2}{2} = as$$

$$\text{or} \quad v^2 = u^2 + 2\, as \qquad \ldots(22)$$

Question

Why have we not put constant of integration K in Eq. (21)? Have we used the initial conditions ?

Example 4

A person driving at 60 km/h finds a child on the road 30 m ahead.

He immediately stops the engine and applies brakes to stop the car within 5 metres of the child. Find retardation and time required to stop the car.

Solution

Initial velocity of the car $u = 60$ km/h

$$= \frac{60 \times 1000}{60 \times 60} \text{ m/s}$$

$$= \frac{50}{3} \text{ m/s}$$

Distance travelled before coming to rest $s = 25$ m

Final velocity of the car $v = 0$

Retardation $= -a$ (?)

Out of the three formulae derived before the formula that gives desired result is

$$v^2 = u^2 + 2as$$

$$a = \frac{-u^2}{2s} = -\frac{2500}{9 \times 2 \times 25} \text{ m/s}^2 = -\frac{50}{9} \text{ m/s}^2$$ (—ve sign shows that vehicle is being retarded)

From the formula

$$v = u + at$$

$$0 = u + at$$

or $$t = \left(\frac{-u}{a}\right) = \frac{50/3}{50/9} \text{ s}$$

$$= 3\text{s}$$

Example 5

Velocity is given by the formula $v = (at + b)$ where $a = 2$ m/s^2 and $b = 1$ m/s. If the particle had started from rest, find the distance travelled when $t = 4$s. Also show that the particle moves with constant acceleration.

Solution

$$v = \frac{ds}{dt} = at + b$$

$$ds = (at + b)\, dt$$

$$\int ds = a \int t\,dt + b \int dt + K$$

$$s = \frac{at^2}{2} + bt + K.$$

at $t = 0$, $s = 0$, above equation gives $K = 0$ and we get

$$s = \frac{at^2}{2} + bt$$

Hence distance travelled in 4 seconds

$$S\Big|_{t = 4s} = (4^2 + 4)\text{ m} = 20\text{ m}$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d}{dt}(at + b)$$

$$= a\frac{dt}{dt} + 0 = a = \text{constant}$$

4 Motion under gravity

When a ball is thrown in the upward direction or when it is dropped from some height, as you know, it is acted upon by a constant acceleration (due to gravity) whose magnitude varies from place to place.

If g stands for acceleration due to gravity, the equations of motion given by Eq. (16d) will be slightly modified by using $a=g$. The formulae will be

$$v = u + gt \qquad \text{..(22a)}$$

$$v^2 = u^2 + 2gh \qquad \text{...(22b)}$$

$$h = ut + \tfrac{1}{2} gt^2 \qquad \text{..(22c)}$$

Where h stands for the height from the earth's surface. Normally positive sign is used for vectors in the vertically upward direction whereas negative sign is used for vertically downward vectors.

Let us study the motion of a ball thrown vertically upwards from the top of a cliff with a velocity of 29.4 m/s as shown in Fig. 5. Equations (22a) and (22c) will give its velocity and height at every instant of time, t and equation (22b) will give its velocity at each coordinate. You may assume g to be constant (magnitude 9.8 m/s^2). Since the cause of acceleration due to gravity, in this example, is pull of the earth, it always acts in downward direction which is considered negative.

Therefore $g = -9.8$ m/s^2. As the ball goes up, this deacceleration slows it down until it reaches the highest point where velocity is zero. This is shown by arrows of gradually decreasing lengths in Fig. 5.

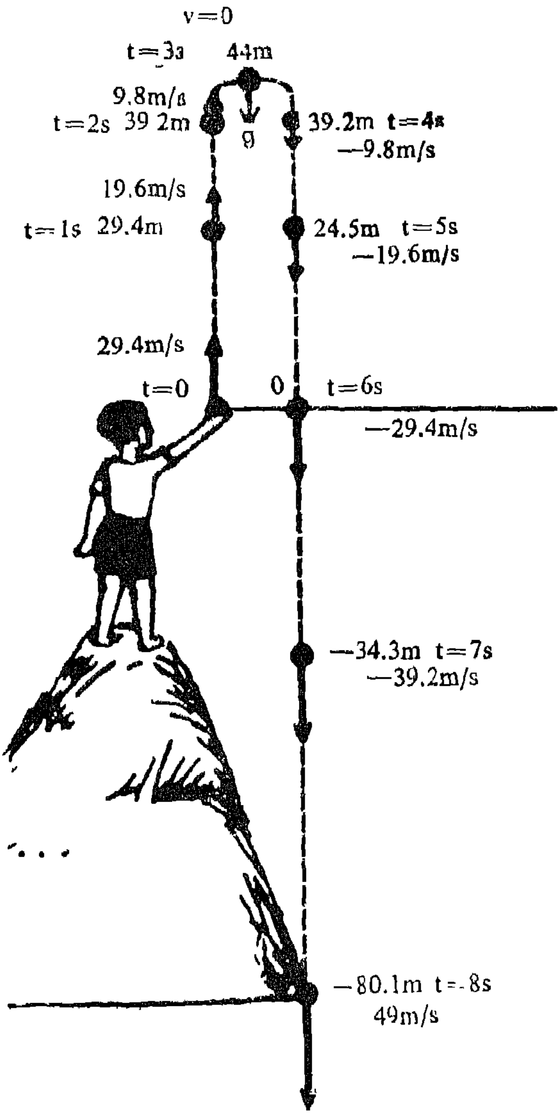

Fig. 5. A ball is thrown vertically up with a speed of 29.4 m/s. Its velocity (represented by arrow) and height (using arbitrary scale) at different times are shown At its highest point $t=3$s velociy is zero but acceleration remains the same at all points. The acceleration vector is shown only at $t=3$s

Question

Is acceleration zero at highest point ? Ans. No

Beyond the highest point both acceleration and velocity are downwards and the ball's speed increases as time passes Let us compute the speed and height one second after the ball leaves the hand. Using Eq. (22a) and Eq. (22c) we get

$$v = 29.4 \text{ m/s} + (-9.8 \text{ m/s}^2)\ 1\text{s} = 19.6 \text{ m/s}$$

and

$$h = (29.4 \text{ m/s})\ 1\text{s} + \tfrac{1}{2}\ (-9.2 \text{ m/s}^2)\ (1\text{s})^2 = 24.5\text{m}$$

Question

Compute the velocity and height at $t = 2$s. Verify your answer from Fig 5.

To determine the height at which velocity is zero (maximum height) you may plug numbers in Eq. (22b) which gives

$$0 = (29.4 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)\ h$$

$$\text{or} \quad h = \frac{(29.4)^2}{2 \times 9.8} \text{ m} = 44.1 \text{ m}.$$

Next let us determine the velocity at $t = 7$s. Eq. (22a) gives

$$v = 29.4 \text{ m/s} + (-9.8 \text{ m/s}^2)\ 7\text{s}$$
$$= -39.2 \text{ m/s}$$

The negative sign implies that velocity vector is downward. We can determine the height at $t = 7$s from Eq. (22c)

$$h = (29.4 \text{ m/s})\ 7\text{s} + \tfrac{1}{2}\ (-9.8 \text{ m/s}^2)(7\text{s})^2$$
$$= -34.3 \text{ m}.$$

Question

What is the significance of this negative sign ?

Fig. 5 shows complete motion of the ball. The velocity changes at the rate 9.8 m/s in every second. But Eq. (22c) is a quadratic. What is the physical interpretation of two roots. As the ball falls down its average speed increases. Therefore, as you can see from Fig. 5 for example, the ball travels a longer distance during the fourth second than it travels in the third second.

Question

Compute the distance travelled during the fourth and the third seconds.

Example 6

A ball is thrown vertically upwards with a velocity of 30 m/s. When will it come back to the ground ? What is the maximum height attained by the ball ? (Take $g = 10$ m/s^2)

Solution

Velocity of projection, $u = 30$ m/s

Maximum height attained by the ball $= h_{max}$

Velocity at maximum height, $v = 0$

Acceleration due to gravity, $g = -10$ m/s^2 (negative sign indicates that g is always down) To determine the time taken to come back to the ground you notice that for this journey $h = 0$. If T is the total time, Eq. (22c) gives

$$0 = (30\text{m/s})\ T - \tfrac{1}{2}\ (10\ \text{m/s}^2)\ T^2$$

which gives $T = 0$s or $T = 6$s

How will you interpret these two answers ?

Applying the formula $v^2 - u^2 = 2hg$ to motion from start to the highest point, we get

$$0 = (30\text{m/s})^2\ - 2\ (10\text{m/s}^2)$$

or $h_{max} = 45$m

Question

1. Show that for a ball thrown in the vertically upward direction with a velocity u
   (a) Maximum height attained is $u^2/2g$
   (b) Time taken to attain maximum height is (u/g)

ASSIGNMENT

1 A boy throws a ball over a wall 10 m high. If the boy stands very near to the wall, what is the least speed at which the ball can leave his hand?

Ans. 14 m/s

2. Find the area under the curve $y = 2x^2 + 3x + 5$ from $x = 0$ to $x = 1$

Ans. 5.8 units

3. Determine the velocity and acceleration of a body at time $t = 3$s whose distance s varies with time according to the formula.

$s = at^2 + bt + c$ Ans. $(6a + b)$ and $2\ a$

4. A ball is dropped from a tower 100m high. At the same instant another ball is thrown up with a velocity 20 m/s. When and where will they meet ? ($g = 10$ m/s²)

Ans. After 2 32s at 73.2 m from the ground

5. Evaluate the integral $\int (ax^2 + bx + c)\, dx$

$$Ans.\ \frac{ax^3}{3} + \frac{bx^2}{2} + cx + constant$$

SELF ASSESSMENT

1. A stone is dropped from a rising balloon at a height of 300 m above the ground and it reaches the ground in 10s. What was the velocity of the balloon at the moment the stone was dropped ?

   (Hint : If the balloon was rising with velocity u, initial velocity of the stone is $- u$)

2. (a) Show that time required to attain the maximum height for a stone thrown upwards with a velocity u is same as the time taken to come back to earth's surface from the maximum height.

   (Hint : Assume t and T to be time of ascent and descent respectively and apply the formula $h = ut + \frac{1}{2} gt^2$, where resultant displacement h is zero. (Why ?)

   (b) If a ball thrown in the upward direction comes back to its point of projection after 6 seconds, determine the velocity with which the ball was thrown up, and maximum height attained by the ball.

3. The distance travelled by a body varies with time t according to the formula

$$s = at + bt^2$$

   where $a = 20$ m/s and $b = 9.8$ m/s²

   Determine the velocity and the acceleration when $t = 3$ seconds (Given at $t = 0$, velocity $= 0$).

4. If acceleration a of a body varies with time t according to the relation $a = \sqrt{t+1}$ determine the velocity of the body when $t = 2$ seconds (Given at $t = 0$, velocity $= 4$ units).

5. A person driving his mobike at a speed of 65 km/h sees a barrier on the road ahead at a distance of 15 m. He immediately applies brakes thereby producing a retardation of 2 m/s². Will he be able to save himself from colliding with the barrier ?

6. Derive the equation [illegible] using definite integrals.

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UNIT 4

Physics for Class XI

LAWS OF MOTION

F-15/87

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

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PREFACE

This unit is fifth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in these units are the expected learning outcomes, stated discretely so that the pupil will know what to master Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project. Shri Anil Kumar, PGT (physics), Centre School, Delhi Cantt, gave some suggestion for improvement. At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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New Delhi
30 May 1982

INTRODUCTION

It is a common experience that in order to catch a moving bus, one has to run for sometime in the direction of motion of the bus before boarding it. One always jumps out of a moving bus, in the direction of motion of the bus. A swimmer pushes the water backward to move forward. When you play carrom board you see that the striker gives motion to the lower wooden coin whereas the coin lying over it continues to be at rest. Do you know the explanations of all these observed phenomena ? In the present unit you will study the laws involved in the above phenomena, which are called Newton's laws of motion. In the preceding units, you have studied the motion of a particle, with particular emphasis on motion along a straight line. In the present and few subsequent units we will discuss the causes of motion of a particle, if the particle with known initial velocity and characteristics (i e. mass, charge, etc.) is placed in a surrounding of which we have a complete description By surrounding, of course, we mean other bodies which can effect the motion of the particle, e g. for a ball rolling on a surface, surrounding is the rough surface and air. Newton's laws to be studied in the present unit cannot be tested piecemeal. Its importance however, is due to the reasons that results yielded by these laws agree with experiments and laws are simple in form.

OBJECTIVES

To be able to

1. Apply Newton's laws of motion in various problems.
2. Define force qualitatively and quantitatively on the basis of Newton's first and second laws respectively.
3. Define inertial and gravitational masses.
4. Distinguish clearly between sliding and rolling friction and apply laws of friction in solving problems.

SUGGESTED READING MATERIAL

1.1. Newton's first law of motion

Suppose you leave a book on a table after reading, the book on its own will not move unless it is disturbed by somebody else in your home. Let us take a rough plane and allow a smooth ball to roll over it. It will move through a rather small distance before coming to rest. If we polish the surface, it will move through greater distance. It is observed that the more the polishing is, the greater will be the distance covered by the ball This leads us to conclude that had the surface been perfectly smooth, ball would have been rolling for ever These observations, if generalised, can be stated as follows:

"A body at rest or in uniform motion in a straight line continues to preserve its state until and unless it is acted upon by some external agency (or force)."

This was first discovered by Galileo and adopted by Newton as the first of his three laws. The continuation of state of rest by a body in the absence of an external agency is very easy to believe whereas preservation of the state of motion involves making a body free from all the influences of its surrounding as in case of smooth ball rolling on smooth surface but we can't make it vanish. Hence a little extrapolation (or use of inductive logic) of the observation of 'smooth ball rolling on a smoother surface' is needed to understand it.

The fact that bodies stay at rest or retain their uniform linear motion in the absence of applied forces is often described by assigning a property to matter called *inertia.* The inertia preventing the body at rest to change its state is called 'inertia of rest' and inertia preventing the body in uniform motion to change its state is called 'inertia of motion'. First law is often called as 'law of inertia'. Perhaps, an 'activity' will make the concept clear.

Take a tumbler and put a card board over its mouth. Now put a coin on this card board as shown in Fig. 1. Give a hard horizontal blow (sudden motion) to the card board with the help of your fingers. What do you observe ? You will observe that the coin, due to inertia of rest, tends to remain at rest. It does not move horizontally but falls into the tumbler. The sudden motion of the card board on which the coin rests, exerts no large force for appreciable length of time.

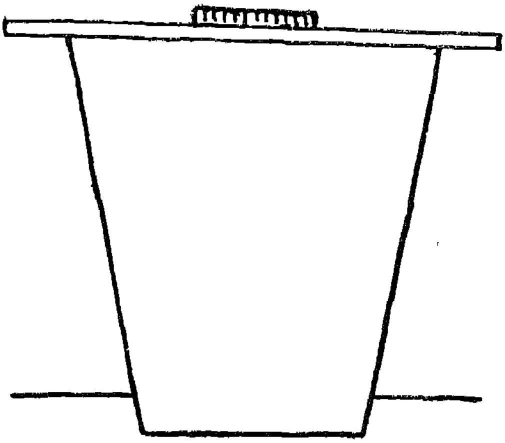

Fig. 1 A coin is placed on a cardboard covering a tumbler.

A person sitting in a stationary vehicle gets pressed against the rear if the vehicle starts off all of a sudden. This happens because the lower part of his body acquires motion along with the vehicle, whereas upper part, due to inertia of rest, tries to remain at rest. A person sitting in a moving vehicle gets pushed towards the front, if brakes are applied all of a sudden. This is due to inertia of motion. Along with the vehicle lower part of the person comes to rest whereas upper part still tries to maintain its state of motion

Questions

1. In order to clean a blanket, why do you strike it with a stick ?
2. While getting down from a moving bus why are you advised to move in the forward direction ?

Few other important facts concerning first law can be listed as follows :

(*a*) There is no distinction between a body at rest and one moving with a constant velocity. Both states are natural in the absence of forces.

(*b*) There is no distinction between the absence of all forces and the presence of forces whose resultant is zero. Hence more general way of stating first law is :

If no net force acts on a body its acceleration is zero, i.e. velocity is constant, zero or finite This means that either the body is at rest or moving with uniform motion.

(c) The first law provides us a qualitative definition of force. "Force is an external agency which changes or tries to change the state of rest or uniform motion of a body."

1.2. Force and mass : Newton's second law of motion.

As it is clear from the foregoing discussions that the effect of application of a force F is invariably to produce some acceleration a

To investigate the relationship between F and a we place a mass m on a horizontal table having negligible friction and attach a spring to it as shown in Fig 2.

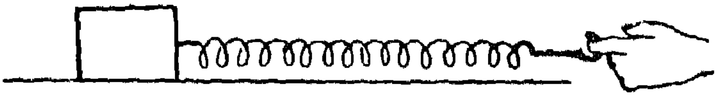

Fig 2 A block is pulled by applying force on a spring

Now we pull the spring horizontally to the right and measure a

Next we double the stretch of the spring (doubling the force) and find that the acceleration also doubles. On making the force (or stretch) three times, we find that the acceleration becomes three times. Thus we conclude that the acceleration is proportional to force for a given mass ($a \propto F$, m constant). The next question is what effect would this force have on different objects ? Everyday experience gives us a qualitative answer, i e the same force will produce different acceleration on different bodies, e.g a small ball will be accelerated more by a given force than an automobile. To get a quantitative picture, we consider standard mass m_0 of 1 kg and apply some force to it by spring shown in Fig. 2 (which can be measured by acceleration a_0 produced). Now we take an arbitrary mass m_1 and apply same force to it; (the one that accelerated m_0 by a_0 m/s^2) We measure the acceleration a_1 of mass m_1 to find

$$\frac{m_1}{m_0} = \frac{a_0}{a_1} \ (F \text{ constant}) \qquad \ldots(1)$$

Hence one can define ratio of the masses of the two bodies to be the ratio of accelerations given to these bodies by the same force.

Therefore for the constant force

$$a \propto 1/m \qquad (F \text{ constant}) \qquad (2)$$

Combining Eq (1) and Eq. (2) we get

$a \propto F/m$, where F and m both are variables

or $a = K(F/m)$

we take constant of proportionality $K = 1$ in SI units, thus

$$F = ma \qquad (3)$$

This is taken as the *statement of Newton's second law* Thus Newton's second law provides a quantitative definition of force The unit of force, one newton can be defined using Eq. (3) as that force which in a mass of 1 kg produces an acceleration of 1 m/s^2.

Question

Show that the first law is contained in the second law.

1.3 Newton's third law of motion

Let us take two spring balances , first one being attached to a hook on a wall and the ring of second one is attached to the hook of the first one as shown in Fig 3 On pulling the second balance, we notice that pointers of both the balances will indicate the same reading in opposite directions.

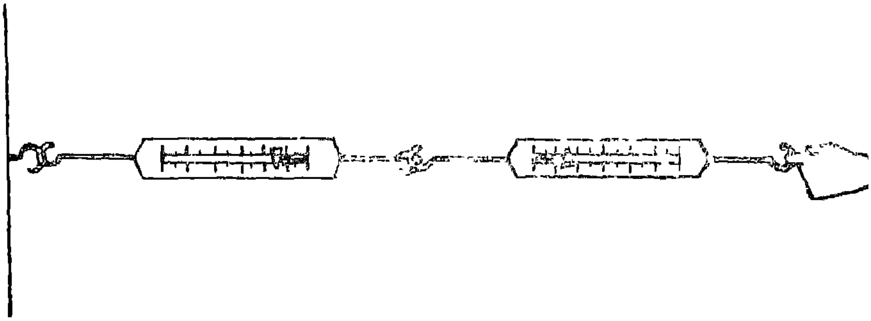

Fig. 3 Two spring balances are hooked together. One end is attached to the wall while the other is pulled by hand Same readings on the two spring balances shows that action and reaction are equal and opposite and act on different bodies.

This shows that the force exerted by the first balance on the second balance (action) is equal and opposite to the force second balance exerts on the first balance (reaction)

This is *Newton's third law* which can be stated as follows, "*To every action there is an equal and opposite reaction.*"

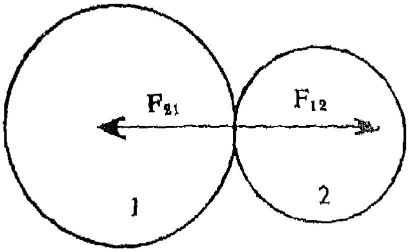

Fig. 4. The force acting on body 2 due to body 1 is F_{12} in the direction shown. The reaction of this is F_{21} on body 1 shown by an arrow in the opposite direction. According to Newton's third law $F_{12} = -F_{21}$.

If body 1 exerts a force F_{12} upon body 2 : body 2 will also exert a force at F_{21} upon body 1 as in Fig. 4 such that

$$F_{12} = -F_{21} \qquad (4)$$

The negative sign indicates that the forces are opposite. *Therefore action and reaction forces always occur in pairs and act on different bodies.* If they were to act on the same body, we could never have accelerated motion because resultant force on every body would have been always zero Hence there is no existance of a single force in nature.

To illustrate what we have stated let us consider an example. A boy walking on the ground, presses the ground with his feet, i.e. he exerts some

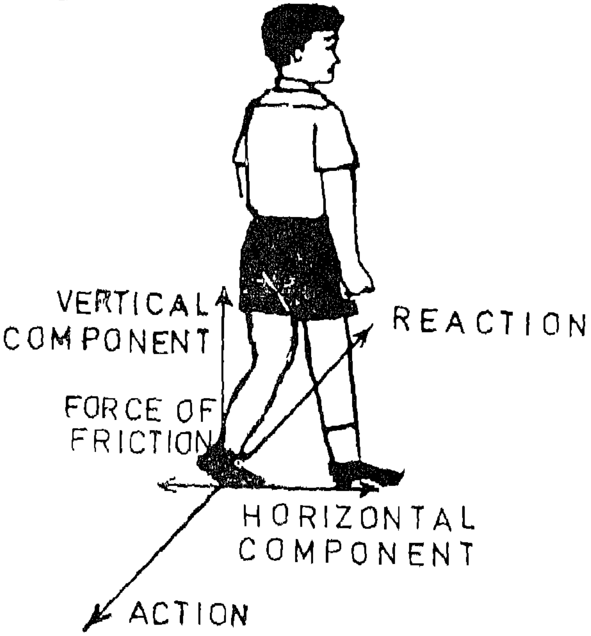

Fig. 5 The boy presses the ground down (action) The reaction of ground is on the boy. This reaction is resolved into horizontal and verticle component. The horizontal component overcomes the force of friction and enables the boy to walk.

force on the ground (action). Ground reacts by exerting same force in the opposite direction (reaction). Reaction can be resolved into two mutually perpendicular components. Vertical component will reduce the effective weight of the boy, whereas horizontal component will enable him to overcome force of friction and to move in the forward direction

Question

Suppose a horse pulls a cart forward. The cart pulls the horse backward as much as the horse pulls the cart forward. How do they ever move? Explain

1.4 Application of Newton's laws

(a) *Motion of a lift*

Let us suppose that a lift moves up with an acceleration a with a man of mass M The man feels a little heavier (i.e his effective weight will increase). Why?

Since the motion is upwards, we should determine the net upward force acting on the man and equate it to Ma as per Newton's second law Let us for a moment isolate the man and list all forces (Fig. 6) acting on him as follows

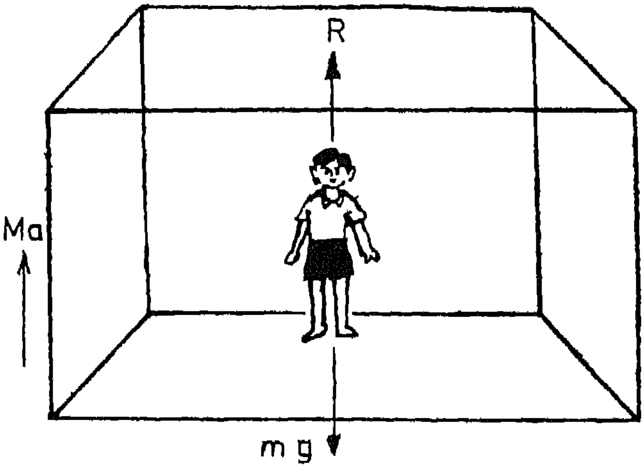

Fig. 6 A man standing on a lift moving upwards with acceleration a The forces acting on the man are his weight Mg and reaction R.

(*a*) The earth pulls the man downwards by a force Mg (this is the force of gravity and is equal to his weight). To this action there is a reaction, i.e. the man pulls the earth upwards (not shown Fig. 6), but this force is on the earth and not on the man.

(*b*) The man is standing on the lift which results in some force (action) acting on the lift. The reaction of this is an upward force R acting on the man.

Therefore, the net upward force on the man is $(R - Mg)$ which equals Ma or

$$R - Mg = Ma$$

Since $R = Mg + Ma$, the man feels heavier.

Similarly when the lift moves down we may write,

$$Mg - R = Ma$$

Question

A man feels lighter when standing on a lift moving down. Explain why?

(b) *Problem of masses and pulley*

Let us consider two masses M_1 and M_2 tied together by a light cord passing over a frictionless pulley (Fig. 7). The mass M_1 moves upwards.

In order to determine the acceleration a of the system let us isolate the mass M_1. Forces acting upon it are T and M_1g where T is tension in the string. But mass M_1 moves in the upward direction. Hence T for the mass M_1 must be greater than M_1g. Thus from Newton's second law

$$T - M_1g = M_1a$$

Similarly isolate the mass M_2. Forces acting upon it are T and M_2g as shown. But mass M_2 moves in the downward direction. Thus from Newton's second law

$$M_2g - T = M_2a \qquad .(6)$$

On solving equations (5) and (6) for T and a we get

$$T = \frac{2M_1M_2}{M_2 + M_1}g$$

$$a = \frac{M_2 - M_1}{M_2 + M_1}g$$

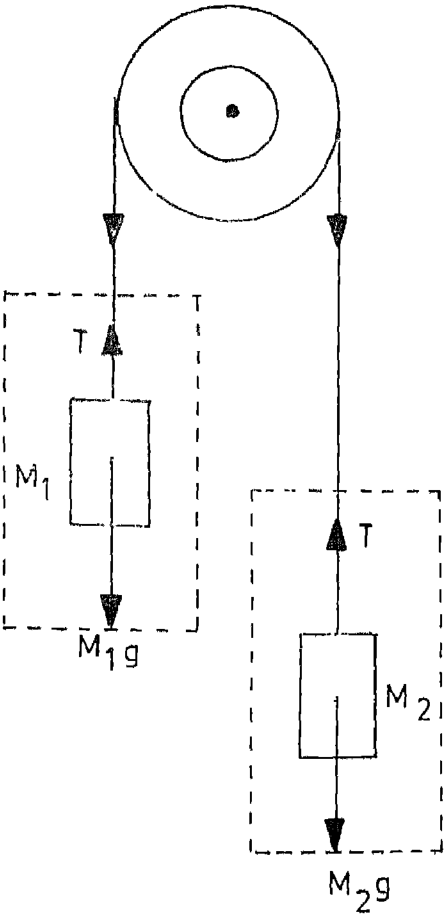

Fig. 7. Forces acting on masses M_1 and M_2 tied together by a string going over a pulley.

Question

What is the minimum force needed to prevent the pulley from falling ?

Example 1

While launching a rocket of mass 2×10^4 kg, a force of 8×10^5 N is applied for 20 s. What is the velocity attained by the rocket at the end of 20 seconds ?

Solution

Mass of the rocket, $m = 2\times10^4$ kg

Force applied, $F = 8\times10^5$ N

Acceleration produced, $a = ?$

From Newton's second law

$$F = ma$$

$$a = F/m = \frac{8\times10^5}{2\times10^4}\ \text{m/s}^2 = 40\ \text{m/s}^2$$

Initial velocity, $u = 0$

Acceleration, $a = 40$ m/s^2

Time, $t = 20$s

Final velocity attained, $v = ?$

Then from the formula

$$v = u + at$$
$$v = (0 + 40\times20)\ \text{m/s}$$
$$v = 800\ \text{m/s}$$

Example 2

A block of mass M is pulled along a horizontal frictionless plane by a rope of mass m as shown in Fig. 8 A force P is applied to one end of the rope.

(*a*) Find the acceleration of block and rope system.

(*b*) Find the force that the rope exerts on the block

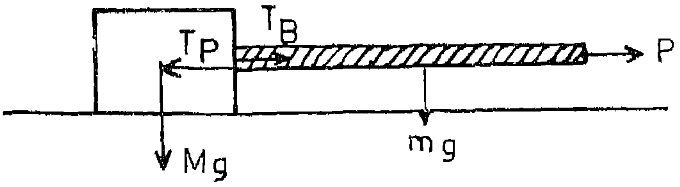

Fig. 8. Force acting on string-block system.

Solution

(*a*) As shown in Fig. 8 the rope pulls the block by a force T_B as a re-action the block pulls the rope by a force T_P such that $T_B = -T_P$. On the rope-block system the net horizontal force is P and its total mass is $(m + M)$. Therefore from Newton's second law

$$P = (m + M)\, a \text{ or } a = \frac{P}{m + M}$$

(*b*) On the block T_B is the only horizontal force acting. Hence $T_B = Ma$

Using the above expression for a, we get

$$T_B = \frac{MP}{m + M}$$

Question

Apply Newton's second law to the rope and write down its equation of motion.

Questions

1. Is it posssible to propel a sail-boat by blowing air from a fan (situated on the boat itself) across its sail ?
2. Under what conditions will the pull of a lift (mass being fixed) on the cable be maximum and minimum ?
3. A goods train has 25 compartments. Will the tension in the coupling between the fourth and fifth compartments be the same as that in the coupling between the twenty first and twenty second compartment ?
4. A mass of 8 kg is being pulled by a puller of mass 2 kg as shown in Fig. 9. Determine the tension T in the string and acceleration a of the system.

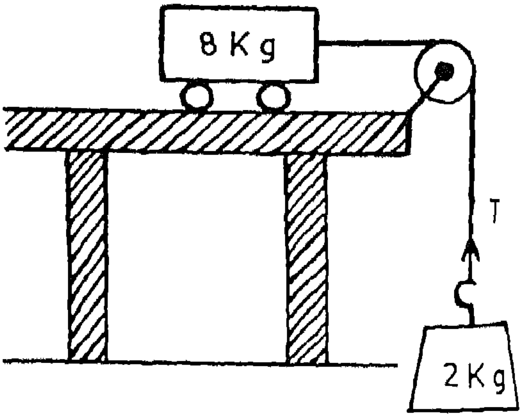

Fig. 9. A 8 kg mass being pulled down by 2kg puller.

2. Inertial and gravitational mass

Inertial mass

The constant m in $F = ma$ is inertial mass Experiment with block being accelerated by springs (article 1.2) had shown m as the unaccelerability of the body concerned, measurable by its F/a. This mass m is a measure of inertia, the tendency of all mechanical systems to oppose change We call it inertial mass and label it m_i.

Gravitational mass

Quite separately from inertial mass we can develop the idea of gravitational mass, the quantity of matter pulled down by a gravitational field We assume that the earth offers the same field to all comers, but we *assign different masses to different victims in proportion to pull on them* This kind of mass is gravitational mass m_g. We say that different objects have different masses to be pulled on by gravitational field. Thus gravitational masses are, by definition, proportional to weights The gravitational mass of an object determines how hard the earth will pull on it

When we compare bodies by weighing we are comparing their gravitational masses When two objects balance on an equal-armed weighing scale, we know their gravitational masses are equal but this observation alone does not tell us whether their inertial masses are equal. To bring out the connection between them, let us proceed with the following argument

We release two bodies A and B to fall freely Each falls with acceleration g The accelerating force on each is its weight W. Applying Newton's second law, $F = ma$, to each falling body.

$$W_A = m^i_A\, g \quad \text{and} \quad W_B = m^i_B\, g$$

Where F is the earth pull W, m is the body's inertial mass m^i and a is the acceleration of free fall g

$$\text{Hence } W_A/W_B = m^i_A/m^i_B$$

But by the definition of gravitational mass

$$\frac{W_A}{W_B} = \frac{m^g_A}{m^g_B} \text{ or } \frac{m^i_A}{m^i_B} = \frac{m^g_A}{m^g_B}$$

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Gravitational masses have the same proportion for A and B, as inertial masses

$$\frac{m^i_A}{m^g_A} \qquad \frac{m^i_B}{m^g_B}$$

Therefore the ratio (gravitational mass/inertial mass) is the same for A and B, and all other bodies. If we choose 1 kg as unit for both kinds of masses, the ratio becomes 1 and gravitational mass becomes equal to inertial mass for anybody of any size, any material, at any place.

3 Static and kinetic friction

Suppose a small pull P is applied on a block lying on a horizontal table (Fig. 10). For small applied force the block does not move, which suggests that between the surfaces in contact there is a horizontal force balancing the applied force. This force is called the force of friction. If P is increased slightly the block still does not move suggesting that the frictional force has also increased. The force of friction increases until the block just starts to move. The maximum value of force of friction is called limiting friction. You know that it is harder to pull heavier block than a lighter block. Experiments have shown that this limiting value of the force of friction is proportional to the reaction of the table on the block called the normal reaction N because it is always normal to the table. Thus

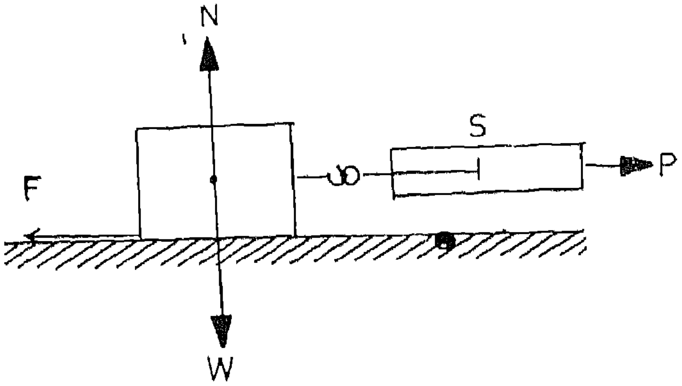

Fig 10. A horizontal force P being applied to a block through a spring balance S. Force of friction F acts in opposite direction

$$F \propto N$$

When two quantities are proportional, they can differ at most by a constant. We may write

$$F = \mu_s N \qquad \ldots(7)$$

Where μ_s is a constant called the coefficient of static friction whose value depends on the nature of the surfaces in contact The force of friction arises due to irregularities of the surfaces in contact and adhesion between molecules constituting the surfaces It always opposes motion and is independent of the areas in contact. When the block is on a horizontal surface as in Fig. 10, $N=W$ and μ_s can be found from equation (7). The values of μ_s are given in Table 1.

When the block starts moving, the force of friction becomes smaller than given by equation (7) It is then given by $F=\mu_k N$ where the new constant μ_k is called the *coefficient of kinetic friction.* For considerable range of low velocities, the force of kinetic friction is independent of velocity. However, for high velocities, friction decreases as speed increases. Tests on automobiles have shown that force of friction between brake lining and brake drum at 75 km/h is only approximately half that at a speed of 25 km/h

4. Motion along an inclined plane

(a) *Uniform motion*

Let us consider a block lying on a wooden plank. At small angles of inclination the block will not slide down Why? At one value of the angle of inclination θ the block will side down with uniform velocity, when all the forces on the block balance. The forces on the block are (a) its weight Mg acting vertically downwards. (b) The reaction of the plane N (c) force of friction F, whose direction is always opposite to the direction of expected motion (Fig 11). For convenience let us resolve the force Mg into two components, one $Mg \sin \theta$ along the inclined plane and other $Mg \cos \theta$ perpendicular to the inclined plane. Why? Since all the forces balance we may write

$$Mg \cos \theta = N$$

$$Mg \sin \theta = F$$

On dividing second equation by the first, we get

$$\tan \theta = \frac{F}{N} = \mu_s$$

which shows that μ_s is independent of the mass of the block. This method is used in the laboratory to determine μ_s.

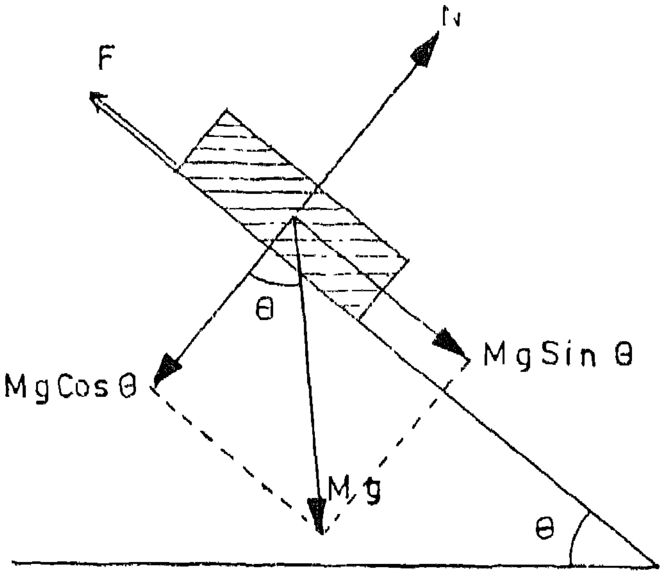

Fig 11. A block slides down an inclined plane The weight of the block mg is resolved into a component $Mg \sin \theta$ along the inclined plane and a component $Mg \cos \theta$ perpendicular to the inclined plane

(b) *Accelerated motion*

If instead of *sliding down* with uniform velocity the body moves down with an acceleration a then still

$$N = Mg \cos \theta$$

But from Newton's second law net force down the plane equals Ma.

$$Mg \sin \theta - F = Ma$$

$$\text{But } F = \mu N = \mu Mg \cos \theta$$

Using this the acceleration of the body is

$$a = g (\sin \theta - \mu \cos \theta)$$

5. Static, sliding and rolling friction

We know from our experience that it requires a lot of force to drag a heavy box along the ground, but a much smaller force is required to do so if we place rollers under the box. This clearly shows that rolling friction is much less than sliding friction

This knowledge has been made use of in attaching wheels to vehicles instead of runners as in sledges. The rolling friction of wheels is far less than sliding friction of runners. Instead of sliding on the ground, the wheel pivots about the point of contact with the ground.

Fig. 12 shows how rolling friction acts. As the wheel presses the ground, the surface of the latter is deformed as shown in Fig. 12 A small hill is formed continually and the wheel has to ride over it. Thus the rolling friction is caused by the deformation produced where the wheel presses against the surface on which it rolls.

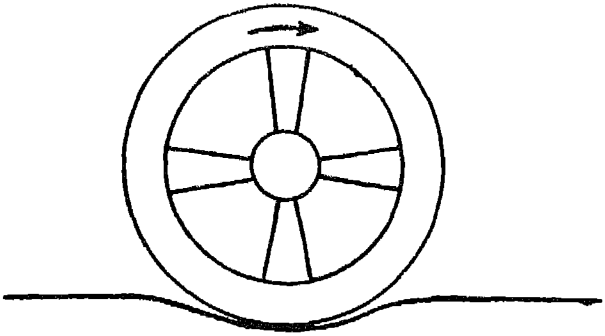

Fig 12 Rolling friction between the ground and the wheel The surface of contact and uphill roll is exaggerated

The deformation produced is less if the rim of the wheel as well as the road surface are hard. So in this case, rolling friction is reduced too. Similarly a cycle moving on a concrete road with its tyres inflated hard experiences much smaller rolling friction.

Friction can always be minimised by using lubricants, e g, mobile oil and grease, etc.

Given below is a table showing coefficient of kinetic and static friction for various substances

TABLE 1

	coefficient of friction	
Surface	Kinetic	Static
Steel on steel	0.18	0.25
Wood on wood	0.25	0.50
Stone on concrete	0 45	0.75
Car tyre on concrete	0.40	0.60

Questions

1 A man is at rest in the middle of a frozen pond on perfectly frictionless ice, how can he get himself to the shore ?

2. Explain why friction is one of the factors which makes it impossible to verify Newton's laws by experiment.

3. Explain why sliding friction is always greater than rolling friction.

Example 3

A block rests on a plane surface. The surface has to be tilted through 30° to get the block moving with constant velocity.

(a) Calculate the coefficient of sliding friction.

(b) If the surface is inclined to horizontal at angle 45°, calculate the final velocity attained by the block starting from rest in 3 seconds.

Solution

(a) For block to move with constant velocity

$$\mu = \tan \theta$$

$$\mu = \tan 30^\circ$$

$$= 1/\sqrt{3}$$

If the inclination is 45°, acceleration is given by the formula

$$a = g\,(\sin 45^\circ - \mu \cos 45^\circ)$$

$$\text{where } g = 9.8 \text{ m/s}^2$$

Hence $u = 9.8 \times \dfrac{1}{\sqrt{3}} \left(1 - \dfrac{2}{\sqrt{2}} \right) \text{m/s}^2 = 2.94 \text{ m/s}^2$

(b) Initial velocity, $u = 0$

$$a = 2.94 \text{ m/s}^2$$

$$v = ?$$

$$t = 3\text{s}$$

from the formula $v = u + at = (0 + 2.94 \times 3)$ m/s

$$= 8.82 \text{ m/s}$$

ASSIGNMENT

1. A saturn rocket develops an initial thrust of 0.3×10^1 N and has to lift off 2.8×10^6 kg. Find the initial acceleration of rocket at lift off.
Ans. 1.1×10^{-3} m/s^2

2. A tractor exerts a force of 1200 N to pull a fallen tree along the ground. If the tree's total mass is 320 kg and the coefficient of sliding friction is 0.35, find (*a*) the force of sliding friction (*b*) the acceleration.
Ans. 1098 N, 0.32 m/s^2

3. (*a*) Two objects of equal masses must rest on opposite pan of a trip scale. Does the scale remain balanced when it is accelerating up or down in an elevator ? Explain.

   (*b*) How can you measure the approximate weight of a spring without taking help of any other material ?

4 (*a*) Distinguish clearly between inertial mass and gravitational mass.

   (*b*) Why does an athlete run for some distance before taking a jump ? Explain

5. Two objects of masses 8 kg and 5 kg respectively are connected by a light cord passing over a horizontal frictionless pulley.

   (1) What is the acceleration of the system ?

   (2) What is the tention in the cord ?

(Given $g = 9.8$ m/s^2)

Ans. 2.2 m/s^2, 60 N

SELF ASSESSMENT

1. A block of mass 0 2 kg rests on the top of a block of mass 0.8 kg The combination is dragged along a level surface at constant velocity by a hanging block of mass 0.2 kg as shown in Fig. 13 (*a*). The 0.2 kg block is removed from 0.8 kg block and attached to hanging block as shown in Fig. 13(*b*). What is the acceleration of the system ? What is the tension in the string attached to 0.8 kg block in Fig. 13(*b*).

2. A 60 kg man stands on a spring weighing machine inside a lift The lift start to descend with an acceleration of 2.5 m/s^2. What is the reading on the weighing machine ?

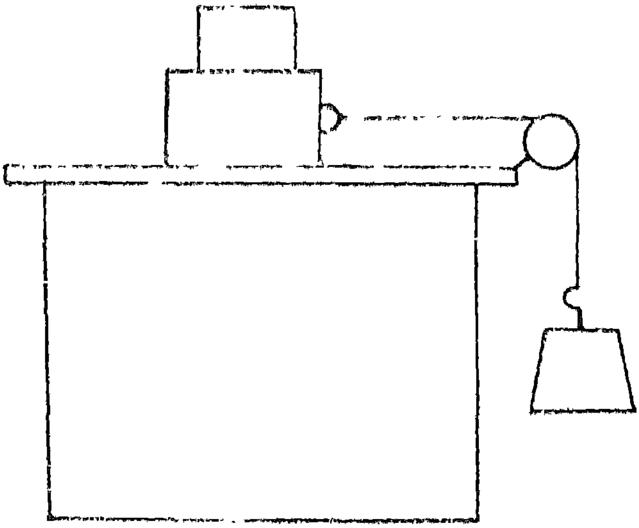

Fig. 13a A 0.2 kg block rests on a 0.8 kg block lying on a table. A string is tied to this block which goes over a pulley and supports another 0.2 kg mass tied to its other end.

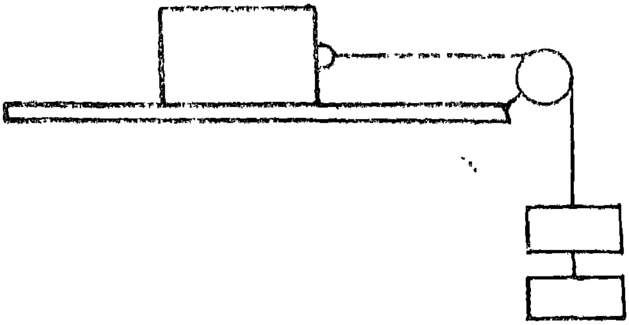

Fig. 13b. The 0.2 kg block (Fig 13 a) is removed from 0.8 block and tied to the other 0.2 kg block.

3. A block of 2 kg is projected along a level surface at a speed of 1.5 m/s. It travels 2.0 m before coming to rest. Calculate (a) retarding force (b) coefficient of sliding friction and (c) the inclination to the horizontal of the same surface for which the block will just slide smoothly without speeding up.

4. A block of mass *M* is suspended by a thread A from the ceiling and another thread B is attached to the bottom of the block (Fig. 14).

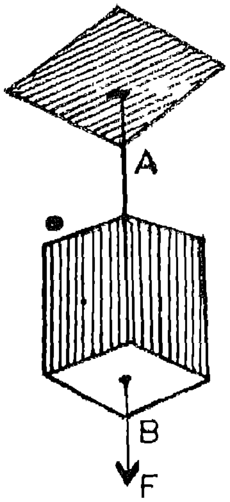

Fig 14 Thread A ties the mass to the ceiling One end of thread B is tied to the mass while a force F is applied on its loose end

Explain the following .

If the force F is a slow and steady pull, the thread will always break at A whereas if it is a sudden jerk, it will always break at B.

5. A 900 kg car starts from rest and reaches a speed of 30 m/s in 40 seconds. Calculate (a) acceleration (b) driving force.

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UNIT 5

Physics for Class XI

PROJECTILE MOTION

F-15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is fifth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this units are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project. Dr. V. Sharma, PGT (physics), Moti Lal Nehru School of Sports, Rai (Haryana), gave some useful comments on the initial draft. At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
N.C.E.R.T.

New Delhi
30 May 1982

INTRODUCTION

You have already studied the motion of objects moving in straight lines. In this unit you will apply this knowledge to study the motion of a stone thrown from a catapult, bomb dropped from a bomber, etc. which describes curved paths You will also understand why during wars, for correct bombardment, a bombardier releases the bomb before actually reaching over the target. The study of such motion is immensely useful in modern gunnery and ballistics. You will study the effect of actual impact of these objects in flight in the next unit

OBJECTIVES

To be able to

1. Derive expressions for the range and maximum [height of a projectile
2. Use physical independence of vertical and horizontal motions to solve problems involving projectile motion.

SUGGESTED READING MATERIAL

1. Description of projectile motion

If you throw stones or coins or crumpled sheet of paper outwards you will find that they follow curved paths Of course, with crumpled sheet of paper and with stone paths will be as shown in Fig. 1.

Any object that is given an initial velocity and which subsequently follows a path determined by the force of gravity is called a *projectile*. Thus the term is applied to a missile shot from a gun, a rocket after its fuel is exhausted, a bomb released from an aeroplane, and thrown or batted cricket ball, The path described by the projectile (stone), shown in Fig. 1 is called its trajectory.

If the projectile is moving very slowly air resistance can be neglected. But for fast moving projectiles air resistance modifies their paths tremendously. Presently we will consider trajectories which are sufficiently short

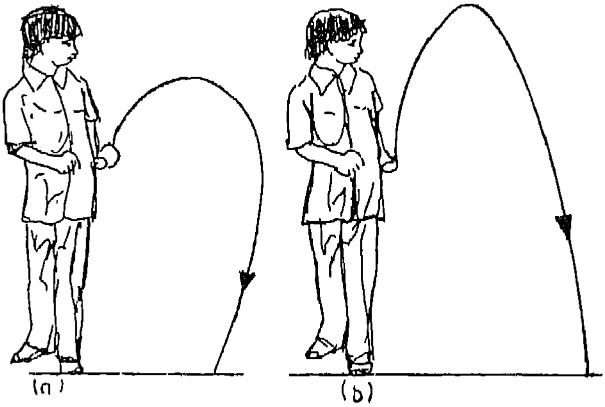

Fig 1. (a) Trajectory of a crumpled sheet of paper. (b) Trajectory of a stone.

ranged so that the gravitational force can be considered constant in magnitude and direction. The motion will be referred to axes with respect to the earth assuming it to be non-rotating, and air resistance will be assumed to be negligible.

Since the only force on a projectile in this idealized case is its weight, the motion is best referred to a set of rectangular co-odinate axes. We shall take X-axis horizontal and Y-axis vertical and the origin at the point where projectile starts its free flight. X-component of the force on projectile is zero and Y-component of the force is the weight of the projectile, *mg*. Consequently, from Newton's second law X and Y components of acceleration are given by

$$\left.\begin{aligned} a_x &= \frac{F_x}{m} = 0 \\ a_y &= \frac{F_y}{m} = \frac{-mg}{m} = -g \end{aligned}\right\} \quad \ldots(1)$$

Since zero acceleration means constant velocity the resultant motion can be described as combination of horizontal motion with constant velocity and vertical motion with constant acceleration. These two motions are independent of each other which can be illustrated considering an example. Take two carrom coins and put them near the edge of a table. Strike one coin by the other. You will find that the coins will have different horizontal ranges but both of them strike the ground at the same time as in Fig. 2. When the coins just leave the table they have different horizontal velocities, but vertical velocity for both of them is the same and equal to zero. Hence although due to different horizontal velocities their horizontal ranges are

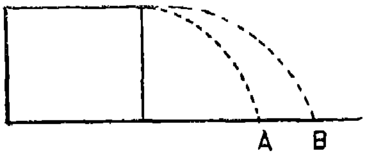

Fig 2. Trajectories of two coins leaving a horizontal table. Coin *B* had greater speed than coin *A* when leaving the table.

different, same vertical velocities ensure same time of fall from the table for both of them

Questions

1 List all the forces acting on a projectile in a real problem.

2. Give an example to show the independence of vertical and horizontal motions.

3. A gun and a coin are initially at the same height. At the time the bullet is fired horizontally, the coin is allowed to drop vertically down Will the bullet and the coin hit the ground at the same time ? Explain.

2. Equation of motion of the projectile

Let a body be projected from the origin O of the XY co-ordinate axes with an initial velocity v_0 making an angle θ_0 with the X axis as in Fig. 3 The components of v_0 along these two directions are ·

$$v_{0x} = v_0 \cos \theta_0$$

$$v_{0y} = v_0 \sin \theta_0$$

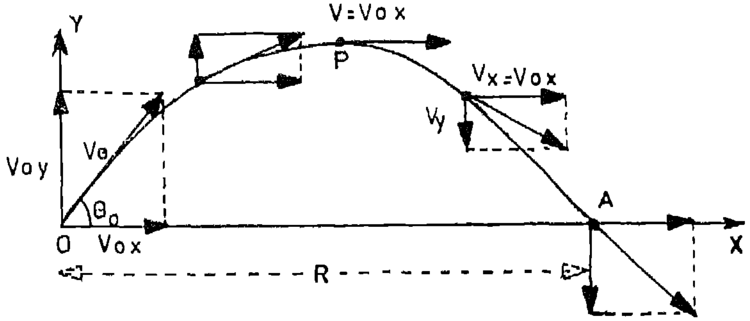

Fig. 3. Trajectory of a projectile. Variation of velocity v and its horizontal component v_{0x} and vertical component v_{0y} is shown. At the highest point *P*, v_{0y} is zero.

Since there is no acceleration along OX, the horizontal component of the velocity $v_0 \cos \theta_0$ remains constant ; the vertical component of the velocity $v_0 \sin \theta_0$ is subjected to a downward acceleration g due to gravity. Hence during ascent the vertical component goes on decreasing and finally at maximum height it becomes zero. Therefore, at maximum height, the motion of the body will be momentarily horizontal since it has only the horizontal component of velocity at that moment. During descent, of course, vertical component gradually increases in the downward direction under the action of downward acceleration g, while the horizontal component remains unchanged

Question

Are the components of velocity at A the same as at O ?

At any instant of time t from start, the velocity components v_y and v_x can be calculated by applying the formula $v = u+at$ in X and Y directions independently as below.

$$v_y = v_{0y} + a_y t$$

$$\text{or } v_y = v_0 \sin \theta_0 - gt \qquad \text{...(2}a\text{)}$$

$$v_x = v_{0x} + a_x t$$

$$\text{or } v_x = v_0 \cos \theta_0 \qquad \text{...(2}b\text{)}$$

The actual velocity is given by

$$\left.\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ \tan \theta &= v_y/v_x \end{aligned}\right\} \qquad \text{...(3)}$$

The coordinates at time t can be calculated by applying the formula $s = ut + \frac{1}{2} gt^2$ independently in X and Y direction as below:

$$x = v_{0x}\, t + \tfrac{1}{2} a_x t^2$$

$$\text{or } x = (v_0 \cos \theta_0)\, t \qquad \text{...(4}a\text{)}$$

$$y = v_{oy}t+\tfrac{1}{2}a_yt^2$$

or $$y = (v_0 \sin \theta_0)t-\tfrac{1}{2}gt^2 \qquad ..(4b)$$

The velocity at a given height can be determined by applying the formula $v^2-u^2 = 2as$ in y direction to get

$$v_y^2 = v_{oy}^2-2gy$$

or $$v_y^2 = (v_o \sin \theta_o)^2-2gy \qquad .. (5)$$

Question

Write down the formula which relates X component of velocity with height.

Since at the highest point $v_y = 0$, *maximum height*, y_{max}, using Eq. (5) is given by

$$y_{max} = \frac{v_0^2 \sin^2 \theta_0}{2g} \qquad ...(6)$$

Similarly, if the time taken to attain the maximum height is T', from Eq. (2*a*) it will be given by

$$T' = \frac{v_0 \sin \theta_0}{g} \qquad ...(7)$$

It can be shown that equation of the trajectory of the projectile is a parabola From Eq. (4*a*) we solve for t and get

$$t = \frac{x}{v_0 \cos \theta_0} \qquad (8)$$

Substituting from Eq. (8) in Eq. (4*b*), we get

$$y = (v_0 \sin \theta_0)\frac{x}{v_0 \cos \theta_0}-\tfrac{1}{2}g\frac{x^2}{v_0^2 \cos^2 \theta_0}$$

$$y = (\tan \theta_0)\, x - \Big[$$

Substituting $p = \tan \theta_0$, and

$$q = \frac{g}{(2v_0^2 \cos^2 \theta_0)}$$

where p and q are constants for a given projectile and given place

We get $y = px - qx^2$ ..(9)

Eq. (9) is the equation of a parabola.

3. Range of a projectile

In Fig. 3, O is the point of projection and A is the point where the trajectory cuts the X-axis. The distance OA is called the horizontal range or simply the *range* of the projectile and is denoted by R.

Let T be the *time of flight* from O to A, Eq. (4*a*) gives

$$R = (v_0 \cos \theta_0)\, T \quad \text{.. (10)}$$

Since at A, $y = 0$ we get from Eq (4*b*)

$$0 = v_0 \sin \theta_0\, T - \tfrac{1}{2} g T^2$$

which gives

$$T = 0$$

$$\text{or } T = \frac{2 v_0 \sin \theta_0}{g} \quad \text{.(11)}$$

Question

Interpret the root $T = 0$

Substituting the value of T from Eq (11) into Eq. (10) we get

$$R = (v_0 \cos \theta_0) \left(\frac{2 v_0 \sin \theta_0}{g} \right)$$

$$\text{or } R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \text{...(12)}$$

Where $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$

Question

(*i*) Compute the value of θ_0 that gives maximum range.

(*ii*) Show that $T' = T/2$

Ans. 45°

Example 1

A bomber is flying with a constant horizontal velocity of 300 ms^{-1} at an elevation of 2000 m towards a point directly above the target. At what angle of sight should a bomb be released to strike the target ?

(Take $g = 10\ \text{ms}^{-2}$)

Solution

We choose a reference frame fixed with respect to the earth, its origin O being the bomb release point. The motion of the bomb at the instant the bomb is released is the same as that of bomber. Hence the initial projectile velocity v_0 is horizontal and its magnitude is 300 m/s The angle of projection θ_0 is zero.

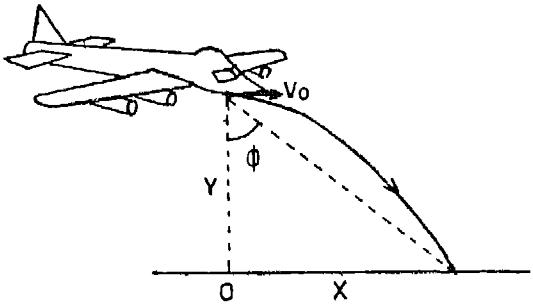

Fig. 4. Trajectory of a bomb released from a bomber in level flight. ϕ is the angle of sight.

The time of fall is obtained by the relation

$$y = (v_0 \sin \theta_0)\, t - \tfrac{1}{2} g t^2$$

Thus $\theta_0 = 0$

$g = 10\ \text{m/s}^2$

$y = -2000$ m

hence $T = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2 \times 2000}{10}}\ s = 20\ s$

The horizontal distance travelled by the bomb in this time is given by

$$x = v_{ox}T = 300\ \text{m/s} \times 20\,\text{s} = 6000\ \text{m}$$

Angle of sight $\phi = \tan^{-1}(x/y)$, how ?

$$\phi = \tan^{-1}\left(\frac{6000}{2000}\right)$$

$$= \tan^{-1}(3)$$

Example 2

A bullet is fired at an angle of 45° with the horizontal and with a velocity of 900 ms^{-1} How high will it rise and what will be its horizontal range.

Solution

Let us resolve the velocity 900 ms^{-1} into its two components as below:

(*i*) Velocity component along horizontal direction

$$v_{ox} = v_0 \cos\theta_0$$

$$= 900\ \text{ms}^{-1} \times \cos 45^\circ = \frac{900}{\sqrt{2}}\ \text{ms}^{-1}$$

(*ii*) Velocity component along the vertical direction

$$v_{oy} = v_0 \sin 45^\circ$$

$$= \frac{900}{\sqrt{2}}\ \text{ms}^{-1}$$

Considering the vertical direction, let the time taken to reach the highest point be T'.

$$v_y = v_{oy} + gT'$$

$$0 = \frac{900}{\sqrt{2}}\text{ms}^{-1} - 9.8\ \text{ms}^{-2}\ T'$$

or $$T' = \frac{900\ s}{9.8\sqrt{2}} = 64.9s$$

Now in the vertical direction

$$v_y^2 - v_{oy}^2 = 2\,gh$$

$$\left(\frac{900}{\sqrt{2}}\text{ms}^{-1}\right)^2 = 2(-9.8\ \text{ms}^{-2})\ y_{max}$$

$$y_{max} = \frac{900 \times 900 \text{ m}}{19.6 \times 2} = 2\ 1 \times 10^4 \text{ m}$$

The total time taken by the bullet for going up and coming down

$$T = 2T' = 2 \times 64.9 \text{ s}$$

$$= 129\ 8 \text{ s}$$

The distance travelled along the horizontal direction or horizontal range R is given by

$$R = v_{ox} \times 2T$$

$$= \frac{900}{\sqrt{2}} \times 129.8 \text{ m}$$

$$= 8.3 \times 10^4 \text{ m}$$

Example 3

A cliff is 20 m high and a ball is thrown up at an angle of 45° with the horizontal from the top of the cliff with a velocity of 8 ms^{-1}. (a) How long will it take to reach the horizontal plane at the foot of the cliff? (b) How far from the foot of the cliff will it strike the ground ? (c) What will be its speed as it strikes the ground ?

Solution

(a) We are given

$$v_0 = 8 \text{ ms}^{-1};\ \theta_0 = 45°$$

$$v_0 \sin \theta_0 = \frac{8 \text{ m/s}}{\sqrt{2}} = 4\sqrt{2} \quad \text{m/s}$$

$$g = 9.8 \text{ ms}^{-2}$$

We have to calculate 't' when the ball strikes the ground and so we use the equation $y = v_{0y}t + \frac{1}{2}a_y t^2$. Since effective vertical displacement, $y = -20$ m, $a_y = -g$ we have

$$-20 \text{ m} = 4\sqrt{2} \quad \text{ms}^{-1}\ T - \frac{1}{2} \times 9.8 \text{ ms}^{-2} \times T^2$$

$$T = \frac{4\sqrt{2} \pm \sqrt{32 + 80 \times 4.9}}{2 \times 4.9} \text{ s}$$

$$= 2.7 \text{ s}$$

(b) The distance travelled in the horizontal direction in time T can be calculated by the relation

$$x = (v_0 \cos \theta_0)\ T$$

$$x = (4\sqrt{2}\ ) \times 2.7\ \text{m}$$

$$x = 15\ \text{m}$$

Thus the ball strikes the ground 15 m from the foot of the cliff.
(c) Applying the equation $v = u + gt$, we can calculate the vertical velocity after time T. We know

$$v_y = v_{0y} + gT,\ \text{where } T \text{ is the time of flight}$$

Hence

$$v_y = (4\sqrt{2}\ - 9.8 \times 2.7)\ \text{ms}^{-1}$$

$$= -21\ \text{ms}^{-1}$$

Question

How will you interpret the negative sign here ?

As the horizontal component of the velocity is always constant, i.e. $4\sqrt{2}$ ms^{-1}, the resultant velocity v is given by

$$v = \sqrt{(4\sqrt{2}\ )^2 + (-21)^2}\ \text{m/s}$$

$$\text{or } v = 22\ \text{ms}^{-1}$$

The speed of the ball as it strikes the ground is 22 ms^{-1}

ASSIGNMENT

1. A bomb is dropped from a bomber when it is directly above a target at a height of 1500 m. The bomber is going horizontally with a speed of 200 kmh^{-1}. By how much distance will the bomb miss the target ?
   Ans. 2.43×10^3 m

2. A bullet is fired from a rifle at an angle 45° with the horizontal and with a velocity of $30\sqrt{2}$ ms^{-1}. How high will it rise and what will be its horizontal range ?
   Ans. 46 m, 184 m

3. A body is projected with an initial velocity of v_0 making an angle θ_0 with the horizontal. Derive the equation for the trajectory of the projectile. Also calculate the range of the projectile.

4. A ship is sailing westward at 8 m/s. While trying to fix at the top of the mast, a sailor drops the bolt. If the mast of the ship is 19.6 m high where will the bolt hit the deck ?

Ans. At the foot of the mast

5 A lunatic driving too fast on a bridge, skids and crashes through the railings along the side of the bridge, and lands (Man+Car) in the river 6 m below the level of the roadway of the bridge (Fig. 6). The police find that car is not vertically below the break in the railings, but 10 m beyond it horizontally.

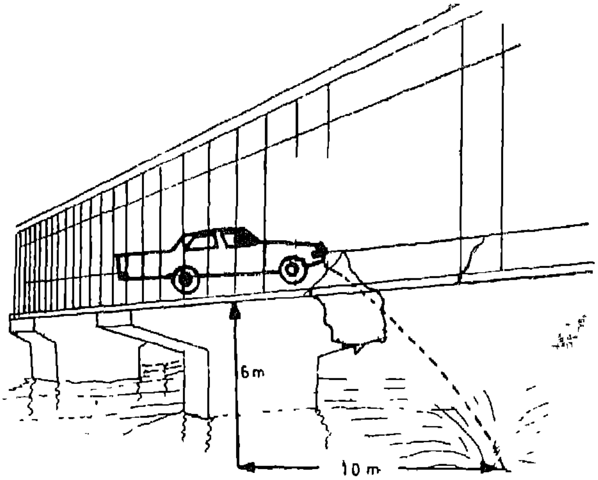

Fig. 6

(*a*) Estimate the speed of the car before crash.

Ans. 9 ms^{-1}

(*b*) State clearly the properties of falling bodies that you assumed in making the calculation of (*a*)

6. A man aims at a monkey hanging on a tree with his gun. The moment he fires, the monkey starts falling down under gravity. Show that only in this case the monkey will get hit by the bullet.

SELF ASSESSMENT

1. (*a*) Show that the maximum height reached by a projectile having an initial velocity v_0 and angle of projection θ_0 is

$$y_{max} = \frac{(v_0 \sin \theta_0)^2}{2g}$$

(*b*) Find the angle of projection at which the horizontal range and the maximum height of a projectile are equal.

2. A shell is fired horizontally from a gun located 144 m above a horizontal plane with a muzzle speed of 800 ms^{-1} (*a*) How long does the shell remain in the air ? (*b*) What is its range ?

3. A dive bomber diving at an angle of 53° with the vertical, releases a bomb at an altitude of 720 m. The bomb hits the ground 5.5 s after being released.

(*a*) What is the speed of bomber ?

(*b*) How far did the bomb travel horizontally during its flight ?

(*c*) What were the horizontal and vertical components of its velocity just before striking the ground ?

4 A gun fires a shot at some angle to the horizontal with a velocity 100 ms^{-1}. If after 3 seconds the vertical component of the velocity is 20 ms^{-1}, determine the resultant velocity at that instant.

Acc. No. F-15187(5)
Date

UNIT 6

Physics for Class XI

WORK, ENERGY AND POWER

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

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PREFACE

This unit is sixth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education A booklet wherein IGSI is described is available under a separate cover This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in these units are the expected learning outcomes, stated discretely so that the pupil will know what to master Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project.

At this stage this unit is an experimental edition I shall welcome any criticism and suggestion for the improvement of this unit.

The project for which these units are prepared is supported financially by the National Council of Educational Research and Training However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
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New Delhi
20 May 1982

INTRODUCTION

In the previous units we discussed about the motion of bodies under the action of constant forces. In the present unit you will learn about work done by constant as well as varying forces The word *work*, in everyday life, is applied to any form of activity that requires exertion of muscular or mental effort. In physics, however, the term is used in a very restricted sense If we hold a heavy weight by our hands for a long time we do feel tired, but according to the definition of work in physics, no work is done We will discuss about power, (and its units such as watt), energy, etc. in the present unit Work-energy theorem which is valid whether the force acting on the body is constant or variable will be discussed. The broader form of principles of conservation of energy and mass—principle of conservation of energy-cum-mass will also be discussed. Finally little bit of Einstein's mass-energy equivalence formula will be stated and explained, which has played a major role in the development of atomic bombs and nuclear reactors. In the very next unit we will discuss about the impulsive forces which are used in rockets, and their effects.

OBJECTIVES

To be able to

1. Deduce work-energy theorem in case of constant as well as variable forces.
2. Solve problems concerning work, energy and power.
3. State and apply Einstein's mass-energy equivalence formula in various problems.
4. Frame questions based on the textual content of this unit.

1. Work done by a constant force

Consider a particle acted on by a force F. In the simplest case F is constant and the motion takes place in a straight line in the direction of the force. In such a situation we define *the work done W by the force on particle as the product of the magnitude of the force F and the distance S through which the particle moves.* Thus

$$W = FS \qquad .(1)$$

However, the constant force acting on a particle may not always act in the direction in which the particle moves. In this case we define the work *done by the force on the particle as the product of the component of the force along the direction of displacement and the distance through which the body moves.*

In Fig. 1, a constant force F makes an angle ϕ with the X-axis and acts on a particle whose displacement along the X-axis is S. If W is the work done, then

$$W = (F \cos \phi)\, S \qquad \ldots(2)$$

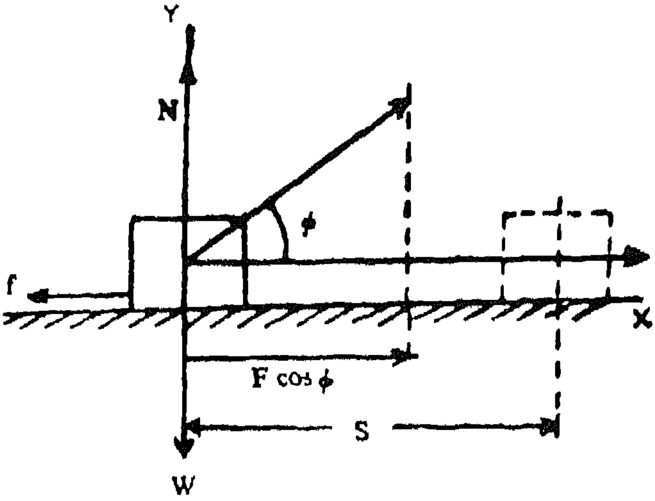

Fig. 1 A block is moved a distance S on a horizontal surface by an applied force F making an angle ϕ with the horizontal. The horizontal component of force, $F \cos \phi$ does work equal to $(F \cos \phi)\, S$. The other forces on the block are (i) the force of friction f, (ii) its weight $W = Mg$ and (iii) the normal reaction of the surface on the block, N.

If other forces are acting on the body, e g. friction, etc. then work done by each of them is to be calculated separately. *Eq.* (2) refers only to the work done on the particle by force F. The total work done on the particle will be the sum of works done by the separate forces which is equal to the work of the resultant force. This statement will be explained shortly.

Work is a scalar, although the two quantities involved in its definition, force and displacement are vectors. In Unit 1, we defined the scalar product of two vectors as a scalar quantity that we find when we multiply the magnitude of one vector by the component of a second vector along the direction of first. Eq (2) shows that work is such a quantity. Hence in the terminology of vector algebra one can write the right hand side of Eq. (2) as a scalar product to get,

$$W = \mathbf{F.S} \quad . (3)$$

Hence by now you must have realised that work is done when a force is exerted on a body while at the line moves in such a way that force has a component along the line of motion of its point of application. If the component of the force is in the same direction as the displacement the work W is positive. If it is opposite to the displacement, the work is negative. If the force is at right angles to the displacement, it has no component in the direction of displacement and the work is zero.

Question

When a body is lifted, the work of lifting force is positive or negative? What is the sign of the work of gravity in this case? Ans+,—

The unit of work is [the work done by a unit force in moving a body through a unit distance in the direction of the force. In SI-Units, the unit of work is 1 Newton-meter (N-m), called 1 joule.

Let us refer back to Fig. 1 where forces W ($= Mg$) and N (Normal reaction) are both at right angles to the displacement. Hence work done by weight W is $W_w = 0$. Similarly work done by the normal reaction N is $W_N = 0$.

The frictional force f is opposite to the displacement so the work of the frictional force is

$$W_f = -fS \quad . (4)$$

Since work is a scalar quantity, the total work W of all forces on the body is the algebraic sum of the individual works.

$$\begin{aligned} W &= W_F + W_w + W_N + W_f \\ &= (F \cos \phi)\, S + 0 + 0 - fS \\ &= (F \cos \phi - f)S \quad \ldots(5) \end{aligned}$$

But $(F \cos \phi - f)$ is the resultant force on the body. Hence *the total work of all forces is equal to the work done by the resultant force*

Questions

1. A man holds a 10 kg object stationary at arm's length, how much is the work done ? Ans. 0

   If he walks 5 m along a level floor while carrying the object, how much will be the work done by gravity ? Ans 0

2. Give a few examples of positive and negative work.

3. A man, rowing a boat up stream, is at rest with respect to the shore (*a*). Is he doing any work ? (*b*) If he stops rowing and moves down with the stream, is any work being done on him ?

Example

In Fig. 1, $F = 50\ N$, $f = 15\ N$, $\phi = 37^\circ$ and $S = 20\ m$. Determine the total work W.

Solution

$$W_F = (F \cos \phi)\, S = 50 \times 0.8 \times 20 \text{ joule} = 800 \text{ joule}$$

$$W_f = -fS = -15 \times 20 \text{ joule} = -300 \text{ joule}$$

$$W_R = W_W = 0$$

$$W = W_F + W_f + W_N + W_W = (800 - 300) \text{ joule} = 500 \text{ joule}$$

2 Work done by a varying force

In the preceding section we defined work of a constant force In many important cases, however, work is done by a force which varies in magnitude and/or in direction during the displacement of the body on which it acts. For example, when a spring is stretched slowly, the stretching force increases steadily as the spring elongates. When a body is falling, the gravitational force exerted on it by the earth varies inversely as the square of its distance from the earth's centre.

The work of a varying force can always be found graphically as follows :

In Fig. 2 (*a*) the curved line is a graph of the component F cos ϕ, for a force which varies in some arbitrary way (in magnitude and direction)

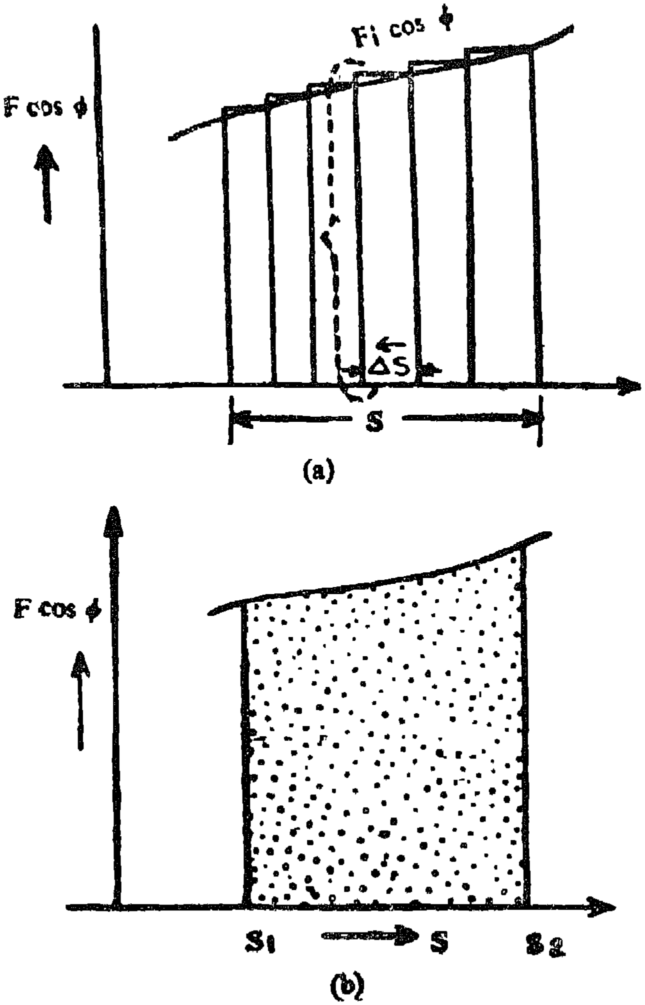

Fig 2 (a) The area under ($F \cos \phi$) vs. S curve is divided into large number of rectangular strips. The area of the i^{th} strip is ($F_i \cos \phi_i$) $\triangle S_i$.

(b) The shaded area under the curve represents the work done by a variable force.

with the distance S. Let the distance be subdivided into very short segments $\triangle S_1$, $\triangle S_2$,...... ., $\triangle S_i$ ...., etc. and approximate the varying force component by one that remains constant at the value $F_1 \cos \phi_1$ over the distance $\triangle S_1$, $F_2 \cos \phi_2$ over $\triangle S_2$ and so on. The total work W would be

$$W = (F_1 \cos \phi_1) \triangle S_1 + (F_2 \cos \phi_2) \triangle S_2 + \ldots + (F_i \cos \phi_i) \triangle S_i + \ldots$$
$$= \sum_i (F_i \cos \phi_i) \triangle S_i \qquad \ldots(6)$$

The products $(F_1 \cos \phi_1)\, \Delta S_1$, $(F_2 \cos \phi_2)\, \Delta S_2$ ...,etc. are the areas of various vertical strips and the total work is equal to the total area of these strips. As the subdivisions are made smaller and smaller, the total area of these strips becomes more and more nearly equal to the shaded area between the smooth curve and the horizontal S-axis, bounded by vertical lines at the ends of the displacement as in Fig. 2 (*b*). Thus in the language of calculus when $\Delta S_i \to 0$ the sum in Eq. (6) can be replaced by integral.

$$\text{Thus } W = \int_{S_1}^{S_2} (F \cos \phi)\, dS = \int_{S_1}^{S_2} \mathbf{F} \cdot d\mathbf{S} \qquad (7)$$

If force F is constant it can be put outside the integration giving us,

$$W = (F \cos \phi)\,(S_2 - S_1)$$

Hence work done by a constant force is a special case of Eq. (7)

It is clear that the work of any variable force can be determined by computing the integral given by Eq. (7). We now illustrate the concept of work done by a varying force taking a suitable example.

Example

Consider a spring attached to a wall. Let the axis of the spring be chosen as X-axis, and the origin, $x = 0$, coincides with the end point of the spring in its normal, unstretched position. Calculate the work done when the end point moves to a position x.

Solution

The force exerted by the spring on the person who is stretching it is $F = -kx$ (k is force constant of the spring).

Thus to stretch it to x, a force $F' = -F$ must be applied (Why ?).

Hence work done

$$W = \int_{x=0}^{x=x} F'\, dx = \int_{x=0}^{x=x} (kx)\, dx = k \int_{0}^{x} x\, dx$$
$$= \tfrac{1}{2} k x^2$$

This is the work done in stretching the spring so that its end point moves from unstretched position to x.

Questions

1. In writing the expression for the force F exerted by the spring on us, we fix a negative sign i e., $F = -kx$. Why ?

2. Assuming F vs. x curve to be a straight line passing through the origin, shows that the area under this curve gives the work done.

3 Should the work to compress a spring by x be same as that to stretch it by x ? Explain.

Example

A small object of weight W hangs from a string of length l A variable horizontal force F which starts at zero and gradually increases, is used to pull the object very slowly (so that equilibrium exists at all times) until the string makes an angle θ with vertical. Calculate the work of the force F

Solution

The forces acting on the object are its weight W, tension in the string T, and the external force F as shown in Fig. 3 (a). Let us resolve T into its horizontal component $T \sin \theta$ and vertical component $T \cos \theta$

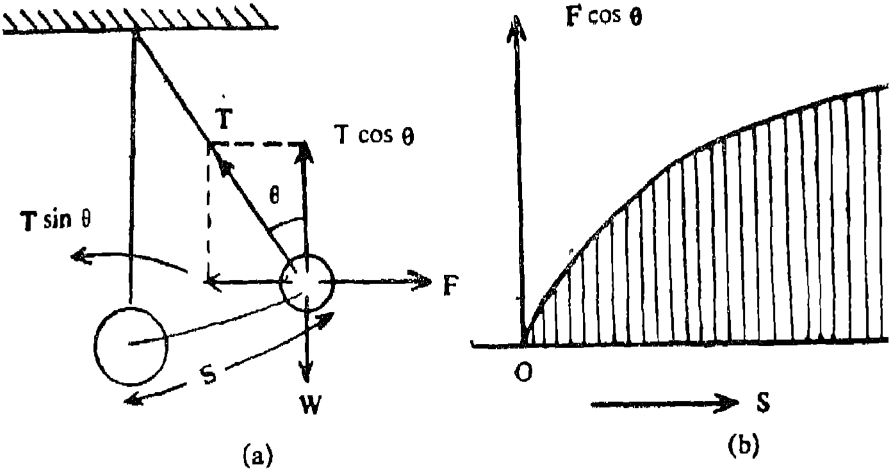

Fig. 3 (a) Tension T in the string is resolved into components $T \sin \theta$ and $T \cos \theta$
(b) Work of the force F is represented by the shaded area.

Since the object is in equilibrium, the horizontal forces balance each other and so do vertical forces.

$$F = T\sin\theta$$

$$W = T\cos\theta$$

$$\text{Thus } F/W = \tan\theta,\ \text{or } F = W\tan\theta$$

The point of application of F moves through the arc (S) shown in Fig 3 (a).

$$S = l\,\theta$$

$$dS = l\,d\theta$$

$$\text{and work} = \int F.\,dS$$

$$= \int F\cos\theta\,dS$$

(Since $F\cos\theta$ is the component of F in the direction of motion)

$$\text{Work} = \int_0^\theta W\tan\theta\cos\theta\,l\,d\theta$$

$$= \int_0^\theta W\sin\theta\,l\,d\theta$$

$$= W\,l\int_0^\theta \sin\theta\,d\theta$$

$$= W\,l\,[-\cos\theta]_0^\theta$$

$$= W\,l\,[-\cos\theta + 1]$$

$$= W\,l\,[1 - \cos\theta]$$

In Fig. 3 (b), $F\cos\theta$ is plotted against S ($= l\,\theta$). The work during any displacement is represented by the area under this curve.

Problem

(a) A force of 1.5 N is applied at an angle of 30° to the direction in which a body moves. If the body moves by 80 cm, find the work done.

(b) Frame a question and problem based on whatever you have studied so far in the present unit.

3. Work-energy theorem

When we apply force and do some work, we say we have lost energy We may, therefore, tend to equate work done to change in some kind of energy. Let us consider the motion of a particle of mass *m* acted upon by a constant force F along X-axis. We have seen in Unit 3 that the acceleration of this particle is given by

$$a = \frac{v-u}{t} \qquad .(8)$$

where u = Initial velocity (at $t = 0$)

v = Final velocity (at time t)

and distance travelled along X-axis is

$$s = \left(\frac{v+u}{2}\right) t \qquad .(9)$$

Thus the work done is

$$\begin{aligned} W &= Fs = mas \\ &= m\left(\frac{v-u}{t}\right)\left(\frac{v+u}{2}\right)t \\ &= \tfrac{1}{2}\, m\,(v^2-u^2) = \tfrac{1}{2}\, mv^2 - \tfrac{1}{2}\, mu^2 \qquad ...(10) \end{aligned}$$

We call one-half the product of the mass of a body and the square of its speed the kinetic energy (K.E.) of the body which by Eq. (10) can be interpreted as the ability to do work If we represent kinetic energy by the symbol T, then the initial K.E., $T_i = \frac{1}{2}\, mu^2$ and final K E., $T_f = \frac{1}{2}\, mv^2$

$$\text{Therefore } W = T_f - T_i \qquad .(11)$$

Eq. (11) can be stated as *the work done by the resultant force acting on a particle is equal to change in kinetic energy of the particle.*

Although we have provided the result for constant force only, it holds whether the force is constant or variable. Let the resultant force vary in magnitude (but not in direction) for example. Take the displacement to be in the direction of the force. Let this direction be the X-axis. The work

Efficiency can never be greater than one as in that case it will violate principle of conservation of energy (How ?)

Question

1. Does the work done in raising a box on to a platform depend on how fast it is raised ?

Solved Example

An automobile uses 100 H.P and moves at a uniform speed of 72 km/hr. What is the forward thrust exerted by the engine on the car ?

Solution

$$P = \frac{W}{t} = \frac{F.d}{t} = F\,v \quad \left(\text{where velocity } v = \frac{d}{t}\right)$$

The forward thrust F is in the direction of motion given by v so that $P = Fv$ (Why ?).

$$F = P/v = \frac{100 \text{ HP}}{72 \text{ km/hr}} = \frac{100 \times 746}{75 \times (5/18)\text{m/s}}$$

$$= 3730 \text{ N}$$

6. Energy

We have seen (in article 3) that the kinetic energy T of a body is the work that a body can do by virtue of its motion and we express it as $T = \frac{1}{2} mv^2$ In some cases we notice that the ability of a body to do work remains the same after completion of a round trip, e.g. in lifting a mass M through a height h and again bringing it back to ground level, does not alter the capacity of a body to do work again. Such systems are called 'conservative', and forces acting in such systems (e.g. in this example it is force of gravity) are called conservative forces. On the other hand if after the completion of a round trip the body returns back with less ability to do work or less kinetic energy, the forces acting in such case (e.g. one such force is friction) are called non-conservative.

In case of action of conservative forces (no force of friction, etc. acting) kinetic energy decreases during first half of the motion, becomes zero and then increases during second half of the motion, and kinetic energy of the system, when it regains its initial configuration returns to its initial value. Under these circumstances one can introduce the concept of energy of configuration, or potential energy, U and keeping in view the principle of conservation of energy it follows that when T changes by dT then U for the system changes by an equal but opposite amount so that sum of the two changes is zero, or

$$dT + dU = 0 \qquad (17)$$

The above equation implies that

$$T + U = \text{constant (How ?)} \qquad \ldots(17a)$$

The potential energy of a conservative system represents a form of stored energy which can be fully recovered and converted into kinetic energy. We cannot associate a potential energy with a non-conservative force such as force of friction, because the kinetic energy of a system in which such forces act does not return to its initial value when the system returns to its initial configuration. Combining work-energy theorem with Eq (17), one can write work done W equal to the change in potential energy with a negative sign Mathematically

$$W = dT = -dU \qquad (18)$$

An example will make the idea clear.

Suppose a ball of mass m is taken from a height h_1 (From an arbitrary reference line) to a height h_2. Its displacement is $(h_2 - h_1)$. The work of gravity will be

$$\begin{aligned} W_{grav.} &= -mg\,(h_2 - h_1) \\ &= (mgh_2 - mgh_1) \end{aligned}$$

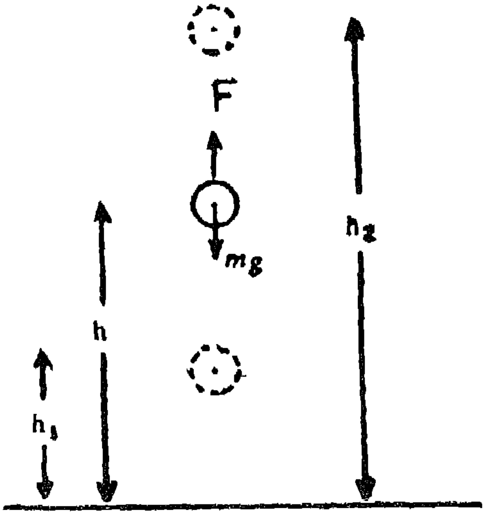

Fig. 4. Lifting a ball up by using force F. The potential energy of the ball at height h is $+\ mgh$.

The R.H.S. is change in a form of energy. This energy is called *gravitational potential energy* and is defined as

Potential energy $= mgh$ ..(19)

$W_{grav.} = -$ (change in potential energy)

Let us now assume that the ball in Fig 4 is thrown up from a height h_1 with an initial velocity u and at height h_2 its velocity is v. This motion is described by the equation

$$v^2 - u^2 = -2g\,(h_2 - h_1)$$

Multiplying both sides by m and rearranging terms, we get

$$1/2\ mv^2 + mgh_2 = 1/2\ mu^2 + mgh_1 \tag{20}$$

This equation simply means that the sum of the kinetic energy and potential energy, called the *mechanical energy* is same at the initial and final points i.e., the *energy is conserved.* In general mechanical energy is conserved when the only force that does work is the force of gravity.

Questions

1. Is the sum of kinetic energy and potential energy same at all points on the path of the ball ?
2. Does Eq. (20) hold in the case of a simple pendulum ? Explain.

Taking help of Eq. (12)

$$du = -w = -\int_{x_0}^{x} F(x)\,dx$$

for the particle moving from x_0 to x Recalling ideas of integration introduced in Unit 3, we can write

$$F(x) = -\frac{dU(x)}{dx} \qquad ...(21)$$

Thus the potential energy is a function of position whose negative derivative gives the force. From Eq. (19) *we find that the potential energy of a system is the work that the system can do by virtue of its position.* If a body of mass m is let fall from a height h it will do work equal to mgh.

Whatever we have discussed so far can even be extended to non-conservative forces which produce heat Q and change into other forms of energy. Thus Eq (17) in fact can be generalised and written as

$$dT+dU+dQ+\text{(Change in other forms of energy)}=0 \qquad (22)$$

In other words, the total energy—kinetic plus heat plus all other forms—does not change. Energy may be transformed from one kind to another but it cannot be created or destroyed; the total energy is constant This is the *principle of conservation of energy.*

There is yet another conservation principle called principle of conservation of mass stated by Lavoisier and which forms the very basis of chemistry As matter can be converted to energy and vice versa, we mention a generalised principle called the principle of conservation of energy-cum-mass, which states that mass and energy are interconvertible but as a whole their total amount remains conserved. Einstein has, infact, established the equivalence between mass and energy by the equation

$$E = mc^2 \qquad (23)$$

Where E is the energy obtained when mass m is completely converted into energy, c is the velocity of light and equals 3×10^8 m/s. Because c^2 is very large, the energy liberated is tremendous. A change in mass of 1 g will require an energy of 9×10^{10} joule.

Questions

1. Is Eq. (23) dimensionally homogenous? Prove your statement.
2. A spring is kept compressed by tying its ends together tightly It is then placed in acid where it dissolves. What happened to its stored potential energy?

Example

If a hydrogen atom has a rest mass of 1 0081 atomic mass unit (amu) and a helium atom has a rest mass of 4 0039 amu, calculate the energy released in each fusion process in which four hydrogen atoms are transformed into a helium atom with the emission of energy in the form of radiation

(Give : 1 amu $= 1.66\times10^{-27}$ kg)

Solution

Masses of four H-atoms before fusion $= 4\times 1.0081$ amu

$= 6.694\times10^{-27}$ kg

Mass of one He-atom after fusion = 6.646×10^{-27} kg

Mass difference (Δm) $= (6.694 \times 10^{-27} - 6.646 \times 10^{-27})$ kg

$= 0.048 \times 10^{-27}$ kg

Energy released $E = (\Delta m)c^2$

$0.048 \times 10^{-27} \times 9 \times 10^{16}$ joule

4.32×10^{-12} joule

Therefore energy released in each fusion process is 4.32×10^{-12} joule

ASSIGNMENT

1. A ball is dropped from rest at a height of 12 m If it looses 25% of its kinetic energy on striking the ground, what is the height to which it bounces? How do you account for the loss in kinetic energy?

   Ans. 9 m

2. A truck and a trailer of mass 1200 kg move at a steady speed of 10 m/s on a level road. The tension in the coupling is 1000 N. What is the power expended on the trailer?

   Ans. 10,000 W

3. Show that when friction is present in an otherwise conservative mechanical system, the rate at which mechanical energy is dissipated equals the frictional force times the speed at that instant

4. A net force of 5 N acts on a 15 kg body initially at rest. Compute the work done by the force in first, second and third seconds and the instantaneous power exerted by the force at the end of the third second.

   Ans. 0.83 J, 3 3 J, 7.5 J, 5 W

5. A deutron is made of one neutron and one proton. The mass of the deutron is found to be 2.01360 amu. Calculate the energy that must be added to the deutron to break it up into proton and neutron.

   Given Mass of proton = 1.00731 amu

   Mass of neutron = 1.00867 amu

   1 amu = 1.66×10^{-27} kg

   Ans 3.6×10^{-13} J

6. A cord is used to lower vertically a block of mass M a distance d at a constant downward acceleration of g/4. Find the work done by the cord on the block.

   Ans $\frac{Mgd}{4}$

7. Frame a problem based on work-energy theorem.

SELF-ASSESSMENT

1. State the law of conservation of energy Prove the conservation of mechanical energy by taking the case of a freely falling body
2. Assuming force to be constant in horizontal direction, show that the work done is equal to the change in the kinetic energy of a system
3. Define power and deduce its relation to the force applied and the velocity of the body.
4. A bullet of mass 0.03 kg moving with a speed of 400 m/s penetrates 12 cm into a fixed block of wood. Calculate the average force exerted by the wood on the bullet.
5. An electron has a mass equal to 9.1×10^{-31} kg and velocity 10 m/s, calculate its kinetic energy.
6. If the loss in mass of the reactants during a nuclear reaction is equal to 1/30 g, calculate the amount of energy released.
7. Explain how you will find the work done in the case of a variable force moving a body through a distance in the direction of force.
8. A man whose mass is 90 kg is 100 m up a hill inclined at 10° to the horizontal and moves with a velocity of 1.3 m/s. Calculate (a) the work he does and (b) his power.

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UNIT 7

Physics for Class XI

IMPULSE AND MOMENTUM

F-15187

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is seventh in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing the draft of this unit I was assisted by some teachers and Shri Partha Goswami, who worked as a Junior Project Fellow on this project.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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Reader
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New Delhi
20 May 1982

INTRODUCTION

In the preceding units, you learnt how to describe motion of a body in terms of vectors such as force, velocity, acceleration etc. In the present unit you will be learning about an alternative technique of describing the motion of a body, namely, in terms of impluse (force × time) and momentum (mass × velocity). In unit 4 you have studied about the Newton's laws of motion. The second law of motion is very intimately related to the concept of momentum. In fact, Newton in his famous book *Principia* expressed his second law as "The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of the force." Thus whenever force acts, it changes the momentum of the body. In the present unit you will also learn about the method of finding velocities of interacting bodies applying the principles of conservation of momentum remains one of the most fundamental and rigid conservation laws of physics,

OBJECTIVES

To be able to

1. State and derive the law of conservation of linear momentum.
2. Relate the impules of force with the change in momentum and use it in solving numerical problems.
3. Apply the laws of conservation of momentum and energy to elastic and inelastic collisions and related problems.

SUGGESTED READING MATERIAL

1. Momentum and conservation of linear momentum

Why do you get hurt, when you are hit by a stone ? On the other hand a crumpled paper does not hurt you, when both of them are separately hurled at you. This trivial experience clearly indicates that impact of a body is significant only when it possesses susbtantial mass and high velocity.

Impact of a particle depends on its momentum vector *p* which for a single particle is defined as the product of its mass *m* and its velocity v. That is

$$\mathbf{p} = m\mathbf{v} \qquad ...(1)$$

Momentum, being the product of a scalar by a vector, is itself a vector. Newton expressed momentum as *quantity of motion*. He stated his second law as ; *the rate of change of momentum of a body is equal to the impressed force.*

or

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad ...(2)$$

One of the basic postulates of Newtonian mechanics is that mass *m* is assumed to be independent of velocity and a constant, hence Eq (2) can be written as

$$F = m\frac{d\mathbf{v}}{dt} \text{ (Since } \mathbf{p} = m\mathbf{v}) \qquad ...(3)$$

Since acceleration ***a*** of the body equals $d\mathbf{v}/dt$ Eq. (3) can be written as

$$\mathbf{F} = m\mathbf{a}$$

Therefore, in Newton's mechanics the relation $\mathbf{F} = d\mathbf{p}/dt$ and $\mathbf{F} = m\mathbf{a}$ are completely equivalent.

In SI Units, unit of momentum is kg m/s.

So far we have discussed the motion of a single particle. Let us now consider a system of *n* particles with masses $m_1, m_2 \ldots m_n$. Let us assume that the total mass *M* of the system remains constant with time, i.e. no mass either enters or leaves the system. We have

$$M = m_1 + m_2 + \ldots\ldots + m_n \qquad ...(4)$$

The particles of the system interact with each other and an external force may also act on them. Let $\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_n$ be the velocities and $\mathbf{p}_1, \mathbf{p}_2, \ldots \mathbf{p}_n$ be the momenta of these particles respectively.

The total linear momentum *P* of a system is given by the vector sum of the linear momenta of all the particles in the system, i.e.

$$P = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots + \mathbf{p}_n \qquad ...(5)$$

On differentiating we get

$$\frac{d\mathbf{p}}{dt} = \frac{d\mathbf{p}_1}{dt} + \frac{d\mathbf{p}_2}{dt} \ldots\ldots + \frac{d\mathbf{p}_n}{dt.} \qquad \ldots(6)$$

The rate of change of momentum is the force. Thus $\frac{d\mathbf{p}_1}{dt}$ is the force F_1 exerted on a particle of mass m_1, $\frac{d\mathbf{p}_2}{dt}$ is the force F_2 exerted on particle of mass m_2 and so on.

$$\frac{dP}{dt} = (\mathbf{F}_1 + \mathbf{F}_2 + \ldots \mathbf{F}_n) \qquad . \ (7)$$

$$= \mathbf{F}_{\text{external}} + \mathbf{F}_{\text{internal}}$$

The right hand side of Eq (7) is the vector sum of all the forces acting on the particles These forces acting on the system include both the external as well as internal forces.

The internal forces do not contribute anything because they occur in equal & opposite pairs, according to Newton's 3rd law. Therefore Eq. (7) can be written as

$$\mathbf{F}_{\text{external}} = \frac{d\mathbf{P}}{dt}, \ \therefore \ \mathbf{F}_{\text{internal}} = 0 \qquad \ldots(8)$$

Where $\mathbf{F}_{\text{ext}}$ is the total external force acting on the system. This is the generalized statement of Newton's second law of motion.

$$\text{If } \mathbf{F}_{\text{ext}} = 0, \ \frac{d\mathbf{P}}{dt} = 0 \ \therefore \mathbf{P} = \text{ constant} \qquad \ldots(9)$$

Therefore if no external force acts on a system, its total linear momentum (vector sum) remains constant.

Therefore principle of conservation of linear momentum can be stated as follows.

If two or more bodies are moving under their mutual action and reaction and if no external force acts on the system, then vector sum of their momenta will always be a constant

In the derivation of principle of conservation of linear momentum Newton's third law was used [Ref Eq. (8)]. In some cases third law is not valid, but conservation of momentum is valid. Hence conservation of momentum theorem is one of the most fundamental principles. Another fundamental principle is "conservation of energy principle" about which you have studied in unit 6.

Example 1

Consider a gun and a bullet constituting a system. Initially both the gun and the bullet are at rest. Hence before the gun is fired, the total momentum is zero. When the gun fires the bullet moves forward with linear momentum. The bullet also reacts upon the gun causing it to recoil backward. The gun, therefore acquires a backward momentum while the bullet acquires a forward momentum. These two momenta are equal and opposite and add to zero. Thus total linear momentum is conserved, i.e. the total linear momentum of the system before the gun fires equals the total linear momentum of the system after the gun fires.

Let M = Mass of the gun

m = Mass of the bullet

V = Velocity of recoil of the gun

v = Forward velocity of the bulle

Net momenta of gun and bullet before firing = 0

Net momenta of gun and bullet after firing = $(-MV)+(mv)$

(Forward direction (of bullet) is taken as + ve and backward direction (recoil of gun) is taken as − ve). The principle of conservation of momentum can be written as.

Total momentum of the bullet and gun before firing

= Total momentum of the bullet and gun after firing

or $$0 = -MV + mv$$

$$MV = mv \qquad (10)$$

You also notice that kinetic energy of gun-bullet system before firing was zero whereas after firing, kinetic energy of gun and bullet is

$$\tfrac{1}{2}MV^2 + \tfrac{1}{2}mv^2$$

Hence in this case kinetic energy is not conserved.

Questions

1. Show mathematically how **P** is constant when $d\mathbf{P}/dt = 0$
2. Explain how conservation of momentum applies to a handball bouncing off a wall.

3. How can you reconcile the sailing of sailboat into the wind with the conservation of momentum principle ?

2. Impulse and impulsive force

In cricket, when a batsman strikes the ball with his bat, the bat is in contact with ball for a very short time during which it exerts a very large force on the ball. The magnitude of this force is generally not uniform. This type of force is called impulsive force (i e. a very large force acting upon a body for a very short interval of time) The variation of impulsive force $F(t)$ as a function of time t is shown in Fig 1 We, of course, assume that the force has a constant direction. The force starts acting at time t_1 and stops acting at time t_2. From Newton's second law $\left(\mathbf{F} = \frac{d\mathbf{p}}{dt}\right)$ we can write change in momentum $d\mathbf{p}$ of a body in a time (dt) during which a force $\mathbf{F}$ acts on it as

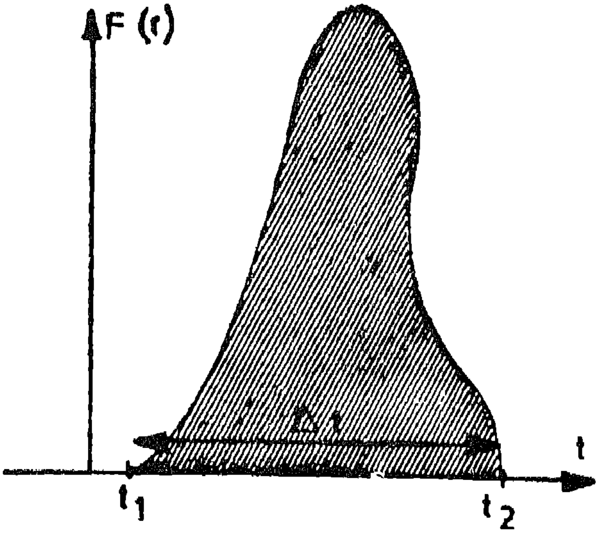


Fig. 1. Variation of impulsive force $\mathbf{F}(t)$ with time. The area under the curve gives impulse of the force.

$$d\mathbf{p} = \mathbf{F}\,dt \qquad ..(11)$$

Since during the time (t_2-t_1), net transfer of momentum takes place, we can find the change in momentum of the body by integrating over the time of contact, i.e.

$$\int_{\mathbf{P}_1}^{\mathbf{P}_2} d\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}\,dt \qquad ...(12)$$

Where P_1 and P_2 are momenta at time t_1 and t_2 respectively. Thus

$$\mathbf{P}_2-\mathbf{P}_1 = \int_{t_1}^{t_2} \mathbf{F}\, dt$$

or

$$\mathbf{P}_2-\mathbf{P}_1 = \mathbf{J} = \overline{\mathbf{F}}\, \Delta t \qquad \ldots(13)$$

Where $\overline{\mathbf{F}}$ is average value of force F during the time interval $\Delta t = t_2-t_1$. J is the integral of the force F over the time Δt during which it acts upon the body. J is called the impulse of the force F. Both impulse and momentum are vectors and both have the same units and dimensions.

As it is clear from Eq. (13), impulse J is represented in magnitude by the shaded area under the force-time curve shown in Fig. 1. For impulsive force, F is very large where as Δt is very small but for weak force e.g. force of gravity etc. F is rather small but Δt is very large i e. such weak forces act for larger interval of time to produce considerable impulse.

Questions

1. How do you explain the motion of a body in the vertically upward direction projected with some velocity ?

   (*Hint* : Apply the concept of impulsive force).

2. If a body A strikes another body B and time of contact between them is not small enough, can we apply the principle of conservation of momentum even then ? Explain.

Example 2

A vessel at rest explodes, breaking into three pieces. Two pieces having equal masses, fly off perpendicular to one another with the same speed of 30 m/s as shown in Fig. 2. The third piece has three times the mass of each other piece. What is the direction and magnitude of its velocity immediately after the explosion.

Solution

Since the vessel was initially at rest, total initial momentum is zero.

Let us resolve the momenta along X and Y axes.

Let m = Mass of smaller pieces

$3\,m$ = Mass of bigger piece

v = Velocity of bigger piece

θ = Angle between the direction of v and + ve direction of X-axis.

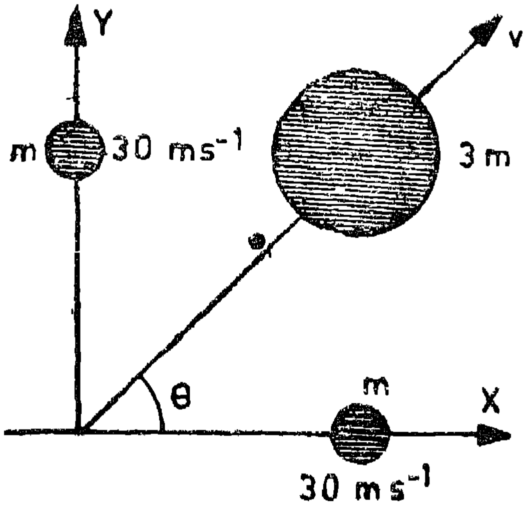


Fig. 2.

Momentum along X-axis before explosion = 0.

Momentum along X-axis after explosion

$= 30\,m + 3\,m\,(+\,v\cos\theta)$

Since momenta are in different directions their equality can be ensured by equating the sums of their X and Y components separately. Equating the sums of X components before and after the explosion we get

$$30\,m + 3m\,v\cos\theta = 0$$

$$30 + 3\,v\cos\theta = 0$$

or $$v\cos\theta = -10 \qquad \text{...14(a)}$$

Momentum along Y axis before explosion = 0

Momentum along Y axis after explosion $= 30\,m + 3\,m\,v\sin\theta$

Equating the sums of Y components of momenta before and after explosion we get

$$30\,m + 3\,m\,v\sin\theta = 0$$

$$v\sin\theta = -10 \qquad \text{...14(b)}$$

From Eq. (14 a) and Eq. (14 b) we get

$$v\cos\theta = v\sin\theta$$

$$\cos\theta = \sin\theta = \cos(90^\circ - \theta)$$

$$\theta = 90^\circ - \theta;\ \theta = 45^\circ \qquad \text{...14(c)}$$

Substituting this in Eq. (14b) we get

$$v\sin 45^\circ = -10$$

$$v\left(\frac{1}{\sqrt{2}}\right) = -10$$

$$v = -10\sqrt{2}\ \text{m/s}$$

—ve sign of the velocity indicates that third piece will move in the direction opposite to the direction shown in Fig 2 and naturally at an angle of 135° to the + ve direction of X axis.

Example 3

A ball of mass 0.25 kg moving horizontally with speed of 20 m/s is struck by a bat for 10^{-2}s. After leaving the bat, the speed of the ball is 40 m/s in a direction opposite to its original direction of motion. Calculate the average force exerted by the bat.

Solution

Since the initial and final velocities are in opposite directions, one can be considered positive and the other negative. Now

$$\begin{aligned}\text{Impulse} &= \text{Change in momentum}\\ &= \text{Final momentum} - \text{Initial momentum}\\ &= [\,0.25\,(-40) - 0.25\,(20)]\ \text{kg m/s}\\ &= (-10-5)\ \text{kg m/s}\\ &= -15\ \text{kg m/s}\end{aligned}$$

Using Eq. (13) we get

$$F\,\Delta t = 15\ \text{kg m/s}$$

or

$$F = \frac{15}{10^{-2}}\ \text{N} = 1500\ \text{N}$$

Problems

1. What is the momentum of 1500 kg car whose speed is 40 km/hr ? At what speed would a 4000 kg truck have the same momentum ? The same kinetic energy ?

2. A 90 kg man standing on a surface of negligible friction kicks forward a 0.04 kg stone lying at his foot such that it acquires a speed of 3 m/s. What velocity does the man acquire as a result ?

3. The Boeing 747 Emperor Ashoka has a total mass when fully loaded of 320,000 kg Its engine generates a total thrust of 6.4×10^4 N Determine the impulse of the force produced by the engine in 30 seconds of operation.

3. Collision (In two dimensions and one dimension)

We have many examples of collisions, carrom coin colliding with another, an automobile colliding with another, molecules in a gas colliding with one another and so on.

The principle of conservation of linear momentum is very useful in dealing with collision of bodies When two bodies come into contact at the onset of the collision each exerts a force on the other. These forces are equal in magnitude and opposite in direction during the complete duration of the collision. Hence impulse of each force is identical in magnitude but opposite in direction. Hence *there is no change in the momentum of the system of two bodies in their interaction.* Therefore when bodies, free to move in a place, interact in a collision, their directions of motion before and after the interactions are generally different but their momenta in a particular direction remains conserved.

During collisions, the transfer of kinetic energy takes place from one body to the other *If there is no loss of kinetic energy during the collision, it is called an elastic collision. Otherwise, it is called an inelastic collision.* In actual practice, some kinetic energy is lost in the form of sound and heat during the collision. Hence, the collisions between most bodies are not perfectly elastic.

Let us consider an example of elastic collision in two dimensions. Let a particle of mass m_1 moving with velocity v_1 collide with another particle of mass m_2 at rest. After collision, they move with velocities v_{1f} and v_{2f} along directions inclined at θ_1 and θ_2, respectively, with the initial direction of motion of m_1. Initial direction of motion of m_1 is taken as X axis and the direction at right angles to it is taken a Y axis.

From principle of conservation of momentum, vector sum of momenta must be constant in the absence of external force, hence Σp = constant, which implies that the sum of scalar components of momenta along X and Y axes must also be separately constant, hence one gets

$$\Sigma P_x = \text{constant}$$

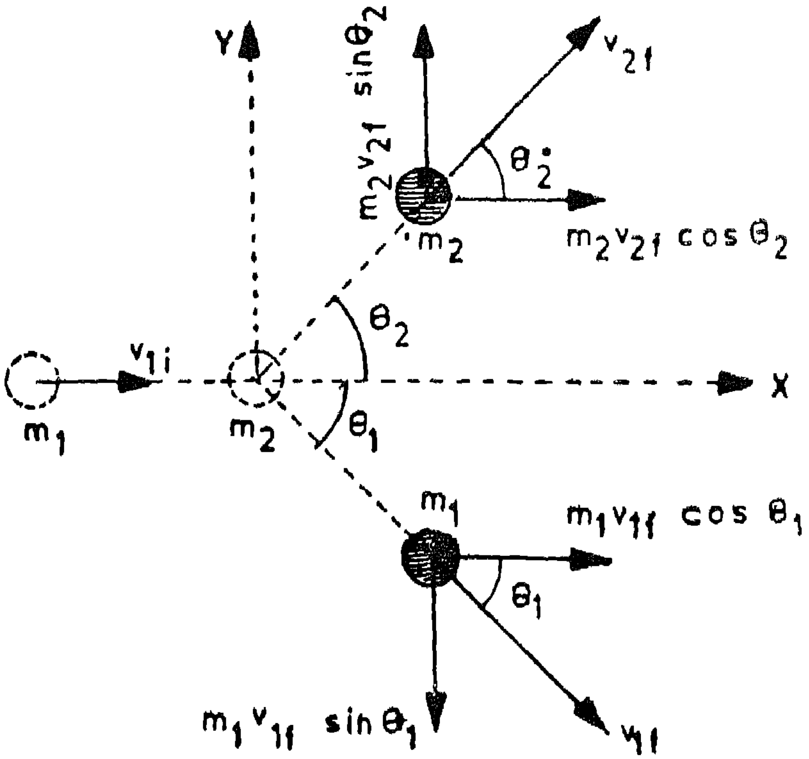
$$\Sigma P_y = \text{constant}$$

Here Σ notation stands for summation. Equivalently, conservation of momentum implies that

$$(\Sigma P_x) \text{ before collision} = (\Sigma P_x) \text{ after collision} \quad .\,(15)$$

and $$(\Sigma P_y) \text{ before collision} = (\Sigma P_y) \text{ after collision} \quad ..(16)$$

Along X axis



Y
$m_2 v_{2f} \sin\theta_2$
v_{2f}
θ_2
m_2
$m_2 v_{2f} \cos\theta_2$
θ_2
v_{1i}
X
m_1
m_2
θ_1
m_1
$m_1 v_{1f} \cos\theta_1$
θ_1
$m_1 v_{1f} \sin\theta_1$
v_{1f}

Fig. 3. Collision of two balls in 2-dimensions.

(ΣP_x) = Sum of X components of momenta of the system (2 particles) before collision

$$= m_1 v_{1i} + 0$$

(ΣP_x) = Sum of X components of momenta of the system (2 particles) after collision

$$= m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2$$

Using these in Eq (15) we get

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \tag{17}$$

Similarly,

$$(\Sigma P_y)_{\text{before collision}} = 0$$

$$(\Sigma P_y)_{\text{after collision}} = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2$$

which using Eq. (16) gives

$$0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2 \qquad .\ (18)$$

Since the collision is elastic, the kinetic energy is conserved, i.e. Kinetic energy before collision = Kinetic energy after collision.

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad ...(19)$$

If we know the values of m_1, m_2, v_{1i} and θ_1 ; then values of v_{1f}, v_{2f} and θ_2 can be found out, as we have three equations [viz (17) (18) and (19)] and three unknowns

Special case : Head on collision

Let us now consider a special case when $\theta_1 = 0$ and $\theta_2 = 0$. In this case, the particles move along the same direction before and after the collision and it reduces to a case of collision in one dimension.

Hence from conservation of linear momentum

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad ...(20)$$

And, of course, from the conservation of energy principle

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v^2_{2f} \qquad ...(21)$$

Let us now find out values of v_{1f} and v_{2f} in terms of m_1, m_2 and v_{1i}.

from Eq (20)

$$v_{2f} = \frac{m_1 \ v_{1i} - m_1 \ v_{1f}}{m_2} = \frac{m_1}{m_2}(v_{1i} - v_{1f}) \qquad ..(22)$$

Substituting Eq. (22) in Eq. (21) and setting up a quadratic in v_{1f} one can get the expression for v_{1f} (on solving the quadratic equation) as

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad ...(23)$$

Substituting Eq. (23) in Eq (22) one gets the expression for v_{2f} as

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad ...(24)$$

Few interesting consequences of this case are discussed below:

(a) If the mass of the incident particle in a collision is greater than the stationary mass, the two masses will move off in the same direction after collision, simply, because in Eq. (23) since $m_1 > m_2$ if v_{1f} is + ve than v_{2f} is + ve. Hence two particles must move in the same direction.

(*b*) If the incident mass in a straight line collision is very much smaller than the stationary mass, it bounces back with nearly the same velocity.

The reason is, that if $m_1 << m_2$; in the Eq. (23) one can drop m_1 in comparison with m_2 i.e.

$$m_1 - m_2 \simeq -m_2$$

$$m_1 + m_2 \simeq m_2$$

Therefore from Eq. (24) $\quad v_{1f} \simeq \left(\frac{-m_2}{m_2}\right) v_{1i}$

$$\boxed{v_{1f} \sim -v_{1i}}$$

This means that lighter particle rebounds from the heavier particle almost with the same velocity.

Example 4

A railway wagon of mass 12,000 kg moving at 2 m/s collides with a second stationary wagon of mass 18,000 kg. Determine the velocity of the wagons after they collide and couple together.

Solution

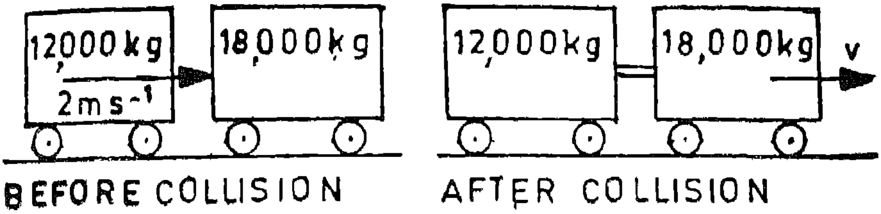


Fig 4.

Let the velocity of both the wagons after collision be V.

Momentum before the collision $= 12{,}000\ \text{kg} \times 2\ \text{m/s} + 18{,}000\ \text{kg} \times 0\ \text{m/s}$

$= 24{,}000\ \text{kg m/s}$

Momentum after the collision $= (12{,}000 + 18{,}000)\ \text{kg} \times V$

From law of conservation of momentum

Total momentum before collision = Total momentum after collision

or $24{,}000\ \text{kg m/s} = 30{,}000\ \text{kg} \times V$

or $V = 0.8\ \text{m/s}$

Example 5

A 10 g carrom coin striker moving at 2 m/s collides head on with a stationary 5 g carrom coin. After collision its velocity in same direction becomes 1 m/s. Determine the velocity of the carrom coin after collision.

Solution

Let the final velocity of the coin be V m/s

Total momentum before collision = Total momentum after collision

$$10 \times 2 + 5 \times 0 = 10 \times 1 + 5V$$

or $V = 2$

Example 6

A 5 g carrom coin moving with a velocity of 10 cm/s strikes a sationary coin of 6 g as shown; m and M fly off at an angle of 20° and 40°. Determine their velocities after collision.

Solutions

Let velocity of mass *m* be *U* and that of mass *M* be *V*.

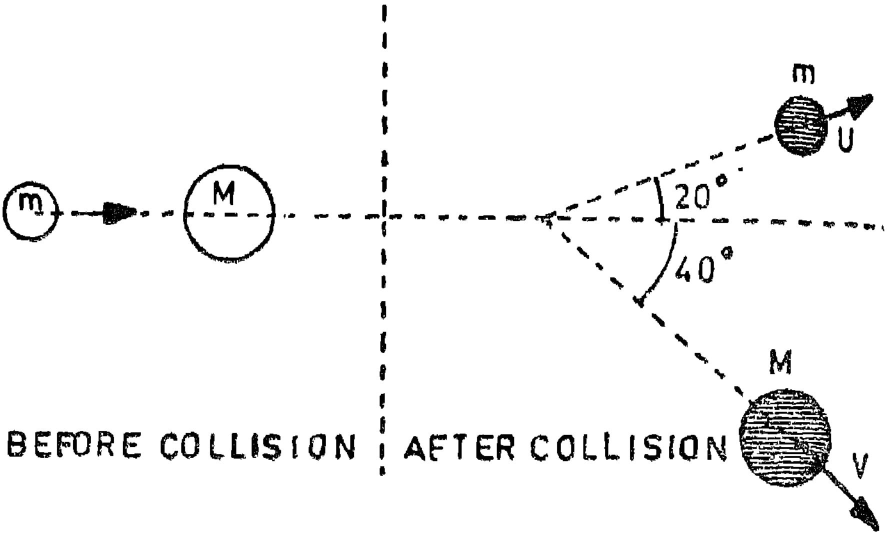


Fig. 5.

In this case the collision is in two dimensions. We must equate the momentum components along the horizontal direction and then along vertical direction.

In horizontal direction

$$5\text{ g} \times 10\text{ cm/s} = (5\text{ g})(U\cos 20^\circ) + (6\text{ g})(V\cos 40^\circ)$$

and in the vertical direction

$$0 = (5\text{ g})(U\sin 20^\circ) - (6\text{ g})(V\sin 40^\circ)$$

These equations are obtained by resolving the momenta of *m* & *M* after collision along and perpendicular to the direction of original motion.

Solving the above two equations simultaneously we get.

$$U = 7.5\text{ m/s}$$

$$V = 3.3\text{ m/s}$$

Do the following activities

(1) Release the air from a filled a balloon suddenly.

What do you observe ? Explain the reason.

(2) Throw a ball towards the wall, observe the velocity of ball and wall after impact. Explain ?

(3) Throw a large football towards a small tennis ball. Observe their motion and explain.

(4) Observe a jet aeroplane. How does it gain its thrust ?

Example 7

A 0.01 kg bullet is fired horizontally into a 4 kg block at rest on a horizontal surface The bullet gets embedded in the block and the combination moves 20 m before coming to rest ($\mu_k = 0.25$). With what speed did the bullet strike the block ? (Assume $g = 10\ \mathrm{m/s^2}$). Is kinetic energy conserved during the collision ?

Solution

(a) Deacceleration due to friction is

$$= \frac{\text{Force of friction}}{\text{Mass of the block and bullet}} = \frac{\mu_k Mg}{M} = \mu_k g$$

Using the relation $v^2 - u^2 = 2a\,s$, the velocity of the block just after the collision is

$$\begin{aligned} u^2 &= 2(\mu_k\ g)\ (20\ \mathrm{m}) \\ &= 2\times 0.25\times 10\times 20\ \mathrm{m^2/s^2} \\ &= 100\ \mathrm{m^2/s^2} \end{aligned}$$

Therefore $u = 10\ \mathrm{m/s}$

During the collision momentum is conserved. Therefore the velocity of the bullet v_b is given by

$$(0.01\ \mathrm{kg})\ v_b = (4.01\ \mathrm{kg})\ (10\ \mathrm{m/s})$$

or

$$\begin{aligned} v_b &= \frac{40.1}{0.01}\ \mathrm{m/s} \\ &= 4\times 10^3\ \mathrm{m/s} \end{aligned}$$

(b) Total K.E. of the system before collision

$$\begin{aligned} &= \tfrac{1}{2}\ (0.01\ \mathrm{kg})\ (4\times 10^3\ \mathrm{m/s^{-1}})^2 \\ &= 8\times 10^4\ \text{joule} \end{aligned}$$

Total K.E. of the system just after the collision

$$\begin{aligned} &= \tfrac{1}{2}\ (4.01\ \mathrm{kg})\ (10\ \mathrm{m/s})^2 \\ &= 2\times 10\ \text{joule} \end{aligned}$$

The ratio of K.E.'s is $\dfrac{8\times 10^4}{2\times 10^2} = 400$

ASSIGNMENT

1. A tennis ball of mass 50 g strikes a volley ball of mass 500 g with an initial velocity of 20 m/s. After the collision the velocity of the tennis ball is 10 m/s in the same direction. Determine the velocity of the volley ball. Ans. 1 m/s

2. A 0.1 kg ball moving with a velocity of 100 m/s collides with a 0.2 kg stationary ball. After the collision the 0 1 kg ball travels with 50 m/s making 45° angle with the initial direction of motion. Determine the velocity of 0 2 kg ball after collision.

3. The two bodies shown in Fig 6 are the two sections of a space vehicle. *M* is 10,000 kg and *m* is 2000 kg. After separation *V* is observed to be 5 m/s. Determine v and the velocity of *m* relative to *M*. Ans. 25 m/s, 30 m/s.

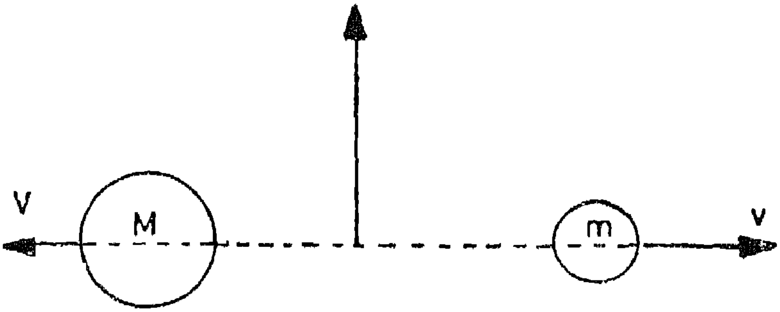


Fig. 6

4. Fuel is consumed at the rate of 100 kg per second in a rocket. The exhaust gases are ejected at a speed of 4.5×10^4 m/s What is the thrust experienced by the rocket. Ans. 4.5×10^6 N

5. Two balls of different masses have equal kinetic energy, which ball (heavier or lighter) will posses greater momentum ?

   Ans. heavier ball.

6. A bullet weighing 0.01 kg is fired with a velocity 800 m,s into a *ballistic pendulum* shown in Fig 7. The bullet remains embedded in the pendulum which weighs 5 kg and is suspended from a cord 1 metre long. Compute (a) the vertical height through which the

pendulum rises (b) kinetic energy of the bullet pendulum system just after the collision (c) Is K.E. conserved in this case

Ans 13 cm, 6.4 J, No

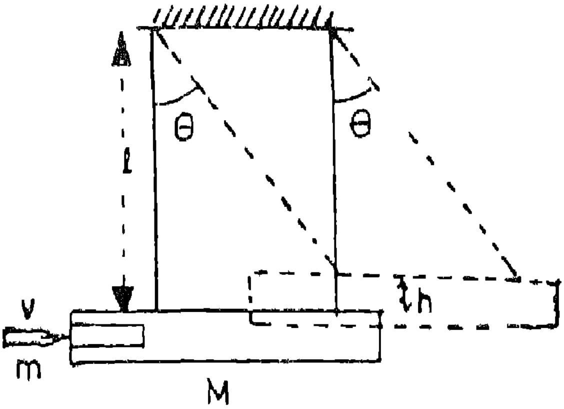


Fig. 7 : The ballistic pendulum.

SELF-ASSESSMENT

1. Give examples of elastic and inelastic collisions. How will the velocity of ball change when it strikes a rigid wall elastically ?
2. (a) Show that the impulse equals change in momentum
   (b) Use Newton's third law to derive the law of conservation of momentum.
3. An impulse of 300 Newton second is exerted on a body whose mass is 2 kg. Determine the change in the velocity of the mass.
4. A momentum of 22 kg m/s is directed at an angle of 30° relative to the X axis. Determine the X component and the Y component of this momentum.
5. A ball of mass m and velocity V makes an elastic head on collision with a ball of equal mass, initially at rest. Find the velocities of two balls after the collision.

UNIT 8

Physics for Class XI

CIRCULAR MOTION

F-15/87

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is eighth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
N.C.E.R.T.

New Delhi
20 May 1982

INTRODUCTION

You must have read or heard much about the motion of planets around the sun, the rotation of planets about their own axes, the motion of man made and natural satellites, motion of electrons round the nucleus, etc. These are some examples of circular motion. In this unit you will learn the basic principles of circular motion, some of its applications, its use in the study of motion of satellites and some related topics. Equations of motion, similar to the ones derived in Unit 3, will be derived in the case of circular motion of point or spherical masses. In the very next unit you are going to study rigid body rotation which involves the circular motion ideas.

OBJECTIVES

To be able to

1. Deduce magnitude and direction of centripetal force in circular motion and work out related problems
2 State Kepler's laws of planetary motion, Newton's laws of gravitation, and work out related problems.
3 Calculate satellite velocity, and escape velocity of a body on earth or any other planet.
4. Frame questions and problems on objectives 1, 2 and 3.

SUGGESTED READING MATERIAL

1. Angular velocity and angular acceleration

Consider a particle revolving in a circular path of radius r with uniform speed. The velocities at different points on the circular path will be directed tangentially as shown in Fig. 1. Why?

Question

When you cycle through sandy soil, sand gets thrown off in a direction tangential to the wheels of the bicycle. Explain why ?

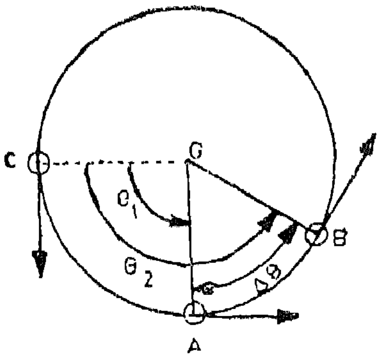


Fig. 1. A particle moving in a circle. The velocity vector, represented by an arrow, is along the tangential direction.

Let the particle be at points C, A, and B at times zero, t_1 and t_2 respectively. Let angular displacements with respect to the starting point be θ_1 and θ_2 at the points A and B respectively. Thus angular displacement in moving from A to B equals $\theta_2 - \theta_1 = \Delta\theta$ during the time interval $t_2 - t_1 = \Delta t$. The average angular speed $\bar{\omega}$ of the body in this time interval is defined as

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \qquad \text{...(1)}$$

The instantaneous angular velocity of the particle is defined as

$$\omega = \lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt} \qquad \text{...(2)}$$

If ω is a constant of time, then the circular motion is called a uniform circular motion. Angular speed has the dimensions of inverse time, T^{-1} its units are commonly taken to be radians per second (rad/s) or revolutions per second (rev/s).

If angular velocity is not constant, then the particle has an angular acceleration. Let ω_1 and ω_2 be the instantaneous angular speeds at times

t_1 and t_2 respectively ; then average angular acceleration $\overline{\alpha}$ of the body is defined as

$$\overline{\alpha} = \frac{\omega_2-\omega_1}{t_2-t_2} = \frac{\Delta\omega}{\Delta t} \qquad ...(3)$$

The instantaneous angular acceleration is the limit of this ratio as Δt approaches zero, or

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \qquad .(4)$$

Angular acceleration has the dimensions of an inverse time squared (T^{-2}); its units are commonly taken to be radians/(second)^2 or revolutions/(second)^2.

Angular velocity and angular acceleration are thus exactly analogous to linear velocity and acceleration (look up Unit 3 for the definition of linear velocity and acceleration)

Angular velocity ω and angular acceleration α are, infact, vectors. The vector angular velocity ω is defined as a vector of magnitude ω, pointing in the direction of advance of a right hand screw which is turned in the direction of rotation of the body. For a body rotating about a fixed axis, the vector ω is parallel to the axis, as shown in Fig. 2.

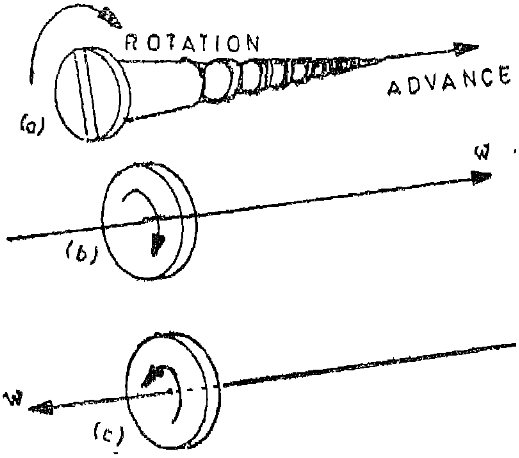


Fig. 2. Determination of the direction of angular velocity.

(*a*) A screw is turned in the direction of rotation. The direction of angular velocity ω is the direction of advance of the screw.

(*b*) Direction of ω is shown for clockwise rotation.

(*c*) Direction of ω is shown for counterclockwise rotation.

Similarly, the vector α is defined as a vector of magnitude α, having at any instant the direction of the vector change in angular velocity $d\omega$. For rotation about a fixed axis, where ω is always parallel to the axis, the vector α is also parallel to the axis

In order to establish a relationship between linear and angular velocities we proceed in the following manner. In Fig. 1, if length of arc AB is Δs, instantaneous linear velocity will be given by the expression

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{...(5)}$$

We define angle $\Delta\theta$ in radians as

$$\Delta\theta = \Delta s/r$$

or $$\frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$$

Taking the limit as $\Delta t \to 0$, we get

$$\frac{ds}{dt} = r\left(\frac{d\theta}{dt}\right)$$

or $$v = r\omega \qquad \text{..(6)}$$

Therefore, $$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \qquad \text{..(7)}$$

Questions

1. Could the angular quantities θ, ω and α be expressed in terms of degrees instead of radians in the kinematical equations ?
2. What are the angular speeds of the second hand and the minute hand of a watch ?

Example 1

A phonograph record on a turn table rotates at 33 rev/min. What is the linear speed of a point on the record at the needle at (a) the beginning and (b) the end of the recording ? The distances of the needle from the turn table axis are 13 cm and 5 cm respectively, at these two positions.

Solution

$$\text{Angular velocity, } \omega = 33 \text{ rev/min}$$

$$= \frac{33}{60} \text{ rev/s}$$

During 1 rev angle subtended is 2π radians.

Thus, $$\omega = \left(\frac{33}{60} \times 2\pi\right) \text{rad/s}$$

The distance of a point from the turn table axis at the beginning of recording, $r_1 = 13$ cm

Linear velocity at that point

$$v_1 = \omega r_1 = \frac{33}{60} \times 2\pi \times 13 \text{ cm/s}$$

$$= \left(\frac{429\pi}{30}\right) \text{cm/s}$$

The distance of a point from the turn table axis at the end of the recording, $r_2 = 5$ cm.

Thus linear velocity at that point

$$v_2 = \omega r_2$$

$$= \frac{33}{60} \times 2\pi \times 5 \text{ cm/s}$$

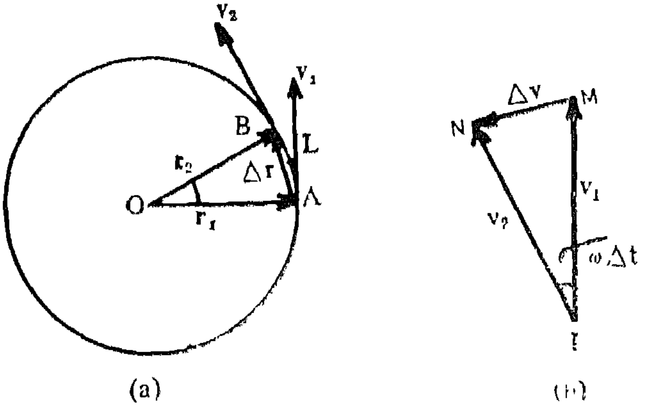
$$= \frac{165\pi}{30} \text{ cm/s}$$

2. Centripetal and centrifugal forces

The most natural force free motion of a particle is uniform motion along a straight line. Now suppose you tie a stone to one end of a string and whirl it overhead in a horizontal circle as shown in Fig. 3. Let us assume that the stone moves with constant speed. It's linear velocity at points A and B, is represented by vectors drawn in tangential directions. When the particle is at B and suddenly we cut the string, it will fly off tangentially in the direction of the arrow at B. Thus if we want to prevent the particle from flying off tangentially and to move in a circle, a pull must be applied continuously towards the centre.

Also, as the body moves in circular path its linear velocity constantly changes in direction although there is no change in its magnitude. Hence from Newton's second law, even though, the body is moving round with uniform linear and angular velocities it must be acted upon by some force which should be such that it only changes the direction of momentum and not its magnitude.

Having established the existence of such a force our next task is to determine its magnitude and direction.



(a) (b)

Fig. 3 Determination of centripetal force
(a) Space diagram showing radius vectors r_1 and r_2 and corresponding velocity vectors v_1 and v_2
(b) Vector diagram showing the difference Δv of vectors v_1 and v_2.

Let us suppose that a particle of mass m is moving along a circle of radius r with uniform angular velocity ω. The linear speed of the particle will also be uniform and will be equal to $r\omega$. Let A be the position (coordinate r_1) of the particle at any instant of time and B be its position (coordinate r_2) after a short interval of time Δt Let the velocities at A and B be v_1 and v_2 as shown in Fig. 3(a).

$$\text{Let} \qquad |v_2| = |v_1| = v$$

Let us determine the acceleration at a point L midway between A and B. For this purpose we will find the change in velocity

$$\Delta v = v_2 - v_1$$

and divide it by Δt. Let us move vectors v_2 and v_1 parallel to themselves so that their tails coincide at L, as in Fig. 3(b). Δv is, thus, given by vector MN Since in Fig. 3(a) $r_1 \perp v_1$, $r_2 \perp v_2$, and $r_2 = r_1 + \Delta r$; triangles OAB and LNM are similar. Equating the ratios of corresponding sides we get

$$\frac{\Delta v}{\Delta r} = \frac{v}{r}$$

Dividing by Δt we get

$$\frac{\Delta v}{\Delta t}=\frac{v}{r}\;\frac{\Delta r}{\Delta t}$$

Taking the limit $\Delta t \rightarrow 0$, we get

$$a_c = \frac{dv}{dt}=\frac{v^2}{r} \qquad \ldots(8)$$

Since v is constant in magnitude, average acceleration is same as the instantaneous acceleration a_c.

Also since $v=r\,\omega$,

$$a_c = \frac{dv}{dt}=r\omega^2 \qquad \ldots(9)$$

In the limit $\Delta t \rightarrow 0$, of course v_1 and v_2 become almost parallel to one another and Δv will be perpendicular to them.

The acceleration being $\lim\limits_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$, will also be perpendicular to the velocity vectors But we know that direction of the velocity at any point on the circle, is along the tangent to the circle at the point. Therefore acceleration will be along the radius through the point directed towards the centre O. Therefore if a body is to move along the circumference of a circle, it should have acceleration directed towards the centre of the circle i.e acceleration will be 'centre seeking type of acceleration' or simply *centripetal acceleration* (since in Greek centripetal means 'seeking centre') Corresponding to centripetal acceleration we have a force called *centripetal force* which is also directed towards the centre and its magnitude is given by

$$F_c=mv^2/r=m\,\omega\,v$$

or $$F_c=m\,\omega^2\,r \qquad \ldots(10)$$

From Newton's third law we know that to every action there is an equal and opposite reaction. Centripetal force should also have a reaction and it is called *centrifugal force* since it is directed away from the centre.

You may note that centripetal force is the resultant inward force on a body in circular motion and action of this force results in a change in the direction of the velocity of the body rather than change in its magnitude.

We now clarify the concept of centripetal force with illustrations and examples from everyday life.

2.1 An automobile taking a turn on level road

Suppose an automobile is negotiating a curve of radius R as shown in Fig 4. Since the road is level, the normal reaction N balances the weight, W The automobile is prevented from skidding by the force of friction between the tyres and the road bed. The required centripetal force is supplied by the force of friction F_f. Thus for safe turn we have

$$F_c = mv^2/R = F_f \;;\; W = N$$

Since $F_f = \mu N$ we get

$$mv^2/R = \mu mg$$

or $$v = \sqrt{\mu g R} \tag{11}$$

Example

What is the smallest radius of a circle at which a bicyclist can travel if his speed is 36 km/hr and the coefficient of static friction between the tyres and the road is 0.32 ?

Solution

From Eq. (11) we get

$$R = v^2/\mu g$$

$$= \frac{(36\ \text{km/hr})^2}{0.32 \times 9.8\ \text{m/s}^2}$$

$$= \frac{100\ \text{m}^2/\text{s}^2}{3.2\ \text{m/s}^2} = 31\ \text{m}$$

2.2 Banking of the road bed

As high speed trains, autos and trucks travel around curves, tremendous centripetal force must be supplied by the road beds to hold them in their curving paths. If the road bed is level this force is derived only from friction between the road and the tyres of the autos, and in case of trains, from friction between the rails and the wheels. The force of friction depends on the weather and tyre conditions. It also produces more wear and tear ; hence a better way of providing the centripetal force is by inclining or banking the road bed. The outer part of the road is raised to a higher level compared to the inner part of the curve.

The forces acting on the vehicle are the normal reaction N of the road bed, which is perpendicular to the road bed and its weight. $W (=mg)$ which is vertically down as shown in Fig 5.

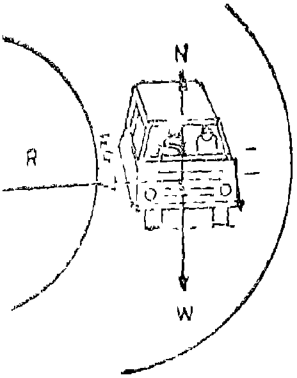


Fig. 4 Road friction supplies the necessary centripetal force to the automobile for negotiating a curve.

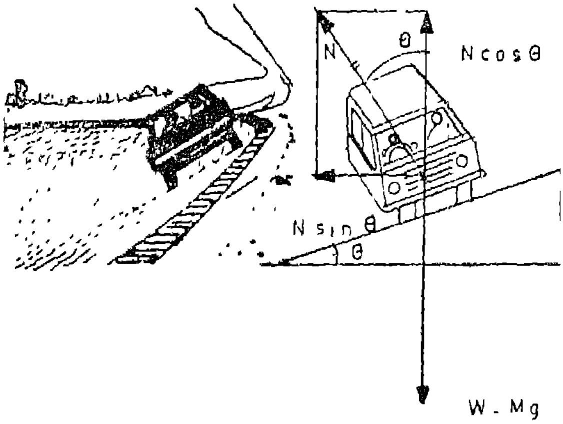


Fig. 5. An automobile negotiating a curve on a banked road bed. The centripetal force is provided by the component $N \sin \theta$ of the normal reaction

Let us resolve N into a vertical component and a horizontal component which is towards the centre of the curve

The horizontal component $N \sin \theta$, where θ is the banking angle, of the normal reaction N provides the centripetal force and verticle component $N \cos \theta$ acts in the direction opposite to weight mg of the vehicle. If R is the radius of the curve, for safe turn

$$N \cos \theta = mg$$

$$N \sin \theta = mv^2/R \qquad \text{(Centripetal force)}$$

which gives $\tan \theta = v^2/Rg$.

Thus if the speed is higher than v (given by $v = \sqrt{Rg \tan \theta}$) then frictional forces have to be relied upon to prevent skidding and if the speed is lower than v then frictional forces have to be relied upon to prevent the body from sliding down the banked roadway.

2.3 Aerobatics

A pilot in a fighter bomber aircraft is at times required to pull out of a dive by attempting a sharp loop (circular path). In this case a tremendous centripetal force will be required to hold the plane in the chosen arc. This force has to be supplied by the reaction of the air against the airfoil surfaces (wing and fuselage). If these forces become too great the wings may be torn off, the pilot himself is likely to collapse (the blood from the brain may get drained, the body is accelerated towards the centre where as the blood, being a fluid, flows along the tangent).

2.4 Centrifuge

The dairy industry makes use of the circular motion in separating the cream from milk with the help of *centrifuge*. The centrifuge is a rotating device capable of 50,000 rpm (revolutions per minute). It is also used in medical researches. It is used for whirling an object with high velocity. The consequent large radial acceleration is equivalent to increasing value of *g*, and such process as sedimentation, which would otherwise take place slowly, can be accelerated in this way.

Questions

1. In Fig. 4 and Fig. 5 some readers may wish to add an outward, centrifugal force to keep the body in equilibrium apart from the forces already shown. Is this a correct point of view ? Explain.
2. From Eq. (12) it is clear that for a given radius no one angle is correct for all velocities. Therefore, how should the curves be banked in the design of highways and railroads ?

Example 1

A small body of mass 200 g revolves in a circle on a horizontal frictionless surface, attached by a cord 20 cm long to a pin set on the surface. If the body makes two complete revolutions per second, find the force exerted on it by the cord.

Solution

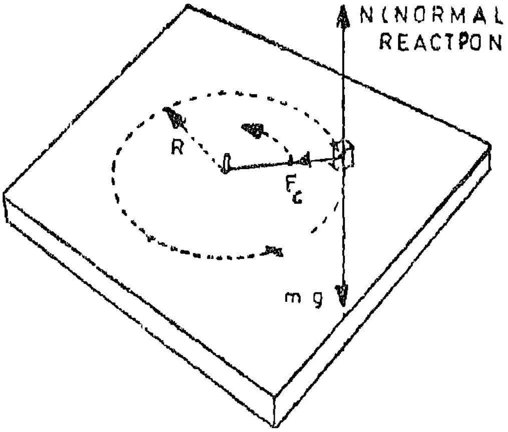


Fig. 6. One end of a cord is tied to a pin at the centre, while the other end is tied to a particle moving in circle The centripetal force is along the string pointing always towards the centre of the circle

The circumference of the circle

$$= 2\pi R$$
$$= 2\pi \times 20 \text{ cm}$$
$$= 40\pi \text{ cm}$$

$$\text{Velocity } v = 2\pi\, R/T$$
$$T = \text{Time period}$$
$$= 1/f\ (f = \text{frequency of revolution})$$
$$f = 2 \text{ rev/s}$$
$$v = \frac{40\pi \text{ cm}}{\frac{1}{2} \text{ rev/s}} = 80\pi \text{ cm/s}$$

The magnitude of centripetal acceleration is

$$a_c = v^2/R = \frac{(80\pi \text{ cm/s})^2}{20 \text{ cm}} = 3150 \text{ cm/s}^2$$

Since the body has no vertical acceleration, the forces N and mg are equal and opposite and the force F_0 is the resultant force.

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Therefore magnitude of F_c

$$F_c = Ma$$

$$= 200\ g \times 3150\ \text{cm/s}^2$$

$$= 6.3\ \text{N}$$

Problems

1. The moon revolves around the earth in a circle (very nearly) of radius $R = 4 \times 10^8$ m and requires 27.3 days to make a complete revolution. What is the acceleration of the moon towards earth ?

2. The total mass of motor cycle and the rider is 150 kg The rider has to negotiate a curve of radius 70 m at a speed of 20 m/s If the coefficient of friction is 0.5, will the rider be able to go around the curve successfully ? At what angle should he lean over so as to avoid skidding ? Acceleration due to gravity is 9.8 m/s².

3. Newton's laws of gravitation

Newton's laws of gravitation are concerned with the interaction of material bodies. The basic essence of this law is the assumption that the force on a particle can be related in a simple way to measurable properties of the particle and of its environment. The laws can be stated as follows

Law I

The force of attraction F between any two particles in the universe with masses M_1 and M_2 is directly proportional to the product of their masses, i.e.

$$F \propto M_1 M_2 \tag{13}$$

Law II

The force of attraction F is inversely proportional to the square of the distance of separation r between them, i.e.

$$F \propto 1/r^2 \quad \ldots(14)$$

Thus from equations (13) and (14)

$$F \propto (M_1 M_2)/r^2$$

or

$$F = G(M_1 M_2)/r^2 \quad .(15)$$

Where G is a universal constant having the *same value for all pairs of particles* and equals 6.67×10^{-11} Nm^2/kg^2. The universal constant G must not be confused with g which is the acceleration of a body arising from the earth's gravitational pull on it The constant G has the dimensions L^3/MT^2 and is a scalar, g has the dimensions L/T^2, is a vector, and is neither universal nor constant. The relation between g and G will be given shortly.

Some important features of these laws are given below

(*a*) The gravitational forces between two particles are action-reaction pair. The first particle exerts a force on the second particle and likewise second particle exerts a force on the first and this forces are directed along the line joining the two.

(*b*) The constant G must be found from experiments Once G is determined for a given pair of bodies, we can use that value in Eq. (15) to determine the gravitational forces between any other pair of bodies

(*c*) Notice that Eq. (15) expresses the force between point mass particles, therefore, if we want to determine force between extended bodies, we must regard each body as decomposed into particles and then the interaction between all particles must be computed. For uniform spheres we can assume that all the mass of the body is concentrated at its centre.

(*d*) As it is clear from Eq. (15), the gravitational force between two particles is independent of the presence of other bodies or the properties of the intervening space

3.1 Gravitational potential

Gravitational potential at a point is the work done in carrying a unit mass from infinity to that point.

In order to calculate it at a point distant r from mass M, let us consider a unit mass placed at a distance x from M.

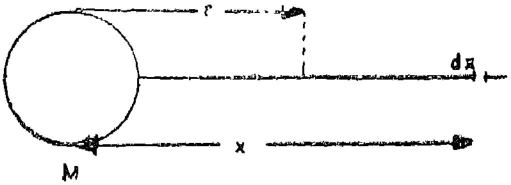

Fig. 7.

Gravitational force at that point

$$= \frac{GM}{x^2}$$

Work done in moving the unit mass through a small distance dx will be

$$dW = + \frac{GM}{x^2} dx$$

Hence total work done in carrying unit mass from infinity to a point distant r will be

$$W = \int dW = + \int_{\infty} \frac{GM}{x^2} \, dx = -GM \left[\frac{1}{x} \right]_{\infty}$$

$$= - \frac{GM}{r}$$

The —ve sign indicates that work is done by the gravitational force of attraction. Thus potential energy of mass m placed at a distance r from M will be

$$U(r) = - \frac{GMm}{r} \qquad \text{(why ?)} \qquad \ldots(16)$$

Questions

1. Why are the laws of gravitation called universal ?
2. Express the laws of universal gravitation in vector form

3.2 Acceleration due to gravity

The gravitational force on a body is proportional to its mass as Eq. (15) shows. Gravitational force F exerted by earth upon a body of mass m will be thus

$$F \propto m$$

$$\text{or} \qquad F = mg \qquad \ldots(17)$$

Where g appears as the constant of proportionality. A comparison with the Newton's second law gives g as the acceleration experienced by the body due to pull of earth.

If M_E is the mass of the earth and R_E its radius, then Eq. (15) gives another expression for F (of course, assuming that the body of mass m is situated over earth's surface) i e

$$F = G\frac{M_E m}{R_E^2} \qquad \text{...(18)}$$

Equations (17) and (18) give

$$mg = \frac{GM_E m}{R_E^2}$$

$$g = GM_E/R_E^2 \qquad \text{...(19)}$$

Equation (19) clearly shows that g is independent of the mass m of the body and it depends only on the mass of the earth (which is of course, constant) and on the value of R_E Therefore g varies with altitude and depth from the earth's surface.

Acceleration due to gravity even varies with latitude, at the equator g is minimum and at poles it is maximum Detailed discussion is beyond the scope of the present unit.

Questions

1 If the force of gravity acts on all bodies in proportion to their masses, why doesn't a heavy body fall faster than a light body.

2. How does the weight of a body vary enroute from the earth to the moon ? Give graphical representation.

Example 2

A body weighs 1000 N on the surface of the earth. How much will it weigh 800 km from the earth Take radius of the earth $R_E = 6400$ km.

Solution

If m is the mass of the body, its weight W on the surface of the earth will be, using Eq. (18), given by

$$W = \frac{GM_E m}{R_E^2} \qquad \text{...(20)}$$

At a distance r_1 from the centre of the earth, the weight W_1 of the body is given by

$$W_1 = \frac{GM_E m}{(r_1)^2} \qquad \text{..(21)}$$

Dividing Eq. (20) by Eq (21) we get

$$\frac{W}{W_1} = \left(\frac{r_1}{R_E}\right)^2$$

or

$$W_1 = W\left(\frac{R_E}{r_1}\right)^2$$

$$= 1000\left(\frac{6400}{6400+800}\right)^2 N = 790\ N$$

3.3 Escape velocity

It is our general experience that if we throw a body upwards it returns back to the earth. However if a body is thrown fast enough it may not return to the earth. The velocity with which a body must be projected upwards so that it neither comes back to the earth nor becomes an artificial satellite, is called the escape velocity. From Eq (16) it is clear that the work needed to take a mass m from surface of the earth to infinity is $\frac{GM_E m}{R_E}$. If the initial kinetic energy of the body is greater than this work done then the body overcomes the gravitational barrier of the earth. Thus the escape velocity v_E is given by

$$\tfrac{1}{2}mv_E^2 > \frac{GM_E m}{R_E}$$

or

$$v_E > \sqrt{\frac{2GM_E}{R_E}} \qquad \text{...(22)}$$

Eq (22) gives the expression for escape velocity of the body, notice that it is independent of the mass of the body to be project Substituting $G = 6.68 \times 10^{-11}$ N m^2/kg^2, $R_E = 6400$ km, $M_E = 6.2 \times 10^{24}$ kg in Eq (22), we get escape velocity equal to 11.2 km/s.

Should a projectile be given this initial speed, it would escape from the earth.

Questions

1. How has the hydrogen present in the earth's atmosphere disappeared long time ago ?
2. Why can't the moon keep any atmosphere at all ?
3. Does a rocket really need the escape speed of 11 2 km/s initially to escape from the earth ?

Example 3

A projectile is fired vertically from the earth's surface with an initial speed of 10 km/s Will it be able to escape from the earth ?

(Assume $g = 10 \text{ m/s}^2$)

Solution

Radius of the earth $R = 6400$ km

$$= 6.4 \times 10^6 \text{ m}$$

Acceleration due to gravity $g = 10 \text{ m/s}^2$

Escape velocity $= \sqrt{\dfrac{2GM}{R}}$, Where M = Mass of the earth

G = Gravitational constant.

$$\text{But } g = \frac{GM}{R^2}$$

$$gR = \frac{GM}{R}$$

Hence escape velocity $= \sqrt{2gR}$

$$= \sqrt{2 \times 10 \times 6.4 \times 10^6} \text{ m/s}$$

$$= \sqrt{1.28} \times 10^4 \text{ m/s}$$

$$= 11.3 \text{ km/s}$$

Therefore projectile will not be able to escape from the earth since its initial velocity of projection is 10 km/s.

4 The motion of planets and satellites

The motion of bodies in the solar system can be deduced from the laws of motion and the laws of universal gravitation. All planets move in elliptical orbits with the sun at one focus. We can understand the planetary motion in simplified manner by considering the orbits to be circular. We will also consider all bodies in the solar system to be spherical and assume their masses concentrated at their centres The ideas developed here apply equally well to the motion of satellites (natural or artificial) around planets.

Let us consider a planet of mass m revolving around the sun of mass M_S in a circular orbit of radius R as shown in Fig 8 Let v be the orbital velocity of the planet and T be its time period Since in time T the planet completes one revolution covering a distance $2\pi R$, we get

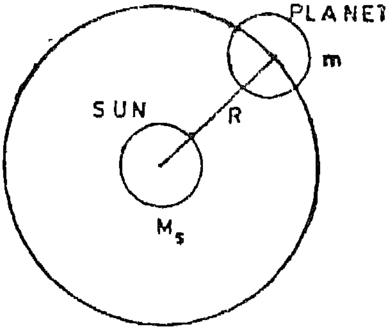

Fig. 8 : Motion of a planet around the sun (not to scale)

$$v = \frac{2\pi R}{T} \quad \text{.. (23)}$$

The centripetal force required to keep the planet moving in a stable circular orbit is supplied by the gravitational force between the sun and the planet. Equating these forces we get

$$\frac{mv^2}{R} = \frac{GM_Sm}{R^2} \quad \text{...(24)}$$

or

$$v = \sqrt{\frac{GM_S}{R}} \quad \text{.. (25)}$$

Which gives the orbital velocity of the planet Substituting for v from Eq. (23) and rearranging terms we get

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_S} = \text{constant for all planets.}$$

This relation was first given by Kepler whose three ***laws of planetary motion*** are given below :

(1) *The law of areas* : Each planet revolves around the sun in an elliptical orbit, with sun at one focus of the ellipse.

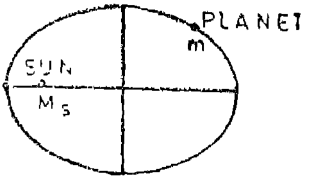

Fig. 9. A planet is shown moving in an elliptic orbit with the sun at a focus (not to scale)

(2) *The law of areas* : The line joining any planet to the sun sweeps out equal areas in equal interval of time

(3) *The law of periods* : The square of the period of any planet about the sun is proportional to the cube of the semi-major axis of the orbit of the planet round the sun.

Instead of considering semi-major axis one can consider the planet's mean distance R from the sun, as the orbits are nearly circular.

Thus, if T is the time period of the planet

$$\frac{T^2}{R^3} = \text{constant (for all the planets.)} \qquad \ldots(26)$$

As given by Kepler these laws were empirical, they simply describe the observed motion of the planets without any theoretical interpretation.

Question

From Kepler's second law and observation of the sun's motion as seen from the earth, we can conclude that the earth is closer to the sun during winter in the northern hemisphere than during summer. Explain.

Example 4

If the time period of Jupiter is 11.6 solar year, how far the Jupiter is from the sun ? Take the distance of the earth from the sun as 1.5×10^{11} m.

Solution

Time period of the earth T_1 = 1 solar yr.
Time period of the jupiter T_2 = 11.6 solar yr.
Distance of the earth from the sun r_1 = 1.5×10^{11} m.
Distance of the jupiter from the sun r_2 = ?

From the Kepler's third law

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$r_2 = \left(\frac{T_2}{T_1}\right)^{2/3} r_1$$

$$= \left(\frac{11.6}{1}\right)^{2/3} \times 1.5 \times 10^{11} \text{ m}$$

$$= 7.68 \times 10^{11} \text{m}$$

ASSIGNMENT

1. In planning to bank a roadway whose radius of curvature is 180 m it is expected that traffic will move around the curve at 72 km/hr. What should be the angle of banking if no dependence on friction is to be placed ? Ans. 13°

2. Determine the mass of the earth from the period T and the radius r of the moon's orbit about the earth, $T = 27.3$ days and $r = 3.84 \times 10^5$ km. Ans. 6×10^{24} kg

3. Mars has a mean diameter of 6,720 km and earth of 12,800 km The mass of the mars is 0.11 M_E (*a*) How does the mean density of mars compare with that of the earth ? (*b*) What is the value of g on mars ? (*c*) What is the escape velocity on mars ?
Ans. 0.76 d_E, 0.4 g_E, 0.46 v_E

4. Calculate the angular velocity of a hour hand and minute hand in a watch. Ans. 1.45×10^{-4} rad/s, 1.74×10^{-3} rad/s

5. A 217 kg satellite travels initially in an approximately circular orbit 600 km above the surface of the earth. Determine its speed and period. Ans. 7.7×10^3 ms^{-1}, 5.7×10^3 s

SELF-ASSESSMENT

1. An artificial satellite of the earth appears to be stationary for an observer fixed on the earth (such a satellite is called Geo-stationary satellite). Find the distance of the orbit from the centre of the earth, if satellite is at the earth's equatorial plane.
   (Hint : Use Kepler's third law. Take period of the moon as 27 days and radius of moon's orbit as 3.84×10^5 km.)

2. How does the earth's rotation affect the apparent weight of a body when the body is moved from equator towards any one of the poles.
   (Hint : The centripetal force necessary for the rotation of the body is provided by the force of gravity)

3. State Kepler's laws of planetary motion and Newton's laws of gravitation.

4. (*a*) Show that to escape from the atmosphere of a planet a necessary condition for a molecule is that it has a speed $v^2 > 2gR$; where g is the acceleration due to gravity of the planet and R is the distance of the molecule from the centre of the planet.

   (*b*) Determine the escape speed from the earth for an atmospheric particle 1000 km above the earth's surface.

   (*c*) Do the same for the moon and the sun

5. Mars has two moons, the larger one orbits at a distance of 6.9 Martian radius from the centre of mars and its period is 30 hrs. The smaller one has a period of 7.6 hrs, find its distance from the centre of mars.

6. (*a*) With what horizontal speed must a satellite be projected at 160 km above the surface of the earth so that it will have a stable circular orbit about the earth ? (Take the earth's radius as 6400 km).

   (*b*) What will be its period of rotation ?

7. Show that centripetal acceleration is given by v^2/r.

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UNIT 9

Physics for Class XI

RIGID BODY ROTATION

F-15187

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is ninth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers

At this stage this unit is an experimental edition. I shall welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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New Delhi
20 May 1982

INTRODUCTION

In the preceding unit you studied about circular motion where a point mass had been considered to move with constant speed In the present unit you will study in detail about the dynamics of rigid body rotation. Here you will come across the fact that distribution of mass round the axis of rotation rather than mass itself plays dominating role in case of rotation. You may have observed that when the blades of a fan rotate the point mass which is farthest moves fastest while the point mass closest to the centre moves slowest In circus you might have watched that an acrobat leaves a swing, with his body almost completely stretched out. Then he quickly folds his arms and legs with the result that his body rotates faster. Again when he stretches his limbs out the angular velocity of his body reduces Obviously while folding and unfolding the limbs the moment of inertia (which gives distribution of mass round the axis of rotation) changes. The details of moments of inertia of a body and one of the most important conservation principles which is widely used in all branches of science and engineering, will be studied in the present unit In the next unit you will study the effects of motion of individual particles of matter

OBJECTIVES

To be able to

1. Derive relations between the angle of rotation, angular velocity and angular acceleration
2. Derive moment of inertia in some simple cases, i e. for a uniform ring and a circular disc about an axis
3 Solve problems using 1 and 2 above.
4 State, derive, and apply the principle of conservation of angular momentum.
5 Solve problem involving torque and kinetic energy.
6. State analogies between linear and angular motions

SUGGESTED READING MATERIAL

1 Equations of motion of a rigid body

In Unit 3 you have studied the translatory motion of a particle. Here we will study the rotation of a *rigid body* about a fixed axis. A rigid body is one whose constituent particles retain their relative positions even in motion, i e. a rigid body can never be deformed Fig. 1 shows a rigid body rotating about a fixed axis passing through point O. Each particle of the body will describe a circle with centre at O As the body rotates particles closer to point O move through smaller distances than the particles away from point O. But if we fix a line OP in the body, all particles on this axis will move through same angle θ with respect of an arbitrary axis OZ fixed in space. Since the body is rigid its position at a given time can be specified by angle θ. The equations of motion appear in simple form when θ is expressed in *radians* (rad).

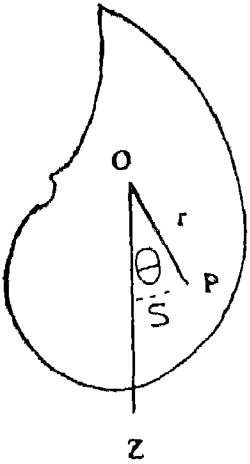


Fig 1. The angular displacement of a rigid body is the angle θ between an axis OP fixed in the body and an axis OZ fixed in space

The angle θ in radian is the ratio of length of the arc s and radius r

$$\theta = \frac{s}{r} \qquad ...(1)$$

In one complete revolution the length of the arc is equal to circumference of the circle $2\pi r$ Therefore, the angle described in one revolution is equal to $\frac{2\pi r}{r}$ rad $= 2\pi$ rad $= 2\times3.14$ rad $= 6.28$ rad. Since one revolution is 360°

$$360^\circ = 6.28 \text{ rad}$$

$$180^\circ = 3.14 \text{ rad} = \pi \text{ rad}$$

$$90^\circ = \frac{\pi}{2} \text{ rad}$$

$$30^\circ = \frac{\pi}{6} \text{ rad}$$

When the length of the arc is equal to the radius ($s = r$), $\theta = 1$ rad. Thus one radian is that angle which subtends an arc equal to the radius.

As the body rotates the axis OP rotates along with it and θ changes. We have defined angular velocity ω in Unit 8 as the rate of change of angle θ.

$$\omega = \frac{d\theta}{dt} \qquad . (2)$$

If the angular velocity is changing, we define angular acceleration α as

$$\alpha = \frac{d\omega}{dt} \qquad . .(3)$$

Since ω is measured in rad/s, α will be measured in rad/s^2

Questions

In Unit 3 you studied the linear motion of a particle. The position of a particle was specified by its distance from the origin of coordinate axis x which is analogus to θ in this unit. The linear velocity v is then analogus to angular velocity ω. What is the quantity analogus to accelaration a, is in this unit

We will assume constant angular acceleration and derive equations of motion just like we did in Unit 3 for linear motion. Let us assume that we start counting time when axis OP coincides with OZ i.e. $\theta = 0$, and angular velocity is ω_0. Mathematically at $t = 0$, $\theta = 0$, and $\omega = \omega_0$. At a later time t, the angle is θ and angular velocity is ω. Rearranging Eq. (3) and integrating we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha \, dt$$

This gives

$$\omega - \omega_0 = \alpha t$$

$$\text{or } \omega = \omega_0 + \alpha t$$

Rearranging and integrating Eq. (2) we get ...(4)

$$\int_0^\theta d\theta = \int_0^t \omega\, dt$$

or using Eq. (4)

$$\int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)\, dt$$

$$= \int_0^t (\omega_0\, dt + \alpha t\, dt)$$

$$= \omega_0 \int_0^t dt + \alpha \int_0^t t\, dt$$

or $$\theta = \omega_0\, t + \tfrac{1}{2}\, \alpha\, t^2 \qquad \ldots(5)$$

Again Eq. (3) can be written as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\ \frac{d\theta}{dt} = \omega\ \frac{d\omega}{d\theta}$$

$$\int_{\omega_0}^{\omega} \omega\, d\,\omega = \int_0^\theta \alpha\, d\,\theta$$

or $$\omega^2 - \omega_0^2 = 2\, \alpha\, \theta \qquad \ldots(6)$$

Question

In the linear case the equation analogus to Eq. (4) is $v = u + at$. What are the equations analogus to Eq. (5) and Eq. (6).

We notice that the relations derived are similar to those of linear motion and are applicable to a rigid body only under constant angular acceleration.

Questions

1. Deduce the relation

$$\theta = (\omega + \omega_0)\, t/2$$

where symbols have their usual meaning.

2. Referring to Fig. 1, we stated that pure rotation of a rigid body can be described by the motion of any one of the particles (such as P) that make it up. Does that mean we can include even the particles that are on the axis of rotation ?

3. Could the angular quantities θ, ω and α be expressed in terms of degrees instead of radians in the kinematical equations ?

Example 1

What is the angular velocity in rad/s of a flywheel making 300 revolutions per minute ?

Solution

In one revolution the angle described is 360° or 2π radians

$$\omega = 300 \frac{rev}{min}$$

$$= \frac{300\ (2\pi \text{ rad})}{(60\text{s})}$$

$$= 10\pi \text{ rad/s}$$

Example 2

A wheel is turned from rest through an angle of 200 radians is 8s. (*a*) What is its angular acceleration ? (*b*) What will be its angular velocity and displacement after 16s from start ?

Solution

(*a*) For motion from start to $t = 8$ s

$$\theta = \omega_0\, t + \tfrac{1}{2}\, \alpha\, t^2$$

$$200 \text{ rad} = 0 + \tfrac{1}{2}\, \alpha\, (8 \text{ s})^2$$

$$\text{or} \qquad \alpha = 6.25 \text{ rad/s}^2$$

(*b*) For motion from start to $t = 16$ s

$$\omega = \omega_0 + \alpha\, t$$

$$\text{or } \omega = 0 + (6.25 \text{ rad/s}^2)\ 16 \text{ s}$$

$$= 100 \text{ rad/s.}$$

To determine the angular displacement we use Eq (6).

$$\omega^2 - \omega_0^2 = 2\,\alpha\,\theta$$

$$(100\text{ rad/s})^2 - 0 = 2 \times 6.25\text{ rad/s}^2 \times \theta$$

$$\theta = 800\text{ rad}$$

Problem

A flywheel makes 240 rmp. Calculate the angular retardation necessary to bring it to rest in 20s, and the angle through which it turns.

2. Rotational kinetic energy and moment of inertia

Fig 2 shows a rigid body rotating with uniform angular velocity ω about an axis. Let the mass points constituting the body be m_1, m_2,...... m_n at distances of r_1, r_2, ...r_n from the axis of rotation. Their linear velocities are v_1, v_2, ... v_n respectively. Kinetic energies of these particles are therefore $\frac{1}{2} m_1 v_1^2$, $\frac{1}{2} m_2 v_2^2$, ... $\frac{1}{2} m_n v_n^2$ respectively. The total kinetic energy of the body T will be the sum of kinetic energies of individual particles constituting the body. Hence

$$T = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \ldots\ldots + \tfrac{1}{2} m_n v_n^2 \qquad \ldots(7)$$

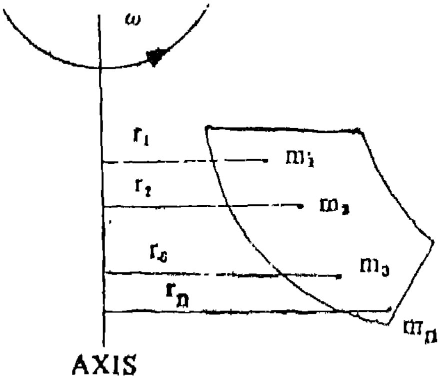


Fig. 2. A rigid body rotating about an axis. The body is made up of a large number of masses m_1, m_2, m_3 ...... having linear velocities v_1, v_2, v_3......

The relationship between linear velocity v and angular velocity ω can be found by differentiating both side of Eq (1)

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} \quad (r \text{ is constant})$$

$$\text{or } \omega = \frac{v}{r} \qquad . \ (8)$$

Therefore

$$\begin{aligned} v_1 &= r_1\,\omega \\ v_2 &= r_2\,\omega \\ &\vdots \\ v_n &= r_n\,\omega \end{aligned} \qquad \ldots(9)$$

Substituting Eq. (9) in Eq. (7), we get

$$\begin{aligned} T &= \tfrac{1}{2}\,m_1\,(r_1\,\omega)^2+\tfrac{1}{2}\,m_2\,(r_2\,\omega)^2+\ \ldots.+\tfrac{1}{2}\,m_n\,(r_n\,\omega)^2 \\ &= \tfrac{1}{2}\,(m_1\,r_1^2+m_2\,r_2^2+\,\ldots\ldots..+m_n\,r_n^2)\omega^2 \end{aligned}$$

$$T = \left(\sum_{i=1}^{n} m_i\,r_i^2\right)\omega^2$$

Let us put $$I = \sum_{i=1}^{n} m_i\,r_i^2 \qquad \ldots(10)$$

Hence $$T = \tfrac{1}{2}\,I\,\omega^2 \qquad ..\,(11)$$

The quantity $I = \sum_{i=1}^{n} m_i\,r_i^2$ is known as the *moment of inertia* of the body about the axis shown in Fig. 2. If there is a continuous distribution of mass points, comprising a body, about the axis of rotation, then considering a small mass dm at a distance of r from the axis of rotation one can say that the moment of inertia of the small mass dm is $r^2\,dm$ Hence moment of inertia of the whole body about the axis of rotation is

$$I = \int r^2\,dm \qquad \ldots(12)$$

Eq. (11) gives the rotational kinetic energy of a body. Eq. (11) is the analogous expression for translational kinetic energy ($= \frac{1}{2}\,mv^2$, where v is the linear velocity and m is the mass of the body which is a measure of inertia in case of translational motion). *One can define moment of inertia of a body as the measure of inertia in case of rotational motion.* It is clear from Eq. (10) that moment of inertia not only depends upon the masses of the particles of which a body is composed but also upon their distances from the axis. Therefore moment of inertia of a body about an axis is constant. However, for the same body, the moment of inertia is different for different axes of rotation. It has the same value for the same body and same axis of rotation irrespective of the state of motion. Its dimensions are ML^2. The units for the moment of inertia are kg m^2 in the SI Units.

Our next task is to calculate the moment of inertia for a few simple bodies.

2.1 Moment of inertia of a uniform ring about an axis through its centre and perpendicular to the plane of the ring

Let us suppose O is the centre of the ring of mass M and radius r through which the axis is passing perpendicular to the plane of the ring.

All mass points m_1, m_2, ...m_n of which ring is composed are at a distance of r from the axis. Moments of inertia of these mass points are $m_1 r^2$, $m_2 r^2$,. .. $m_n r^2$ respectively. Hence, the moment of inertia of the entire ring is given by

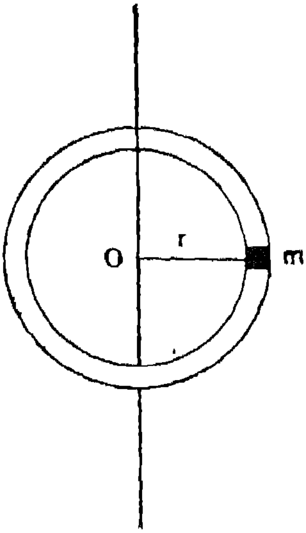


Fig. 3. A ring is made up of large number masses such as m.

$$I = m_1 r^2 + m_2 r^2 + m_3 r^2 + \ldots + m_n r^2$$
$$= (m_1 + m_2 + m_3 + \ldots + m_n) r^2$$

But $m_1 + m_2 + m_3 + \ldots + m_n = M =$ whole mass of the ring

$$\text{Thus } I = Mr^2 \qquad \ldots(13)$$

2.2 The moment of inertia of a uniform circular disc about an axis through its centre perpendicular to the plane of the disc

Consider a uniform circular disc of total mass M, and radius r. Since the disc is uniform we define mass per unit area

$$\varrho = \frac{M}{\pi r^2}$$

Let us consider two circles of radii x and $(x+dx)$ as shown in Fig 4 Then shaded area in Fig. 4 looks like a ring. The area of this ring will be equal to (since thickness of the disc is assumed to be very small that is why we are talking about area, otherwise one must talk about volume).

$$\pi (x+dx)^2 - \pi x^2 \qquad \ldots(14)$$
$$= 2\pi x \, dx$$

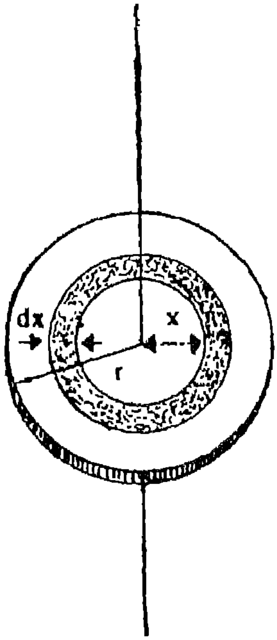


Fig. 4 A ring element of thickness dx is used to determine the moment of inertia of the disc.

Where the small term $\pi\,(dx)^2$ has been neglected

Mass of the circular ring is given by

$$dm = 2\,\pi\,x\,\varrho\,dx$$

Thus from the earlier derivative, one can say that moment of inertia of the ring considered is

$$\begin{aligned} dm\,x^2 &= (2\pi\,x\,\varrho\,d\,x)\,x^2 \\ &= 2\,\pi\,\varrho\,x^3\,dx \end{aligned} \qquad \text{...(15)}$$

The moment of inertia of disc now can be found by dividing the entire disc into many such small rings and then summing out their moments of inertia, which effectively means integration of Eq. (15) within the limits 0 to r.

Hence the moment of inertia of the circular disc about the axis through O is

$$\begin{aligned} &\int_0^r 2\,\pi\,\varrho\,x^3\,dx \\ &= 2\,\pi\,\varrho\,[x^4/4]_0^r \\ &= \tfrac{1}{2}\,\pi\,\varrho\,[r^4 - 0^4] \\ &= \tfrac{1}{2}\,\pi\,\varrho\,r^4 \\ &= \tfrac{1}{2}\,\pi\left(\frac{M}{\pi\,r^2}\right)r^4 \\ &= \tfrac{1}{2}\,M\,r^2 \end{aligned}$$

We considered a circular ring between radii x and $(x+dx)$, because distance between any point in the ring and centre of the disc is constant. The circular ring is a sort of mass point dm.

Questions

1. Is the moment of inertia a scalar or vector?
2. If two circular discs of same weight and thickness are made from metals having different densities, which disc will have the larger rotational inertia about its central axis?
3. A person can distinguish between a raw egg and a hard boiled one by spinning each one on a table. Explain how?

4. In the calculation of I of the circular disc about the axis in Fig. 4, mass point dm was taken to be circular ring within radii x and $(x+dx)$. Can you take mass point of any other shape ? Explain.

Example 3

A dumbell shown in Fig. (5) is rotated with angular velocity 1 rad/s first about axis AB and then about axis CD. Compute its K.E. in each case.

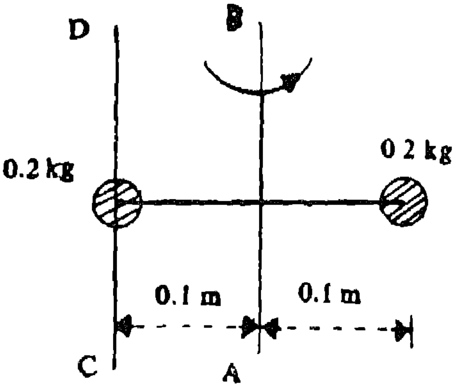


Fig. 5

Solution

Since the system consists of discrete masses, Eq. (10) can be used to compute the moment of inertia about axis AB

$$I_{AB} = \sum_i m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2$$

$$= (0.2 \text{ kg}) (0.1 \, m)^2 + (0.2 \text{ kg}) (0.1 \, m)^2$$

$$= 0.004 \text{ kg m}^2$$

The kinetic energy in this case is

$$K.E. = \tfrac{1}{2} I_{AB} \, \omega^2$$

$$= \tfrac{1}{2} (0.004 \text{ kg m}^2) (1 \text{ rad/s})^2$$

$$= 0.002 \text{ joule}$$

Let us compute the moment of inertia about axis CD

$$
\begin{aligned}
I_{CD} &= \sum_i m_i r_i^2 \\
&= m_1 r_1^2 + m_2 r_2^2 \\
&= (0.2\text{ kg})(0) + (0.2\text{ kg})(0.2\text{ m})^2 \\
&= 0.008\text{ kg m}^2
\end{aligned}
$$

and

$$
\begin{aligned}
K.E. &= \tfrac{1}{2} I_{CD}\,\omega^2 \\
&= \tfrac{1}{2}(0.008\text{ kg m}^2)(1\text{ rad/s})^2 \\
&= 0.004\text{ joule}
\end{aligned}
$$

Example 4

A 50 g meter stick of 2 cm width and negligible thickness is rotated with 2 rad/s about a vertical axis AB passing through its centre (Fig. 6.). Calculate its moment of inertia and notational kinetic energy.

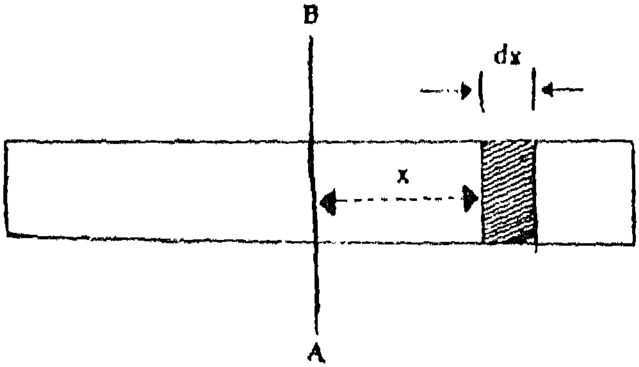


Fig. 6

Solution

Since the mass of the stick has continuous distribution Eq. (12) rather than Eq. (11) will be used to compute the moment of inertia (why ?).

The symmetry of the problem suggests we should consider a rectangular mass element dx at a distance x from the axis. The meter stick can be considered as made up of large number of such rectangles. Since the mass is uniformly distributed, the mass per unit area is

$$\rho = \frac{\text{Mass}}{\text{Area}} = \frac{0.05\text{ kg}}{0.02\text{ m} \times 1\text{m}} = 2.5\text{ kg/m}^2$$

The mass of the element dx is

$$\begin{aligned} dm &= \varrho\ (0.02\ \text{m} \times dx) \\ &= (0.05\ dx)\ \text{kg/m} \\ I_{AB} &= \int dm\ x^2 \\ &= \left[0.05 \int_{-0.50\text{m}}^{0.50\text{m}} x^2\, dx \right] \text{kg/m} \\ &= 0.05 \left[\frac{(0.50\ \text{m})^3}{3} - \frac{(-0.50\text{m})^3}{3} \right] \text{kg/m} \\ &= 4.6 \times 10^{-4}\ \text{kg m}^2 \end{aligned}$$

$$\begin{aligned} \text{Rotational kinetic energy} &= \tfrac{1}{2} I\, \omega^2 \\ &= \tfrac{1}{2} \times 4\ 6 \times\ 10^{-4} \times 2^2\ J \\ &= 9.2\ \text{J} \end{aligned}$$

3. Angular momentum, torque, and conservation of angular momentum

3.1 Angular momentum

While studying linear motion (Unit 7) momentum was defined as product of mass and velocity

$$p = mv$$

Analogously we define angular momentum L of a rigid body (such as a rotating flywheel) as product of moment of inertia and angular velocity.

$$L = I\,\omega \qquad \text{...(16)}$$

In the case of a single particle moving in a circle (Fig. 7) $I = mr^2$ and $\omega = v/r$, therefore, its angular momentum is

$$\begin{aligned} l &= I\,\omega \\ &= (mr^2)\left(\frac{v}{r}\right) \\ &= m\,v\,r \\ &= r\,p \end{aligned} \qquad \text{. (17)}$$

Vectorially l can be expressed as a vector product

$$\mathbf{l} = \mathbf{r} \times \mathbf{p} \qquad \text{..(18)}$$

Thus moment of momentum is called angular momentum.

Question

For a body moving in a horizontal circle of radius r with linear momentum $\boldsymbol{P}$ (= $m\boldsymbol{v}$) Fig. 7, what should be the direction and magnitude of angular momentum ?

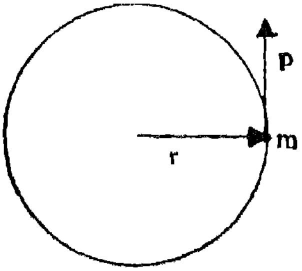


Fig. 7 A mass m moving in a circle of radius r The momentum p is in a tangential direction

Let us now differentiate Eq. (18) with respect to time t, we get

$$\frac{d\boldsymbol{l}}{dt} = \frac{d}{dt}(\boldsymbol{r} \times \boldsymbol{p})$$

$$= \frac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} + \boldsymbol{r} \times \frac{d\boldsymbol{p}}{dt} \qquad \text{...(19)}$$

Here $\frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}$ (velocity of the body). Since $\boldsymbol{v}$ and $\boldsymbol{p}$ are in the same direction

$$\frac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} = \boldsymbol{v} \times \boldsymbol{p} = 0$$

Thus in Eq. (19)

$$\frac{dl}{dt} = \overset{r}{\boldsymbol{r}} \times \frac{dp}{dt}$$

But from Newton's second law, $\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$

Hence

$$\frac{dl}{dt} = \boldsymbol{r} \times \boldsymbol{F} \qquad \text{...(20)}$$

Eq. (20) states that rate of change of angular momentum is the vector product of $\boldsymbol{r}$ and $\boldsymbol{F}$ where $\boldsymbol{r}$ is the distance (vector). The product ($\boldsymbol{r} \times \boldsymbol{F}$) is defined as the torque τ

$$\tau = \boldsymbol{r} \times \boldsymbol{F} \qquad \text{...(21)}$$

The rotational effect which is produced by torque depends not only on the magnitude of the force but also on the distance and orientation of the force.

Hence Eq. (20) can be written as

$$\frac{dl}{dt} = \tau \qquad ...(22)$$

We have seen in linear motion that when a force acts on a body, its momentum changes and rate of change of linear momentum is measure of the force applied. Likewise when a troque acts on a rotating body, its angular momentum L changes and it is clear from Eq. (22) that rate of change of angular momentum of the body is a measure of the torque applied.

3.2 Conservation of angular momentum

Let us again cosider a rigid body made of a large number of particles of masses m_1, m_2, m_3 . for each of which l_1, l_2, l_3 .. are the angular momenta. Hence

$$\frac{dl_1}{dt} = \tau_1; \frac{dl_2}{dt} = \tau_2, \frac{dl_3}{dt} = \tau_3; \qquad (23)$$

where $\tau_1, \tau_2, \tau_3, \ldots$. are the torques acting upon the particles. Since each angular momentum is along the axis of rotation (why ?), the total angular momentum of the body will be

$$L = l_1 + l_2 + l_3 + \,.\,. \qquad ...(24)$$

Taking the time derivative we get

$$\frac{dL}{dt} = \frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} + \ldots\ldots$$

$$\frac{dL}{dt} = \tau_1 + \tau_2 + \tau_3 + \ldots. \qquad ..(25)$$

or

$$\frac{dL}{dt} = \Gamma$$

where the resultant torque $\Gamma = \tau_1 + \tau_2, + \tau_3 + \quad \ldots$

If the resultant torque Γ is zero, then

$$\frac{dL}{dt} = 0$$

$$L = \text{constant.} \qquad .(26)$$

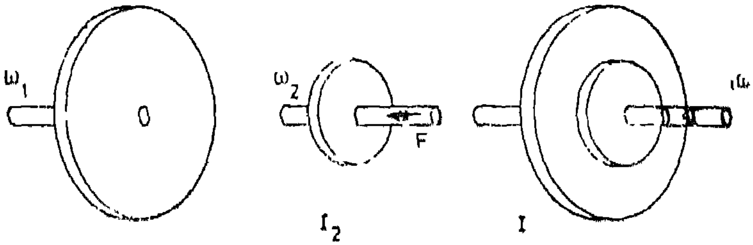
i.e total angular momentum of the system is conserved. This is known as the principle of conservation of angular momentum, which can be stated as follow:

When the resultant external torque acting upon a system of paticles is zero, the total (vector sum) angular momentum of the system remains constant.

It may be noted that this principle is similar to the principle of conservation of linear momentum.

Illustration 1

Let us apply this principle in the case of rotating discs as shown in Fig 8.



(a) BEFORE

(b) AFTER

Fig. 8

Initially two discs are rotating independently with angular velocities ω_1 and ω_2. A force F is applied along the axis of the smaller disc, to join the discs as shown in Fig. 8. Since F does not produce any external torque (why ?), the total angular momentum must be conserved.

Total angular momentum before joining them (see Eq. (16)) is

$$= I_1 \omega_1 + I_2 \omega_2 \qquad (28)$$

Total angular momentum after joining them

$$= I \omega \qquad ...(29)$$

Where in this case I = Moment of inertia of the system of two discs

$$= I_1 + I_2 \qquad ..(30)$$

Hence conservation principle of angular momentum demands

$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega \qquad ..(31)$$

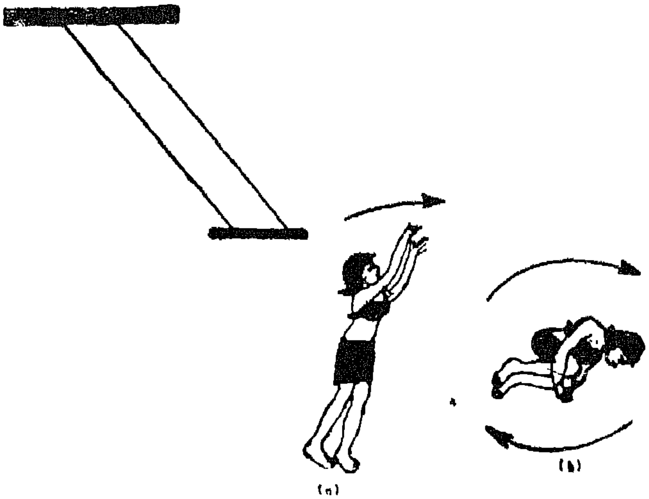


Fig 9

Illustration 2

A person performing acrobatics is also an illustration of conservation angular momentum. If the person stretches out his hands and legs as shown in Fig 9(a), after leaving a swing the value of I increases considerably. Thus his angular speed decreases. Whereas when he draws his hands and legs in the inward direction as shown in Fig. 9(b), the value of I decreases and his angular speed ω increases considerably Thus his angular momentum ($I\omega$) in both the cases remains constant.

Questions

1. What are the dimensions of angular momentum? Can you find any significance in the fact that they are same as those of energy multiplied by time?
2. State Newton's three laws of motion in words suitable for rotating bodies.

Example 5

A 25 kg flywheel has a radius of 0.2 m. It is making 240 rpm. What is the torque necessary to bring it to rest in 20 s? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force?

Solution

Initial angular velocity of the flywheel

$$\omega_i = 240 \frac{\text{rev}}{\text{min}}$$

$$= 240 \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$8\pi \text{ rad/s}$$

Final angular velocity $\omega_f = 0$.

$$\omega_f = \omega_i + \alpha t$$

$$\alpha = -\frac{\omega_i}{t} = -\frac{8\pi \text{ rad/s}}{20\text{s}} = -\frac{2\pi}{5} \text{ rad/s}^2$$

But moment of intertia $= \frac{1}{2} MR^2 = \frac{1}{2} \times 25 \times (0.2)^2 \text{ kg m}^2$

$= \frac{1}{2} \text{ kg m}^2$

Hence torque $\Gamma = I\alpha$

$$= \frac{1}{2}\left(-\frac{2\pi}{5}\right) \text{kg m}^2 \text{ rad/s}^2$$

$$= -\frac{\pi}{5} \text{ Nm}$$

Again force F is applied tangentially, thus magnitude of torque due to the applied force

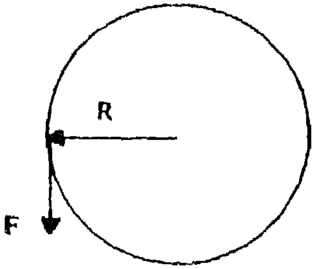


Fig. 10

$$|\Gamma| = Fr \sin 90°$$

$$\text{or } F = \frac{\Gamma}{r}$$

$$= \frac{\pi}{5} \frac{\text{Nm}}{0.2\text{m}}$$

$$= \pi \text{ N}$$

Example 6

What is the horse power transmitted by a shaft turning at 30
torque is found to be 800 Nm.

Solution

$$\omega = 3000 \frac{\text{rev}}{\text{min}}$$

$$\omega = 3000 \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$= 100 \text{ rad/s}$$

$$P = \Gamma \omega$$

$$= 800 \text{ Nm} \times 100 \text{ rad/s}$$

$$= 80{,}000 \text{ watt}$$

$$= \frac{80{,}000}{746} \text{ HP}$$

$$= 107 \text{ HP.}$$

Problem

A cord is wound around the circumference of a wheel of diameter 0.3 m. The axis of the wheel is horizontal. A 0.5 kg mass is attached at the end of the cord and it is allowed to fall from rest. If the weight falls 1.5 m in 4 s, what is the angular acceleration of the wheel? Also, find out the moment of inertia of the wheel.

(Ans. 1.25 rad/s^2, 0.588 kg m^2)

4 Work done by torque, and analogy between rotational motion and translational motion

Let two equal and opposite forces F,—F act tangentially on the rim of a disc from the extremeties of a diameter. Let the disc be free to rotate about an axis through its centre O perpendicular to the plane of the disc. If the couple rotates the body through a very small angle θ, the displacement s of each force is given by $s = r\,\theta$. Since θ is very small the displacement s is almost in the same direction as F. Therefore work done by a single force is $Fs = Fr\,\theta$. The total work done by two forces constituting the couple is $2Fr\,\theta$. But $2rF$ is the moment of the couple or torque Γ.

Hence work done by the torque

$$W = \Gamma \theta \qquad \ldots(34)$$

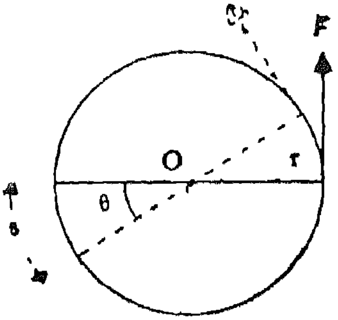


Fig. 11

TABLE

Rectilinear motion		Rotation about a fixed axis	
Displacement	s	Angular displacement	θ
Velocity	$v = \frac{ds}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	M	Rotational inertia	I
Force	$F = Ma$	Torque	$\Gamma = I\alpha$
Work	$W = \int F dx$	Work	$W = \int \Gamma \, d\theta$
Kinetic energy	$\frac{1}{2} m v^2$	Kinetic energy	$\frac{1}{2} I \omega^2$
Power	$P = Fv$	Power	$\Gamma \omega$
Linear Momentum	Mv	Angular momentum	$I \omega$

The power delivered

$$P = \frac{dW}{dt} = \frac{\Gamma\, d\theta}{dt} = \Gamma\omega \qquad (3$$

Eq. (34) is yet another example where analogy between linear and rotational motions are quite apparent

In the table given we compare the translational motion of a rigid body along a straight line with the rotational motion of a rigid body about a fixed axis.

ASSIGNMENT

1. The radial distance from a given axis at which the mass M of a body could be concentrated without altering the rotational inertia of the body about that axis is called the *radius of gyration*. Let k represent radius of gyration. Show that

$$k = \sqrt{\frac{I}{M}}$$

I = Moment of inertia of the body about the given axis

Hence show that a solid cylinder of mass M and radius R is equivalent to a thin hoop of mass M and radius $R/\sqrt{2}$.

2. What is the angular velocity in rad/s of a turbo-jet aircraft engine at 1500 rpm? The engine is accelerated in preparation of take off At the end of 10 s it rotates at a frequency of 8000 rpm. Find the angular acceleration assuming it to be a constant and also the total angle it has turned through during 10 s interval

Ans 157 rad/s, 84 rad/s^2, 4189 rad

3. A rotor can be accelerated from rest to an angular velocity of 50,000 rpm in 0.2 s. The moment of inertia of the rotor about its spin axis is 385 g cm^2 (a) what is the angular acceleration as the rotor is brought up to that speed? (b) what torque is needed to produce this angular acceleration?

Ans. 2.6×10^1 rad/s^2, 1 Nm

4. Compute the moment of inertia of a meter stick of mass 50 g and width 2 cm and negligible thickness, about a vertical axis passing through one of its edges (Fig. 12).

Ans 1.7×10^{-2} kg m^2

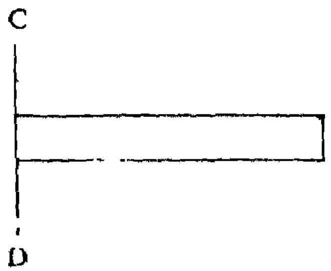


Fig. 12

5 Compute the moment of inertia about axes AB, CD and passing through 10 kg mass perpendicular to the plane of the paper of the system shown in Fig. 13

Ans. 10 kg m^2, 10 kg m^2, 15 kg m^2

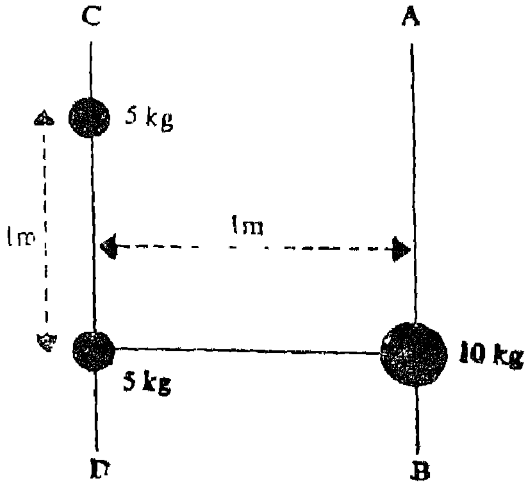


Fig. 13

6 Write down the rotational analogies of the following:

(*a*) $s = ut + \frac{1}{2} gt^2$

(*b*) $F = ma$

(*c*) Power = Fv

(*d*) Work = Fs

7. The oxygen molecule has a total mass of 5.30×10^{-26} kg and a rotational inertia of 1.94×10^{-46} kg. m^2 about an axis through the centre perpendicular to the line joining the atoms. Suppose that such a molecule in a gas has a mean speed of 500 m/s and that its rotational kinetic energy is 2/3rd of its translational kinetic energy. Find the average angular speed.

Ans. 6.75×10^{12} rad/s

8. A man stand at the centre of a turntable, holding his arms extended horizontally with a 5 kg mass in each hand. The turntable is rotated about a vertical axis making 2 rev/s. The man then suddenly drops his hands to his sides. Calculate his new angular velocity. Assume the moment of inertia of the man to be constant and equal to 0.2 kg m^2; original and final distances of the masses from the axis 1 m and 0.3 m.

Ans. 18.5 rev/s

SELF-ASSESSMENT

1. Derive an expression for the moment of inertia of a thin rod about an axis through one end and normal to its length.

2. A 0.1 kg mass tied to one end of a string is whirled around in a horizontal circle of radius 0.5 m with a velocity of 0.1 m/s as in Fig 14. The other end of the string passes through a hollow pipe and a force F acts on it. The radius is suddenly decreased to 0.25 m. Calculate the final velocity of the mass.

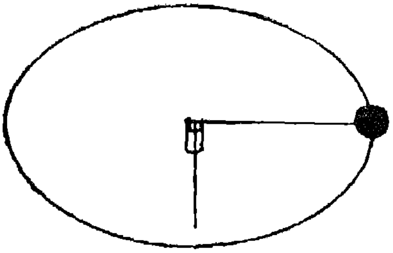


Fig 14

3. An automobile engine develops 100 HP when rotating at a speed of 1800 rpm. What torque does it deliver ?

4. Assume the earth to be a sphere of uniform density

   (a) What is its rotational kinetic energy? Take the radius of the earth to be 6.4×10^3 km and mass of the earth to be 6.0×10^{24} kg. (Given $I = 2/5\ MR^2$).

   (b) Suppose this energy could be harnessed for man's use. For how long could the earth supply 1 kw of power to each of the 4×10^9 persons on the earth?

5. Give the rotational analogies of the following:

   (a) $v = u + at$

   (b) Velocity

   (c) Linear momentum

   (d) Kinetic energy

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UNIT 10

Physics for Class XI

KINETIC THEORY OF GASES I

F-15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

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PREFACE

This unit is tenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr. K. J. George who is working as a Senior Project Fellow helped me in editing this unit.

At this stage this unit is an experimental edition. I shall welcome any criticism and suggestion foi the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Tiaining. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
NCERT

New Delhi
20 May 1982

INTRODUCTION

You know that matter is made up of molecules and due to thermal energy, they are in a state of constant motion. Molecules also interact among themselves. If the thermal agitation is much more dominant than force of interaction between molecules, they fly apart. This phase of matter is called gaseous state, where molecules move randomly. In the present unit you will study about this random motion of molecules in the gaseous states. The direct experimental evidences of this haphazard motion of molecules are phenomena such as diffusion and Brownian motion. In this unit you are also going to apply the laws of mechanics you studied in previous units. Task will not be very gigantic since we will calculate pressure for an assembly of molecules for which detailed life histories of individual atoms are not important. In the next unit we will continue with the same and learn about the concept of temperature in the context of kinetic theory.

OBJECTIVES

To be able to

1. Differentiate between macroscopic and microscopic definitions of an ideal gas.

2 State basic assumptions (microscopic description of ideal gas) for the development of kinetic theory of gases and know their consequences.

3. Distinguish between mean, mean square, and root mean square (rms) velocities and to calculate their values.

4. Derive the expression for pressure exerted by an ideal gas on the basis of assumptions mentioned in objective 2.

5. Answer questions and solve problems based on objectives 2, 3, and 4.

6. Deduce important gas laws from the expression for pressure exerted by a perfect gas.

SUGGESTED READING MATERIAL

1. Ideal gas model—macroscopic and microscopic description

The world of molecules is totally chaotic. Thus if one wants to study the motion of molecules in an assembly of the them, on an individual basis, with the help of laws of mechanics; one is sure to find difficulty. But if one is interested in the collective (or macroscopic) behaviour of molecules, no matter how much weird the behaviour of single molecule be, problem will not be utterly hopeless. At this juncture, the terms *macroscopic* and *microscopic* need a little elaboration. *Macroscopic* simply means, pertaining to larger domains, e.g. a whole assembly of a large number of molecules, problems concerning celestial bodies and so on. Microscopic world is confined to quantities concerning minute entities, e.g. volume occupied by a molecule, mass and charge of subatomic particles and so on Now coming back to a system of an assembly of gas molecules, we know that the over all state of the system can be described by over all pressure, volume, temperature, etc Hence such quantities are macroscopic variables and they are calculated by averaging techniques, averaging, of course, must be carried out over all minute components (molecules) of the system, which are microscopic entities.

However, to initiate a study, a model is needed which helps in explaining and understanding a large body of facts, both qualitatively and quantitatively. The simplest model available is that of an ideal gas, whose macroscopic definition (the definition in terms of macroscopic variables such as pressure, volume, temperature, etc.) will be as follows:

> Given a definite mass (say, n moles) of a gas at pressure P, temperature T, and volume V, experiments have shown that these variables are related to each other by a simple relationship
>
> $$PV=nRT \qquad \ldots(1)$$
>
> (where R is Universal gas constt.$=8.314$ J/mole K)

then the gas is called an ideal gas; which obeys the relation (1) under all conditions. Eq. (1) is called the *equation of an ideal gas* In reality there is no such thing as an ideal gas; but it remains a useful and simple concept connected with reality by the fact that all real gases approach the ideal gas behaviour at very low densities.

From the microscopic point of view, we define an ideal gas by making assumptions to be given shortly, which are also regarded as the basic assumptions of kinetic theory of gases. In here, we will be applying relatively simple mathematical averaging technique to enhance our knowledge about the pressure exerted by a gas.

Following are the fundamental assumptions of kinetic theory of gases (or the definition of an ideal gas from microscopic point of view).

1. Gases are composed of a large number of molecules and molecules of a particular gas are identical in all respects.

2. The size of a molecule is negligible compared with the average distance between two molecules. Thus actual volume occupied by the molecules is extremely small in comparision with the volume occupied by the gas.

3. Gas molecules are perpetually in a state of chaotic motion; they move in all possible directions with all possible velocities.

4. In course of this haphazard motion, molecules collide with themselves and also with the walls of the containing vessel. The collisions are perfectly elastic resulting into no loss of energy or momentum.

5. The time occupied by an encounter between two molecules is very small in comparison with the interval elapsing between successive encounters.

6. The pressure exerted by a gas on the walls of containing vessel is due to bombardment of molecules on the walls.

7. Gas molecules exert practically no force of mutual interaction.

Strictly speaking none of the above assumptions are true for real gases; nevertheless the model thus developed on the basis of these assumptions tolerably satisfactorily explains a large number of experimental facts about gases some of which you are going to study.

2. Expression for pressure exerted by an ideal gas

Consider a cube of side l and (Fig. 1) volume V (where $V = l^3$) enclosing an assembly of n number of gas molecules with mass of each molecule

m. The molecules are in radom motion Their velocities at an instant of time are $c_1, c_2, c_3, \ldots, c_n$. These velocities can be resolved into three

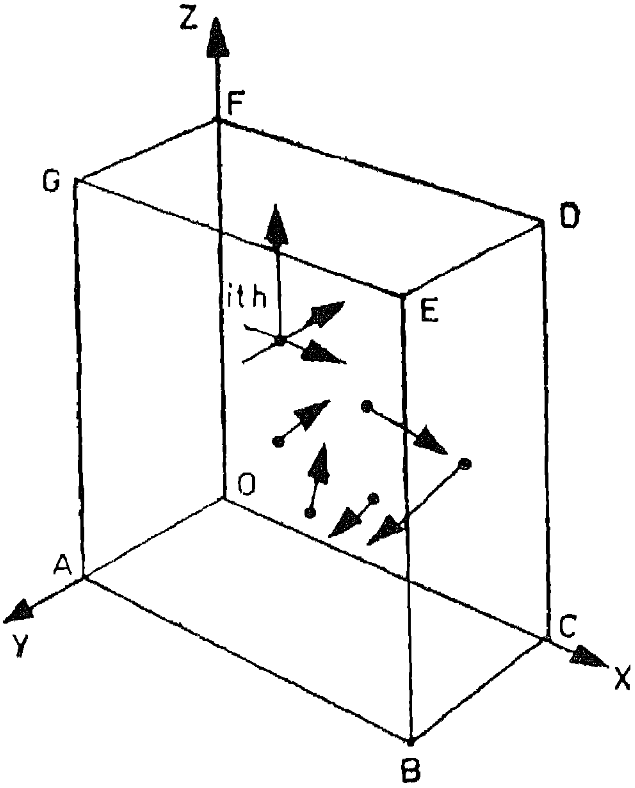

Fig. 1

components along the three mutually perpendicular X, Y and Z axes. Consider i^{th} molecule with velocity c_i whose components along X, Y and Z axes are u_i, v_i and w_i respectively; then (Detailed discussion is given in appendix A Go through it and then proceed further)

$$c_i^2 = u_i^2 + v_i^2 + w_i^2 \qquad \text{.. (2)}$$

or for all the n molecules

$$\sum_{i=1}^{n} c_i^2 = \sum_{i=1}^{n} (u_i^2 + v_i^2 + w_i^2) \qquad \text{..(3)}$$

You have learnt in a previous unit that if velocity resolution is taking place along X and Y axes, then

$$c_i^2 = u_i^2 + v_i^2$$

(Eq. (3) is the generalisation of the above in three dimensions).

Let us now suppose i^{th} molecule starts off the face OAGF, moves straight ahead along X axis, collides with the face BCDE and comes back to its starting point You must have realised that the assumption itself is too unreal; nevertheless let us move ahead with the same. On striking BCDE, change in momentum of the i^{th} molecule along X axis will be equal to

$$mu_i - (-mu_i) = 2\,mu_i \tag{4}$$

(Since on collision, only direction of momentum is reversed, there is no change in the magnitude). In course of its entire journey, molecule describes a distance $2\,l$, assuming that its velocity along X axis remains fixed (i.e. u_i) total time taken to accomplish this voyage is

$$(2\,l/u_i) \tag{5}$$

Hence number of collisions between i^{th} molecule and face BCDE in one second is

$$(u_i/\,2\,l) \tag{6}$$

Since, in one collision change of momentum is $(2\,mu_i)$, for $(u_i/2\,l)$ collisions taking place in one second, change of momentum will be $(2\,mu_i) \times (u_i/2\,l)$ equal to (mu_i^2/l) This will be the change in momentum for i^{th} molecule along X axis in one second. Thus for all the n molecules rate of change of momentum along X axis will be

$$\frac{m}{l}\sum_{i=1}^{n} u_i^2 \tag{7}$$

From Newton's second law this is the force exerted along X axis (Say, F_x)

$$\text{Thus } F_x = \frac{m}{l}\sum_{i=1}^{n} u_i^2 \tag{8}$$

Hence pressure P_x along X axis will be

$$P_x = F_x/l^2 = \frac{m}{l^3}\sum_{i=1}^{n} u_i^2$$

or

$$P_x = \frac{m}{V}\sum_{i=1}^{n} u_i^2 \tag{9}$$

Similarly pressure along Y axis will be

$$P_y = \frac{m}{V}\sum_{i=1}^{n} v_i^2 \tag{10}$$

and that along Z axis will be

$$P_z = \frac{m}{V}\sum_{i=1}^{n} w_i^2 \qquad (11)$$

Let us again make an assumption that

$P_x = P_y = P_z = P$ (say); which is not too bad (why ?).

Then

$$3P = \frac{m}{V}\sum_{i=1}^{n} (u_i^2 + v_i^2 + w_i^2) \qquad (12)$$

From Eq. (3)

$$3P = \frac{m}{V}\sum_{i=1}^{n} c_i^2 \qquad (13)$$

Now mean square velocity $\overline{c^2}$ of the molecules will be defined as (Detailed discussion is given in appendix B)

$$\overline{c^2} = \frac{c_1^2 + c_2^2 + \cdots + c_n^2}{n} = \sum_{i=1}^{n} c_i^2 / n \qquad (14)$$

$$\text{Hence } \sum c_i^2 = n\overline{c^2} \qquad (15)$$

Using this in Eq. (13) we get

$$PV = \tfrac{1}{3}\, mn\overline{c^2} \qquad (16)$$

or

$$P = \tfrac{1}{3}\,\frac{mn}{V}\,\overline{c^2} \qquad (17)$$

$$\text{But } \rho \text{ (Density of the gas)} = \frac{mn}{V} \qquad (18)$$

$$\text{Hence } P = \tfrac{1}{3}\,\rho\,\overline{c^2} \qquad (19)$$

This is the expression for pressure we wanted to calculate.

From Eq (19), we get

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\varrho}} \qquad (20)$$

$\sqrt{\overline{c^2}}$ is called the root mean square (*rms*) velocity of molecules

2.1 Discussion and questions

There are many apparent flaws which you may point out in the derivation of Eq. (19) We shall now try to explain some of them or put as questions.

1. In Eq. (19), $\overline{c^2}$ is the mean square velocity of molecules where we have earlier stated, $c_1, c_2, \ldots c_n$ are the velocities of molecules at an instant of time One may then think that Eq (19) is not valid for all times. Now if $\overline{c^2}$ is calculated over a few molecules and for a short time, then, of course, fluctuations may occur in the expression for $\overline{c^2}$ given by Eq (20). But if the number of molecules is very large and average is taken over a time large enough, then fluctuations can be ignored.

2. In deriving Eq. (19) we have assumed walls of the container to be elastic Actually, walls may be inelastic. In practice this makes no difference as long as the walls are at the same temperature as the gas; why ?

3. We have assumed, $P_x = P_y = P_z$. But along Z axis force of gravity is acting. What justification is there in neglecting the effect of gravity ?

4. In Eq. (19), we have implicitly assumed that the force exerted by the molecules on the walls of the container is steady in time. What justification is there ?

5. We derived Eq. (19) by neglecting collisions between particles is Eq (19) true even when we consider collisions. Answer is *yes*, because of the exchange of velocities in an elastic collision between identical particles there will be always one molecule which will collide with the wall BCDE with momentum mu_i corresponding to one that left the wall OAGF. Also the time spent during collision

is negligible compared to the time spent between collisions Hence neglecting friction introduces no serious error.

6. Our derivation of Eq (19) is for a container cubical in shape. Will the result be still valid if we take container of any shape ?

Answer is obviously yes: to verify it try to rededuce Eq (19) by assuming container to be a cuboid or of any shape

2.2 Consequences of basic assumptions of kinetic theory of gases

In order to understand the various assumptions of kinetic theory of gases in a better way, let us put few questions. You must try to answer them before proceeding further (answers must be qualitative)

1. In one of our assumptions, we regard molecules to be mass points because its size was assumed to be negligible in comparison with the average distance between two molecules. In reality molecules do occupy definite volume, hence how will Eq. (19) get modified?
2. We assumed collisions between molecules to be perfectly elastic and of negligible duration. What effect will you have in Eq. (19) if collisions are inelastic and not of negligible duration ?
3. If you don't have an assembly of large number of molecules, what effect are you going to expect on the expression of pressure given by Eq. (19) ?
4. If force of interaction between molecules is finite, how is the expression for pressure going to get affected ?

Example 1

Calculate the root mean square speed of hydrogen molecule at 0°C and 1 atm pressure, assuming hydrogen to be ideal gas

(Given at 0°C, density of hydrogen $= 9 \times 10^{-5}$ g/cm^3)

Solution

Density of hydrogen $\rho = 9 \times 10^{-5}$ g/cm^3

$$= 9 \times 10^{-2} \text{ kg/m}^3$$

Pressure $P = 1$ atm $= 1.01 \times 10^5$ N/m^2

Hence,

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{9 \times 10^{-2}}}\ m/s$$

$$= 1840 \text{ m/s}$$

Example 2

(*a*) Compute the *rms* speed of an argon atom at room temperature (27°C)

(*b*) At what temperature will *rms* speed be half that value ? Twice that value ?

(Given, molecular mass of argon 40 g/mole, $R = 8.3$ J/mole K)

Solution

We know for 1 mole of a gas

$$PV = \tfrac{1}{3} M\overline{c^2} \text{ (where } M = \text{ Molecular mass)}$$

But $\quad PV = RT$ (for 1 mole)

$$RT = \tfrac{1}{3} M\overline{c^2}$$

$$\text{R M S speed } \sqrt{\overline{c^2}} = \sqrt{\frac{3RT}{M}}$$

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3 \times 8.3 \times 300}{40/1000}} \text{ m/s}$$

$$= 432 \text{ m/s}$$

We notice, $\overline{c^2} \propto T$

At 300 K, $\sqrt{\overline{c^2}}$ is 432 m/s; let at temp. T_1, $\sqrt{\overline{c^2}}$ be (432/2) m/s. Then

$$\frac{300 \text{ K}}{T_1} = \left(\frac{432}{432/2}\right)^2$$

$$T_1 = 75 \text{ K}$$

Similarly at T_2 (say), $\sqrt{\overline{c^2}}$ is (2×432) m/s.

$$\text{Then} \qquad \frac{300\ \text{K}}{T_2} = \left(\frac{432}{432 \times 2}\right)^2$$

$$T_2 = 1200\ \text{K}$$

Example 3

How many molecules are there in 10^{-6} m^3 of air at 300 K having a pressure of 10^5 N/m^2 ?

$$\text{Given, Boltzmann constant } k = \frac{R \text{ (Universal gas constant)}}{N \text{ (Avogadro's number)}}$$

$$= 1.38 \times 10^{-23} \text{ joule/molecule K}$$

Solution

We know that

$$PV = nRT$$

$$\text{where } n = \text{No. of moles} = \frac{m \text{ (mass of the gas)}}{M \text{ (Molecular mass)}}$$

Let n_1 = No. of molecules (to be calculated) present in the given amount of gas

If μ = Mass of each molecule

Then $m = n_1 \mu$

$M = N\mu$; (N = Avogadro's number).

Thus

$$PV = \frac{(n_1)(\mu)}{N\mu} R$$

$$\text{or,} \quad PV = (n_1 N)\, RT$$

$$\text{or} \quad n_1 = \frac{PV}{(R/N)\,T}$$

But $R/N = k$ = Boltzmann constant

$$\text{Hence } n_1 = \frac{PV}{kT}$$

Now $P = 10^5 \text{ N/m}^2$

$V = 10^{-6} \text{ m}^3$

$T = 300 \text{ K}$

$k = 1.38 \times 10^{-23}$ joule/ molecule K

$$\text{Hence } n_1 = \frac{10^5 \times 10^{-6}}{300 \times 1.38 \times 10^{-23}}$$

$$n_1 = 2.5 \times 10^{19}$$

Problems

1 The density of nitrogen is 1.25 g/litre at S.T P. calculate the *rms* speed of its molecules.

2 Given that the universal gas constant (R) is 8.3 joule/mole K and the atomic mass of chlorine 35.5 ; find the *rms* speed of chlorine molecules at 0°C.

3. Show how to find the root mean square speeds of He and Ar molecules at 40°C from that of oxygen molecules (460 m/s at 0°C) The molecular mass of oxygen is 32g/mole, of Ar 40 g/mole, and of He 4 g/mole.

3. Deduction of laws describing behaviour of gases

So far we have set up a model, viz. ideal gas made assumptions from microscopic angle and on the basis of that we derived an expression for pressure exerted by a perfect gas. Let us now examine how far these all explain the various laws describing the behaviour of gases. If we can show them to be valid for our ideal gas model, microscopic definition of ideal gas will be consistent with macroscopic definition.

Gas Laws

One can write Eq. (16) as

$$PV = (2n/3) \tfrac{1}{2} m \overline{c^2} \qquad ..(21)$$

where $(\frac{1}{2} m\overline{c^2})$ is the average kinetic energy of translation per molecule When we heat a gas its molecules tend to move faster suggesting that kinetic energy of molecules depends on temperature. The average kinetic energy of translation per molecule can be assumed to be proportion to temperature, i e.

$$\frac{1}{2}\, m\overline{c^2} \propto T \qquad (22)$$

Hence, we get from Eq. (21) and Eq. (22)

$$PV = \text{Constant} \quad T \qquad .\,(23)$$

(Since $(2n/3)$ is a constant)

Thus at constant temperature (T) we get

$$PV = \text{Constant (Boyle's Law)} \qquad .\,(24)$$

At constant pressure (P), we get

$$V = \text{Constant} \quad T$$

$$V \propto T \text{ (Charle's law)} \qquad (25)$$

At constant volume (V), we get

$$P = \text{Constant} \quad T$$

$$P \propto T \text{ (Law of pressure)} \qquad (26)$$

3.1 Avogadro's hypothesis

Avogadro's hypothesis states that at the same condition of pressure and temperature, equal volumes of all gases will contain equal number of molecules.

To deduce this let us consider two different assemblies of two different types of gas molecules, maintained at the same condition of pressure P and temperature T and their volumes V are same.

Let n_1, n_2 = Number of molecules present in the two different gases.

m_1, m_2 = Masses of each molecule of the two different gases.

$\overline{c_1^2}$ $\overline{c_2^2}$ = Mean square velocities of gas molecules in two different gases.

From Eq. (19),

$$PV = \frac{1}{3}\, m_1 \; n_1 \; \overline{c_1^2} \qquad \ldots(27)$$

and $$PV = \tfrac{1}{3} m_2 n_2 \overline{c_2^2} \qquad \text{...(28)}$$

Hence $$\tfrac{1}{3} m_1 n_1 \overline{c_1^2} = \tfrac{1}{3} m_2 n_2 \overline{c_2^2} \qquad \text{.(29)}$$

or, $$(\tfrac{1}{2} m_1 \overline{c_1^2})\, n_1 = (\tfrac{1}{2} m_2 \overline{c_2^2})\, n_2$$

Since average kinetic energy of translation per molecule $\frac{1}{2} m\overline{c^2}$ is proportional to temperature T and temperature of both the gases are same so

$$\tfrac{1}{2} m_1 \overline{c_1^2} = \tfrac{1}{2} m_2 \overline{c_2^2} \qquad \text{..(30)}$$

Substituting Eq. (30) in Eq. (29), we get

$$n_1 = n_2$$

which is Avogadro's hypothesis.

3.2 Graham's law of diffusion

The law states that relative velocity of diffusion of gases are inversely proportional to the square root of their relative densities.

Suppose the gas 1 is diffusing into the gas 2. When they get thoroughly intermingled, the pressure exerted by gas 1 will be equal to that exerted by gas 2 , let the pressure now be P. Using pressure equation one finds

$$P = \tfrac{1}{3} \rho_1 \overline{c_1^2} \qquad \text{...(31)}$$

and $$P = \tfrac{1}{3} \rho_2 \overline{c_2^2} \qquad \text{...(32)}$$

where ρ_1, ρ_2 = Densities of the gases 1 and 2 respectively.

$\overline{c_1^2}$, $\overline{c_2^2}$ = Mean square velocities of gases 1 and 2 respectively

Thus,

$$\frac{\overline{c_1^2}}{\overline{c_2^2}} = \frac{\rho_2}{\rho_1} \qquad \text{...(33)}$$

or

$$\sqrt{\overline{c_1^2}} \,/ \sqrt{\overline{c_2^2}} = \sqrt{\rho_2/\partial_1} \qquad \text{(34)}$$

Let u_1, u_2 = Rate of diffusion for gas 1 and gas 2. It is known that

$$\frac{u_1}{u_2} = \frac{\sqrt{\overline{c_1^2}}}{\sqrt{\overline{c_2^2}}} \text{ or } \frac{u_1}{u_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad ..(35)$$

which is Graham's law of diffusion.

Important constants

We should like to jot down the values of important constants which we have referred to (in the present unit) and will be referring to (in the next unit)

1 Universal gas constant R = 8.3 J/mole K

2. 1 atomic mass unit (a m.u.) = 1.66×10^{-27} kg

3. Avogadro's number N = 6.02×10^{-23} molecules/mole

4. Boltzmann constant k = 1.38×10^{-23} J/molecule K

5. 1 standard atmosphere = 0.76 m of Hg

= 1.01×10^{6} dynes/cm^2

= 1.01×10^{5} N/m^2

6. 1 N/m^2 = 10 dynes/cm^2.

APPENDIX A

We will show how the square of the velocity of a molecule is equal to the sum of the squares of its components, i.e.

$$c^2 = u^2+v^2+w^2.$$

We know from our concept of vectors in two dimensions that any vector, e.g. the velocity vector c, can be resolved into components along the two axes in such a way that

$$c^2 = u^2+v^2.$$

This is easily understood from Fig. 2a where Pythagoras theorem is applied after completing the parallelogram of velocity components.

Let us see what happens to the same vector in three-dimensions where we have three axes. Let the axes be so chosen as shown in Fig. 2*b*.

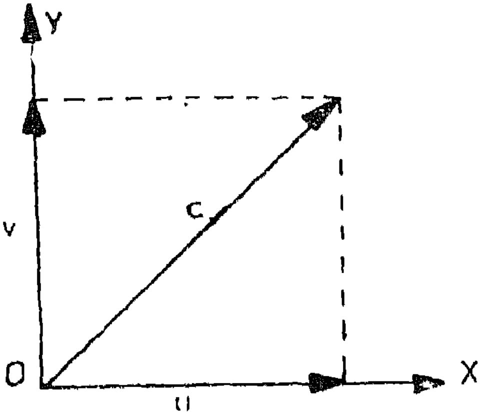

Fig 2*a*

From the tip A of the vector *c* a perpendicular AB is dropped on to the XY plane. OB is joined. This line OB in the XY plane must be perpendicular to AB forming the right angled triangle OBA. AB along Z axis represents the component *w* of *c* along Z axis. From the △ OBA using Pythagors theorem we have,

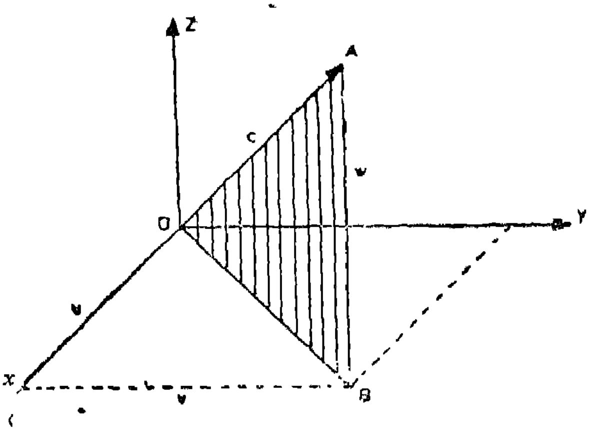

Fig. 2*b*

$$c^2 = (OB)^2 + w^2 \qquad \text{..(A.1)}$$

Again if perpendiculars are dropped from B to X and Y axes then we see from the Fig. 2*b* that

$$(OB)^2 = u^2 + v^2 \qquad \text{...(A.2)}$$

combining (A.1) & (A.2), we get

$$c^2 = u^2 + v^2 + w^2 \qquad ..(A.3)$$

APPENDIX B

We must be able to distinguish between the different terms such as mean, mean square and root mean square (*rms*) speeds. For an assembly of n gas molecules with individual molecular velocities $c_1, c_2, c_3, \ldots c_n$;

the mean speed $\bar{c}$ will be simply

$$\bar{c} = \frac{c_1+c_2 \quad +c_n}{n} = \left(\sum_{i=1}^{n} c_i/n\right) \qquad .(B.1)$$

Mean square speed of gas molecules is the mean of the squares of speeds of individual molecules and is denoted by $\overline{c^2}$. Thus

$$\overline{c^2} = \frac{c_1^2 + c_2^2 + c_3^2 \;.\; c_n^2}{n}$$

$$= \left(\sum_{i=1}^{n} c_i^2 / n\right) \qquad .. (B.2)$$

Root means square speed c_{rms} will then be given by

$$c_{rms} = \sqrt{\overline{c^2}} \qquad . (B.3)$$

It should be noted that *rms* speed and mean speed are different. Perhaps, an example will clarify it.

Consider three molecules with speeds 0 5 km/s, 1 km/s and 2 km/s respectively.

$$\text{Mean speed } \bar{c} = \frac{0\ 5+1+2}{3} = 1.2 \text{ km/s.}$$

$$\text{Mean square speed } \overline{c^2} = \frac{(0.5)^2+(1)^2+(2)^2}{3} = .175 \text{ km}^2/\text{s}^2$$

$$\text{RMS speed } (\sqrt{\overline{c^2}}) = \sqrt{1.75} \text{ km/s} = 1.32 \text{ km/s.}$$

$$\text{Hence } \bar{c} \;.\; \sqrt{\overline{c^2}} :: \;10 \;.\; 11$$

ASSIGNMENT

1. The density of helium at 1 atmospheric pressure and 0°C is 1.78×10^{-4} g/cm^3. What is the *rms* speed of the He molecules ?

   Ans. 1.3×10^5 cm/s

2. (*a*) How is pressure related to density and kinetic energy of molecules ?

   (*b*) Six molecules of a gas have velocities 1 km/s, 2 km/s, 3 km/s, 4 km/s, 5 km/s and 6 km/s respectively. Compute the average speed and *rms* speed of the molecules. Compare the two values.

   Ans 2.5 km/s, 3.9 km/s

3. The mass of an oxygen molecule is 5.31×10^{-23} g and at 0°C it has an *rms* speed of 4.16×10^4 cm/s. How many molecules of the gas are there in 1 cm^3 at 0°C and at 1 atmospheric pressure ?

   2.7×10^{19}

4. At 273°F and 1.00×10^{-2} atmospheric pressure the density of a gas is 1.24×10^{-5} g/cm^3.

   (*a*) Find *Crms* of the gas molecules

   (*b*) Find the moleculai mass of the gas and identify it
   Given Boltzmann constant $k = 1.38 \times 10^{-23}$ J/molecules **K**

   Ans. 4.9×10^4 cm/s, 7×10^{-23} g, chlorine

5. The mass of the H_2 molecule is 3.32×10^{-24} gm. If 10^{23} hydrogen molecules per second strike 2.0 cm^2 of the wall at an angle 45° with the normal when moving with a speed of 10^5 cm/s, what pressure do they exert on the wall ?

   Ans. $2.35 \times 10^3 \dfrac{N}{m^2}$

6. (*a*) What effect does the assumption of extremely small collision time compared to time spent by a molecule between collisions have on the derivation of gas laws ?

   (*b*) What is the consequence of the fact that no appreciable forces act on the molecules except during a collision ?

SELF-ASSESSMENT

1. State the basic assumptions required for the development of the kinetic theory of gases.

2. Derive the expression

$$P = \frac{1}{3} \frac{mN\,\overline{c^2}}{V}$$

on the basis of kinetic theory of gases where P is the pressure exerted by N number of molecules each having mass m contained in a vessel of volume V, $\overline{c^2}$ in the mean square velocity of the molecules.

3. (*a*) What effect do elastic collisions have on the derivation of gas laws.

   (*b*) Why is the assumption "total number of molecules being very large" so important ?

4. (*a*) If the *rms* speed of the molecules of a gas is 400 m/s when the pressure is 700 mm of Hg, what is the density of the gas?

   (*b*) Five molecules of a gas have velocities 1.5 km/s, 1.2 km/s, 1.3 km/s, 1.6 km/s and 1.4 km/s respectively. Compute the average speed, mean square speed and *rms* speed for them.

5. Derive the following gas laws for an ideal gas on the basis of kinetic theory of gases (*a*) Boyle's law (*b*) Charle's law (*c*) Avogadro's law (*d*) Graham's law of diffusion and (*e*) Law of pressures.

6. (*a*) If the *rms* speed of hydrogen molecules at 0°C is 1840 m/s, what is the *rms* speed at 100°C ?

   (*b*) Using the ideal gas equation, determine the value of universal gas constant R. (Given one gram molecule of a gas at S.T.P. occupies 22.4 litres).

7. Oxygen gas at 273 K and 1.00 atmospheric pressure is confined to a cubical container of 10 cm side. (*a*) How long does it take a typical molecule to cross the container ? (*b*) Compare the change in gravitational potential energy of an oxygen molecule falling through the height of the box with its mean kinetic energy.

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UNIT 11

Physics for Class XI

KINETIC THEORY OF GASES II

F-15187

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PREFACE

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In preparing this unit, I was assisted by some teachers. Dr. K. J. George who is working as a Senior Project Fellow.

At this stage this unit is an experimental edition. I shall welcome any criticism and suggestion for the improvement of this unit.

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New Delhi
20 May 1982

INTRODUCTION

In the previous unit we have discussed the derivation of an expression for pressure (a macroscopic variable) exerted by an ideal gas from microscopic considerations. In the present unit we will determine the relation between temperature (yet another macroscopic variable) and the mean kinetic energy of gas molecules. Apart from that we will also discuss about equipartition theorem of kinetic energy, two different specific heats of gases and their relationship The concepts introduced in these two units on kinetic theory of gases will be extremely useful for the subsequent units.

OBJECTIVES

To be able to

1. Set up a relation between temperature and mean kinetic energy of an ideal gas.
2. Find the number of degrees of freedom of different molecules.
3. State and derive the law of equipartition of energy and know its consequences.
4. Derive the relationship between the two specific heats of a gas on the basis of kinetic theory.
5. Solve problems based on objectives 1 and 4.

SUGGESTED READING MATERIAL

1. Relation between temperature and mean kinetic energy of an ideal gas

In the preceding unit we have deduced the expression (for monoatomic gas)

$$PV = \frac{1}{3} mn \overline{c^2}$$

where P Pressure exerted by an ideal gas

V = Volume of the gas

m = Mass of each molecule

n Number of molecules

$\bar{c}^2$ = Mean square velocity of molecules

Let us consider one mole of an ideal gas, then

$n = N$ (Avogadro's number)

Hence equation (1) becomes

$$PV = \frac{1}{3} mN\bar{c^2} \qquad \text{..(2)}$$

For 1 mole of an ideal gas, from equation of state, we know that

$$PV = RT \qquad \text{...(3)}$$

Equations (2) and (3) give

$$RT = \frac{1}{3} mN\overline{c^2}$$

$$RT = (2/3) (\tfrac{1}{2} mN\bar{c^2}) \qquad \text{...(4)}$$

In the previous unit while discussing deduction of gas laws, we have stated that

$\frac{1}{2} m\overline{c^2}$ = Average kinetic energy of translation per molecule

Hence $\frac{1}{2} mN\overline{c^2}$ $= E =$ Average kinetic energy of 1 mole

Thus from Eq. (4)

$$RT = (2/3)\, E;\ \text{or},\ E = (3/2)\, RT \qquad \text{..(5)}$$

Naturally average kinetic energy of translation per molecule

$$\epsilon = \frac{E}{N} = \frac{3}{2}\left(\frac{R}{N}\right)T \qquad \text{...(6)}$$

We know

$$\frac{R}{N} = \text{k} = \text{Boltzmann constant}$$

Hence

$$\epsilon = \frac{3}{2} kT \quad .. (7)$$

Equations (5) and (7) clearly show that the temperature is intimately related to the kinetic energy of a molecule. Whenever a gas gains energy, its temperature rises

Questions

1. The gas kinetic temperature in the upper atmosphere (see Eq. (5)) is of the order of 1000 K. It is quite cold up there. Explain this paradox.

2. Interpret absolute zero in terms of kinetic energy of gas molecules.

Example 1

What is average kinetic energy of a molecule of monoatomic gas at a temperature of 27°C ?

Solution

We know average kinetic energy of a molecule

$$= \frac{3}{2} KT$$

where $k = 1.38 \times 10^{-23}$ joule/molecule K

$T = (27 + 273)$ K $= 300$ K

Hence K.E. of a molecule $= \frac{3}{2} kT$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \text{ joule}$$

$$= 6.2 \times 10^{-21} \text{ joule}$$

Problem

Calculate the total translational kinetic energy of 3 moles of an ideal monoatomic gas at 217°C.

2.1 Degrees of freedom

Suppose an independent particle is in space. Its exact position can be specified by three numbers. In the cartesian co-ordinate system, these three numbers are the familiar X, Y and Z coordinates There is no way in which the position of an independent particle in space can be described by less than three numbers (x, y, z) as shown in Fig. 1.

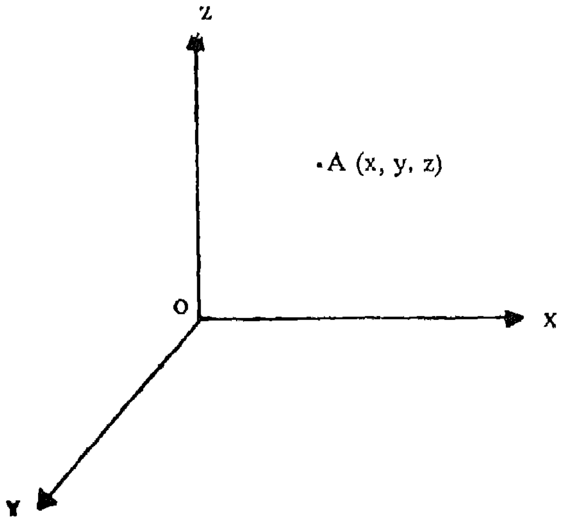

Fig. 1

If this particle happens to be moving then at any time its velocity can be described by three components. These three components are usually denoted by u, v and w (Fig. 2) along the X, Y and Z directions respectively. Once again it is impossible to specify the state of motion (i.e. velocity) of an independent particle in free space without these three numbers (the velocity components). Notice that, if we know the velocity we can always find the position of the point at any instant by the relations.

$$x = u\,t,\ y = v\,t \text{ and } z = w\,t$$

Therefore for an independent particle whether at rest or in motion, we need only three numbers to describe its position at any time.

In order to express these ideas concisely we consider that an independent particle has three degrees of freedom

Now consider two independent particles in space. Obviously considered together they will have six degrees of freedom (x_1, y_1, z_1) for the first and (x_2, y_2, z_2) for the second.

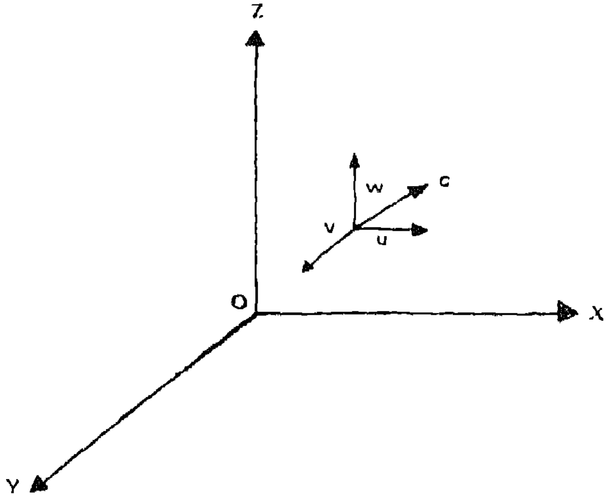

Fig. 2

Suppose the two particles are not independent and one is connected to the other by a rigid rod. If the length of this rod is l, then a relationship exists between the three co-ordinates of one particle and the three co-ordinates of the other. This relationship is

$$l^2 = (x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2$$

or

$$l = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

In order to describe this system of two particles connected by a rigid rod, we require only five numbers. For instance, suppose we know the values of x_1, y_1, z_1, x_2 and y_2 (z_2 is not known). We can always find out z_2 using the relation

$$z_2 = z_1 - \sqrt{l^2 - (x_1-x_2)^2 - (y_1-y_2)^2}$$

So if we know any five of the six co-ordinates $(x_1, y_1, z_1, x_2, y_2, z_2)$, we can always find the sixth. We say, therefore that this system of two particles connected by a rigid rod has five degrees of freedom

In general, we define the number of degrees of freedom of a dynamical system as the minimum number of co-ordinates required to completely specify the position and configuration of that dynamical system

2.2. Rule for finding the number of degrees of freedom

Our example for a two particle system has clearly illustrated that the number of degrees of freedom of a system is actually equal to the total number of co-ordinates required to define the positions of the elements of the system minus the number of independent relations existing between the elements.

Let us see how we can use this rule to find the number of degrees of freedom for a system of two particles and three particles

Two particle system (Fig 3)

Number of particles = 2

Total number of co-ordinates for 1 particle = 3

Total number of co-ordinates for 2 particles = 6

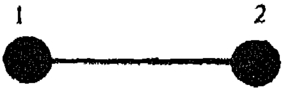

Fig 3

Number of bonds between 2 particles = Number of independent relations = 1

Hence degrees of freedom = 6 — 1 = 5

Three particle system (*Fig. 4*)

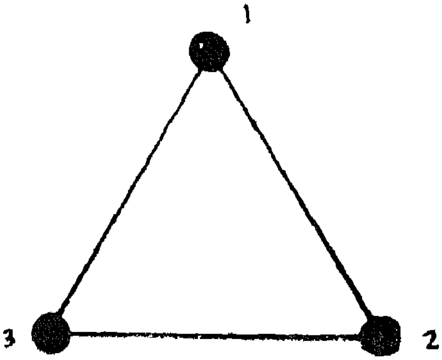

Fig. 4

Total number of co-ordinates for 3 particles = $3 \times 3 = 9$

Total number of bonds = Number of independent relations = 3

Hence degrees of freedom $= 9 - 3 = 6$

3. The law of equipartition of energy

In the preceding unit, the ideal gas model was considered in three dimensions. There are 3 directions of motion (X, Y and Z axes), which are equally probable for the gas molecules. This implies that average value of the components of velocity c (i.e. u, v and w) along the three directions should be equal or for a molecule all the three directions are equivalent.

$$\bar{u} = \bar{v} = \bar{w}$$

or,

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3}\overline{c^2}$$

Since

$$c^2 = u^2 + v^2 + w^2$$

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Multiplying throughout by $\frac{1}{2}m$, where m is the mass of a molecule we have

$$\frac{1}{2}m\,\overline{u^2} = \frac{1}{2}m\,\overline{v^2} = \frac{1}{2}m\,\overline{w^2}$$

But $\frac{1}{2}m\,\overline{u^2} = \epsilon_x$ = Total mean kinetic energy of a molecule along X axis

Therefore $\epsilon_x = \epsilon_y = \epsilon_z$.

But the total mean kinetic energy of a molecule is

$$\epsilon = \frac{3}{2}kT$$

where

k = Boltzmann constant

T = Absolute temperature

and since

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\epsilon_x = \epsilon_y = \epsilon_z = \frac{1}{2}kT$$

Since three velocity components u, v and w correspond to the three degrees of freedom of the molecule, we can conclude that

Total kinetic energy of a dynamical system is equally divided among all its degrees of freedom and it is equal to $\frac{1}{2}$ kT per degree of freedom.

This is the law of equipartition of energy and was deduced by James Clark Maxwell.

So far we have been considering only translational motion Let us now consider rotation too, of course, for a monoatomic molecule, we have only translational motion because they are not capable of rotation (although they can spin about any one of the three mutually perpendicular axes if it is like a finite sphere) Hence for one molecule of a gas total energy $\epsilon = \frac{3}{2}kT$.

For a diatomic molecule we can suppose it to be two spheres joined by a rigid rod Such a molecule can rotate about any one of three mutually perpendicular axes However, the rotational inertia about an axis along the rigid rod is negligible compared to that about an axis perpendicular to the rod, so that rotational energy consists of two terms such a $\frac{1}{2}\ I\omega_y^2$ and $\frac{1}{2}\ I\ \omega_z^2$

Now the spatial description of the centre of mass of a diatomic gas molecule will require three coordinates. Thus for a diatomic gas molecule having both rotational and translational motion,

$$\epsilon = 3\left(\tfrac{1}{2}\ kT\right) + 2\left(\tfrac{1}{2}\ kT\right)$$

$$= \frac{5}{2}kT$$

For triatomic gases, each molecule contains three spheres joined together by rods (we can suppose so), that the molecule is capable of rotating energetically about each of three mutually perpendicular axes. Hence for a triatomic gas molecule having both translational and rotational motion, energy ϵ will be

$$\epsilon = 3\left(\tfrac{1}{2}\ kT\right) + 3\left(\tfrac{1}{2}\ kT\right)$$

$$= 3\ kT$$

(Here three degrees of freedom for the translation of the centre of mass of the molecule and three for rotation along three mutually perpendicular axes).

Example 2

Find the number of degrees of freedom and associated energy of a polyatomic molecule as shown in Fig. 5 (Hypothetical).

Solution

Total number of coordinates required to define the positions of atoms 1,2,3 and 4 = 3 × 4 = 12

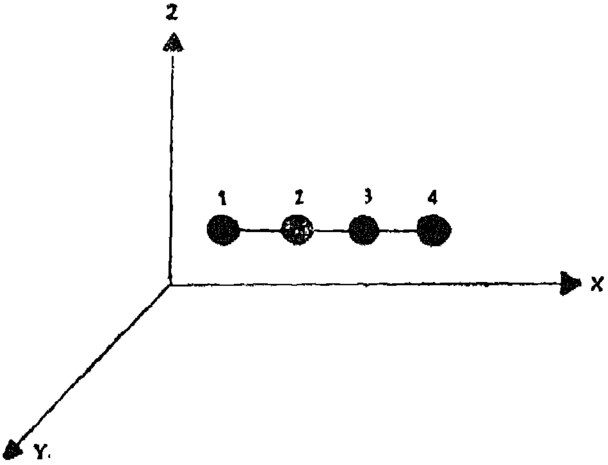

Fig. 5

Number of independent relations = 3.

Number of degrees of freedom = 12—3 = 9.

(Note that for any number of atoms interconnected by rods, there will always be three translational degrees of freedom because any atom moving in a particular direction (with three degrees of freedom) will be responsible for the motion of the other atoms connected with it, in the same direction. The remaining degrees of freedom, therefore, have to be rotational degrees).

Number of translational degrees of freedom = 3

Number of rotational degrees of freedom = 9 — 3 = 6

The energy (ϵ) associated with this polyatomic molecule

$$= 3\left(\tfrac{1}{2}kT\right)+6\left(\tfrac{1}{2}kT\right)$$

$$= \frac{9}{2}kT$$

4. Specific heat of gases (C_p and C_v)

Let us set up a relation between C_p and C_v, where

C_p = Molar specific heat of a gas at constant pressure

C_v = Molar specific heat of a gas at constant volume

C_v is defined as the quantity of heat required to raise the temperature of one mole of a gas at constant volume by one degree, i.e.

If E_T = Total energy of gas at TK

E_{T+1} = Total energy of gas at $(T+1)$ K

Then $C_v = E_{T+1} - E_T$ (At constant volume)

For monoatomic gas, thus

$$C_v = \frac{3}{2}R\,(T+1) - \frac{3}{2}RT = \frac{3}{2}R$$

For diatomic gas

$$C_v = \frac{5}{2}\,F\,(T+1) - \frac{5}{2}\,RT = \frac{5}{2}\,R$$

For triatomic gas

$$C_v = 3\,R\,(T+1) - 3\,RT = 3R$$

Let us consider a cylinder fitted with a smooth movable piston and containing some gas as shown in Fig. 6. Let the volume of the gas be V initially.

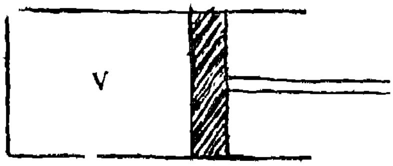

Fig 6

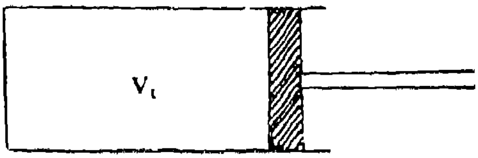

Fig. 7

Now if the piston is moved to a new position as shown in Fig. 7, then the volume of the gas changes to V_1.

Change in volume $= V_1 - V$

If the process of changing the volume of the gas is carried out in such a way that the pressure P remains constant, the work done at constant pressure is given by

$$\text{Work done} = \text{Force} \times \text{Distance}$$

$$W = F \times d$$

$$= P \times A \times d$$

$$\text{Since Pressure } (P) = \frac{\text{Force } (F)}{\text{Area } (A)}$$

But Area × Distance = Change in volume

Therefore $W = P\,(V_1 - V)$.

When volume remains constant in a process, $W = 0$)

Going back to specific heats, we find that the heat required to raise the temperature of 1 mole of a gas through one degree at constant pressure is made up of two parts .

(*a*) Heat required to do an external work to produce a change in volume of the gas

(*b*) Heat required to raise the temperature of the gas through one degree, (C_v)

This means

$$C_p = W + C_v$$

where $W = P(V_1 - V)$

According to the gas law

$$PV = RT \text{ before heating}$$

$$PV_1 = R\,(T+1) \text{ after heating}$$

$$P\,(V_1 - V) = R\,(T+1) - RT = R$$

Hence ~~$C_p - C_v = R$ joules/mole K~~

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Converting joules into calories

$$C_p - C_v = \frac{R}{J}$$

where $J = 4.18$ joules/calorie

Hence for monoatomic gas

$$C_p = C_v + R = \frac{3}{2} R + R = \frac{5}{2} R$$

For diatomic gas

$$C_p = \frac{5}{2} R + R = \frac{7}{2} R$$

For triatomic gas

$$C_p = 3R + R = 4R$$

Example 3

If one mole of helium at 300 K is mixed with 3 moles of oxygen at 400 K and if the mixing takes place at constant pressure, calculate the final temperature.

Solution

C_p of helium (monoatomic) $= \frac{5}{2} R$

C_p of oxygen (diatomic) $= \frac{7}{2} R$

Heat gained (lost) = Mass × Specific heat × Rise (fall) in temperature Let T be the final temperature. Then heat lost by oxygen

$$= 3 \times \frac{7}{2} R \times (400K - T)$$

Heat gained by heliume $= 1 \times \frac{5}{2} R \times (T - 300\ K)$

Heat lost = Heat gained

$$1 \times \frac{5}{2} R \times (T - 300\ K) = 3 \times \frac{7}{2} R \times (400\ K - T)$$

or $5\,T - 1500\,K = 8400\,K - 21\,T$

or $26\,T \qquad = 9900\,K$

or $\quad T \qquad = 381$ K

Problem

Calculate C_p and C_v for argon, given

$$R = 8.3 \text{ J/mole K}$$

ASSIGNMENTS

1. What is the average kinetic energy of a diatomic molecule at 227°C?
2. The average translational kinetic energy of a hydrogen molecule at 27°C is 6.2×10^{-21} joule. What is the average translational kinetic energy at 127°C? If the mass of the hydrogen molecule is 3.1×10^{-27} kg, determine the *rms* velocity of hydrogen molecule at 127°C.
3. A thin rubber ball (with no leak) has air inside. It is thrown fast against a massive vertical wall, and it bounces back. Using your knowledge of kinetic theory, describe the changes in the air in the ball during the process of hitting the wall and rebounding.
4. You are given a certain mass of a gas in a cylinder fitted with a piston. How will you find whether the gas is monoatomic or diatomic or triatomic or polyatomic ?
5. One mole of oxygen is heated at a constant pressure starting at 0°C. How much heat energy must be added to the gas to double the volume?
6. Take the mass of a helium atom to be 6.66×10^{-27} kg. Compute the specific heat at constant volume for it.
7. (*a*) Why for all types of molecules translational degrees of freedom is always three ?

   (*b*) Explain how do you calculate the total number of degrees of freedom of a polyatomic molecule.

SELF-ASSESSMENT

1. (*a*) Define the degrees of freedom of a molecule

   (*b*) On what factors does the number of degrees of freedom of a molecule depend?
2. Considering a diatomic molecule find out the number of degrees of freedom.

3. (*a*) State and derive the law of equipartition of energy.

   (*b*) Why is kinetic energy equally divided per degree of freedom on a molecule.

4. Define specific heat at constant volume (C_v) for a gas. Prove that for a diatomic gas $C_v = (5/2)\ R$.

5. Establish the relationship between C_p, C_v, R and J for a monoatomic gas.

6. Calculate the values of C_p and C_v for oxygen given $R = 8.3$ joules/mole K.

7. Two moles of a monoatomic gas at 573 K are mixed with 1 mole of a diatomic gas at 373 K. Calculate the final temperature of the mixture, assuming that the mixing takes place at constant volume and that there is no exchange of heat with the environment.

8. Ten grams of oxygen are heated at constant atmospheric pressure from 27.0°C to 127.0°C. How much heat is transferred to the oxygen? What fraction of the heat is used to raise the internal energy of the oxygen ?

9 Take $C_v = 0.75$ kcal/kg K for argon and calculate (*a*) the mass of an argon atom and (*b*) the atomic mass of argon.

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3. (*a*) State and derive the law of equipartition

   (*b*) Why is kinetic energy equally divided per
   on a molecule.

4. Define specific heat at constant volume (C_v) fo
   for a diatomic gas $C_v = (5/2)\ R$.

5. Establish the relationship between C_p, C_v, R a
   mic gas.

6. Calculate the values of C_p and C_v for o
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UNIT 12

Physics for Class XI

THERMODYNAMICS I

F. 15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
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New Delhi
20 September 1982

INTRODUCTION

In this unit you will learn about a new branch of physics, called *thermodynamics* which deals with conversion of heat into mechanical work and vice-versa. The relation of heat to other forms of energy such as magnetic, electrical, etc. also come under the scope of thermodynamics. You have already studied the interpretation of macroscopic variables such as pressure and temperature in terms of microscopic entity like mean square velocity of gas molecules on the basis of assumption of random motion and the whole discussion was theoretical, although consistent with experimental observations In contrast, thermodynamics to be studied here and in the subsequent unit, is an intensely practical subject. It concerns itself with a clearly defined system, e.g. a gas contained in a cylinder, a given mass of solid undergoing fusion, etc. Hence the system interacts with surroundings (usually atmosphere) and, by such interaction, useful functions are performed. The branch of thermodynamics is tremendously important from practical point of view. The various principles of thermodynamics find extensive applications in heat engines, refrigerators, thermal radiation, electrochemistry and critical phenomena.

OBJECTIVES

To be able to

1. Distinguish between heat, temperature (given by zeroth law of thermodynamics), work, and internal energy.
2. Compute graphically the work done in *P-V* diagram.
3. Describe Joule's experiment and solve problems relating work and energy.
4. State first law of thermodynamics, discuss its various applications and solve related problems.

5. Compute change in internal energy and work done during isothermal and adiabatic processes.

6. Distinguish between reversible and irreversible processes.

SUGGESTED READING MATERIAL

1. Some important definitions

You know that solids and liquids expand, resistance of a wire increases and so on, when temperature of a body is raised. In such processes flow of heat occurs accompanied by the performance of work. The study of these phenomena and energy conversions involved in the flow of heat and performance of mechanical, electrical or magnetic work come under the domain of thermodynamics. Let us now introduce few important definitions which will be immensely useful in understanding the content of the present unit.

First of all let us define what do we mean by thermodynamic system and surroundings. *The temperature, pressure, etc. are regarded as the thermodynamic coordinates and a system which may be a limited region of space of definite portion of matter, described in terms of these coordinates is called a thermodynamic system*, e g. gas, vapour, surface films, electric cells, stretched wires, thermocouples, etc. And everything outside the system which has a direct bearing on the behaviour of the system is called its surroundings. As already mentioned temperature, pressure, etc. are macroscopic variables called thermodynamic coordinates which are not totally independent, but connected together by a relation known as *equation of state* of the system. Since in order to define a thermodynamic system temperature invariably comes in, let us now define it. Let there be a thermodynamic system consisting of many participants. And if there exists a certain single valued function (it will be clear shortly) f of the state variables P and V, which has the same numerical value T for all the participants, i.e.

$$f(P_1, V_1) = f(P_2, V_2) = f(P_3, V_3) = \dots = T \qquad \text{.. (1)}$$

(where the subscripts 1, 2, ... refer to the different participants).

Then this identical numerical value T is known as the temperature and the system in such a situation is said to be in thermal equilibrium. This is called zeroth law of thermodynamics The Eq (1) defines an empirical temperature T whose SI Unit is K (degree Kelvin) and shows that it is such a thermodynamic coordinate by which the state of the system can be expressed in a more general form as

$$f(P, V, T) = 0 \qquad \ldots(2)$$

which is the *equation of state* for any substance. Now that we have introduced the concept of temperature, we are in a position to define *heat. The flow of heat is a non-mechanical energy transfer, brought about by a temperature difference between two bodies.* Heat can be used only when there is a transfer of energy between two or more systems. *Hence heat can be defined as energy in transit.* The performance of work (to be discussed shortly) and the flow of heat are methods of supplying energy to a body or extracting energy from a body. Conventionally, the quantity of heat energy Q transferred to a body is said to be positive and that transferred from a body is said to be negative. The SI Unit of heat is 1 joule, although till recently it was 1 calorie (where 1 calorie = 4,18 joule).

Let us define now the *internal energy* of a body. Every piece of matter characterized by pressure, temperature, volume, etc. possesses some energy, which depends upon its nature and also on the particular state of the internal condition of the body. This is known as internal energy and is represented by symbol U. Internal energy of a system has two components, viz.

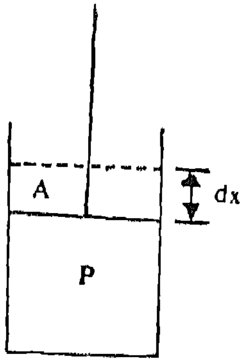
(a) Kinetic energy due to translational, vibrational and rotational motion of the molecules (and all these depend upon temperature) which can be changed by heat transfer.

(b) Potential energy due to inter-molecular forces which depends on the intermolecular distance and it can be changed by the performance of work. This internal energy U is also a *single valued function* of states ,i.e. if a body is subjected to a cycle of operations and finally restored to the original state, then change in the internal energy dU is zero.

2. Work in thermodynamic processes

If a thermodynamical system as a whole exerts force on its surroundings with a consequent displacement, then we say external work has been performed, either by or on the system, e g when a gas in a container is compressed, work is done by the force exerted on it by the moving piston. Now since in thermodynamics we are always concerned with the interaction between a system and its surroundings and hence, the term work will always mean external work. Of course, force acting between parts of the same system are internal forces, the work of these forces is called internal work, which is not being considered in thermodynamics. We calculate now an expression for the work.

Consider a cylinder having a movable piston (Fig. 1) and containing some gas. If the pressure extorted by the gas be P and the area of cross section of the cylinder be A, then the force exerted by the gas on the piston will be PA. If due to this force piston moves through an infinitesimal distance dx, then the corresponding work done will be



Fig, 1

$$dW = PA\,dx$$

$$dW = PdV \qquad ...(3)$$

where dV is the infinitesimal volume change Adx. If the piston steadily moves through a finite distance and volume of the gas increases from V_i to V_f and *if the process is carried out infinitely slowly so that at every* *state system is in thermal equilibrium with the surrounding,* total work done W will be given by

$$W = \int_{V_i}^{V_f} PdV \qquad \ldots(4)$$

Conventionally work done by a system is taken to be positive and work done upon the system is taken to be negative.

The work done by or upon a system can be computed conveniently from what is known as the indicator diagram—*the diagram which represents the variation of state of a thermodynamic system.* The state of a system may be defined in terms of any two thermodynamic coordinates. P, V, T and U (internal energy). Let us consider a $P-V$ diagram shown in Fig. 2. Consider the preceding system of cylinder containing some gas in which gas exerts some pressure on the piston of the cylinder resulting into change in volume V and pressure P. In order to represent the process graphically let the volume V be plotted along abscissa and pressure P along the ordinate. By plotting the values of P and V for successive positions of the piston, the process can be represented by a curve as shown in Fig. 2. Then the shaded area shown in Fig. 2, gives the work done in the process.

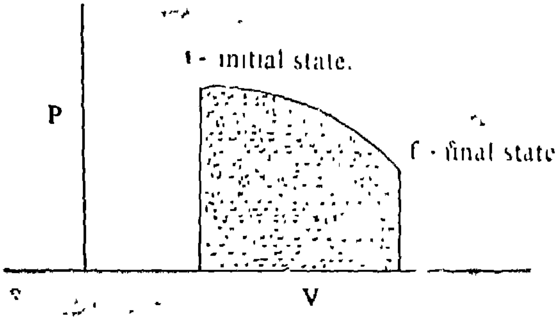


Fig. 2

Now let us discuss a little about what is known as a cyclic process. *A process which brings a thermodynamic system to the same initial state after a cycle of operations (expansions and compressions) is said to be a* *cyclic process.* In P-V diagram given in Fig. 3, we have shown a cyclic process. Here the work done by the system is represented by the area ABCDA

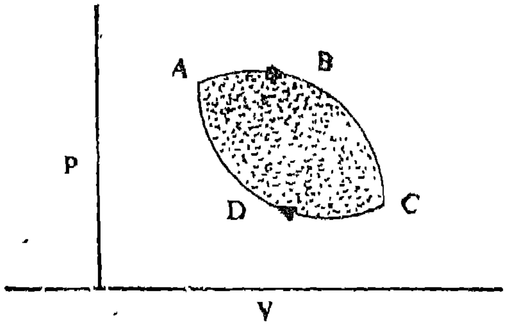


Fig. 3

Example 1

Compute the work done in a process where the pressure and volume at initial state (i) are 4 N/m^2 and 0.1m^3 respectively and those at the final state (f) are 2 N/m^2 and 0.5m^3 respectively as shown in the indicator diagram (Fig. 4).

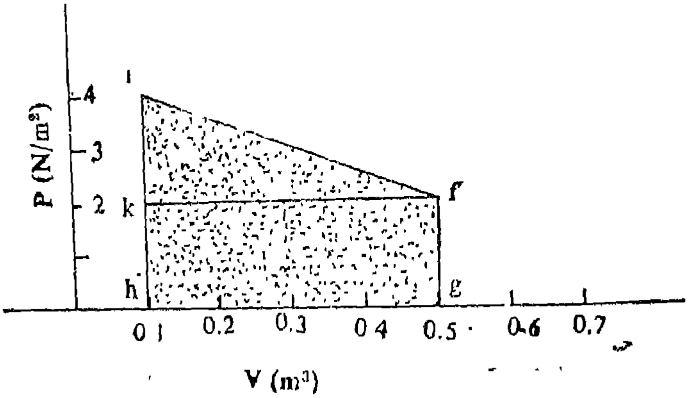


Fig. 4

Solution

Work done

$$\begin{aligned} W = \int_{V_i}^{V_f} PdV &= \text{Shaded area } ifgh \\ &= \text{Area } ikf + \text{Area } khgf \end{aligned}$$

$$\begin{aligned} \text{Area } ikf &= \tfrac{1}{2} \times (4-2) \times (0.5-0.1) \text{ joule} \\ &= \tfrac{1}{2} \times 2 \times 0.4 \text{ joule} = 0.4 \text{ joule} \end{aligned}$$

$$\begin{aligned} \text{Area } khgf &= \text{side } gf \times \text{side } gh \\ &= (2-0) \times (0.5-0.1) \text{ joule} \\ &= 0.8 \text{ joule} \end{aligned}$$

$$\begin{aligned} \text{Hence total work done} &= (0.4 + 0.8) \text{ joules} \\ &= 1.2 \text{ joules} \end{aligned}$$

Problem 1

Compute the work done from the *P-V* (Indicator) diagram shown in Fig. 5, where *i* denotes the initial state and *f* denotes the final state.

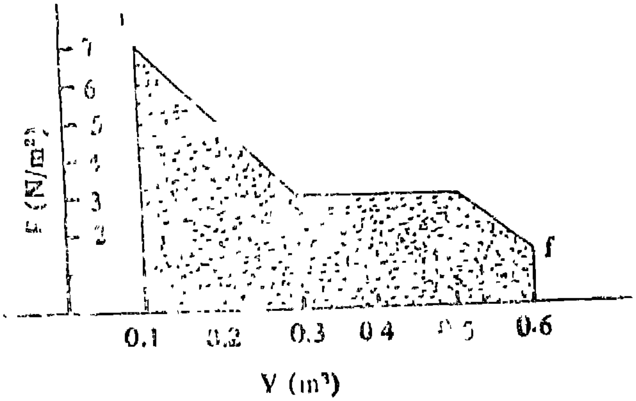


Fig. 5

Problem 2

Calculate the work done during isobaric (constant pressure) case when volume of the given system changes from V_1 to V_2.

Questions

1. Can heat be considered a form of stored (or potential) energy? Would such an interpretation contradict the concept of heat as energy in process of transfer because of a temperature difference?

2. Give examples to distinguish clearly between temperature and heat.

3. Can heat be added to a substance without causing the temperature of the substance to rise? If so, does this contradict the concept of heat as energy in process of transfer because of a temperature difference?

3. Relation between work and heat—Joule's experiment

We have already mentioned that in SI Unit heat and work both are measured in joules. The question arises what is the relation between two quantities It was Joule who showed by his experiment (known as Joule's experiment) that, *when a given quantity of mechanical energy is converted to heat, the same quantity of heat is always developed.* Thus the equivalence of heat and mechanical work as two forms of energy was definitely established. Let us now discuss how Joule arrived at this conclusion.

A rod to which some paddles are attached is mounted in a cylinder containing water. The rod could be revolved by a thread and pulley arrangement as shown in Fig. 6. Two identical weights (mg) are allowed

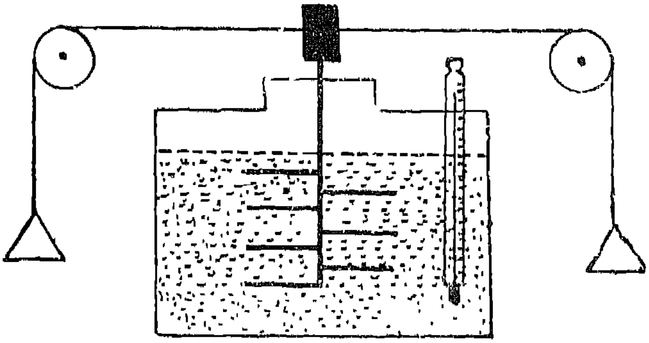


Fig. 6

to fall through a height h, n number of times. Naturally the rod as well as the paddles will start revolving. The water in turn will be set into motion and it gets slightly heated. The rise in temperature $\Delta 0$ of the water can be measured by a sensitive thermometer

The amount of work done in revolving the rod is of course the change in gravitational potential energy of the weights Since weights are allowed to fall, n times, change in gravitational P.E. is $2nmgh$.

Thus work done in revolving the rod $W = 2\,nmgh$. ...(5)

If the mass of water and cylinder are m_1 and m_2 and their specific heat capacities are s_1 and s_2 respectively, then heat absorbed by water and cylinder is

$$[\,(m_1s_1 + m_2s_2)\,\Delta\,\theta\,] \qquad \ldots(6)$$

Of course, specific heat capacity of water $s_1 = 1$ cal.g^{-1}

Thus total heat absorbed

$$H = [\,(m_1s_1 + m_2s_2)\,\Delta\theta + \mathfrak{C}\,] \qquad \ldots(7)$$

where $\mathfrak{C}$ is the necessary corrections for the heat energy lost by radiation, frictional losses, etc.

Now Joule, performing this experiment, concluded that a definite relation exists between the work done and the heat gained. Mathematically, he expressed this relation as

$$W = JH \qquad \ldots(8)$$

where W is measured in joules and H is measured in calories and J is a constant, called *mechanical equivalent of heat*. From his experiment equating From Eq. (5) and Eq. (7), i.e.

$$(2nmgh) = J\,[\,(m_1s_1 + m_2s_2)\,\Delta\,\theta + \mathfrak{C}\,]$$

$$J = \left\{ \frac{2n\,mgh}{(m_1s_1 + m_2s_2)\,\Delta 0 + \mathfrak{C}} \right\} \text{joules-calorie}^{-1}$$

Joule determined its value as 4.18 joules per calorie. It is clear from Eq. (8) that if work and heat both are expressed in joules (as in SI Units) then

$$W = H \qquad \ldots(9)$$

Eq. (8) or Eq. (9) clearly demonstrate the complete equivalence of heat and mechanical work. Later of course, Helmholtz expressed the idea that not only heat and mechanical energy but all forms of energy are equivalent, and that a given amount of one form cannot disappear without an equal amount appearing in some of the other forms.

Question

Is the mechanical equivalent of heat J, a physical quantity or merely a conversion factor for converting energy from heat units (e.g. calories) to mechanical units (e.g joule) and vice-versa ?

Example 2

In a Joule experiment, a mass of 6.00kg falls through a height of 50.0 metres and rotates a paddle wheel which stirs 0.600 kg of water. The water is initially at 15°C. By how much does its temperature rise ?

(Take $g = 10\ \text{m/s}^2$)

Solution

Work done in rotating paddles $W = mgh$

$$= 6 \times 10 \times 50 \text{ joules}$$

$$= 3000 \text{ joules}$$

Heat absorbed by water $H = ms\,\Delta\theta$

$$= (0.6 \times 1000 \text{ calories } {}^\circ\text{C}^{-1})\,\Delta\theta$$

$$= (600 \text{ calories } {}^\circ\text{C}^{-1})\,\Delta\theta$$

(where specific heat capacity of water = 1000/kg °C

We know $W = JH$ where $J = 4.18$ joules calorie^{-1}

$$3000 \text{ joules} = (4.18 \times 600 \text{ joules } {}^\circ\text{C}^{-1})\,\Delta\theta$$

$$\Delta\theta = \left(\frac{3000}{4.18 \times 600}\right){}^\circ\text{C}$$

$$\Delta\theta = 1.17^\circ\text{C}$$

Example 3

A 50g lead bullet of specific heat capacity 0.03 kcal kg^{-1} $°C^{-1}$ is at 20°C. If it is fired vertically upwards with a speed of 420 ms^{-1} to strike a block of ice at 0°C on its return. Calculate the amount of ice melted. Assume that the mechanical energy is completely converted into heat and temperature of the bullet is not changed during its journey before striking the ice block. Latent heat of fusion of ice $L = 80{,}000$ cal kg^{-1}.

Solution

Kinetic energy of the bullet

$$= \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2} \times (50 \times 10^{-3}) \times (420)^2 \text{ joules}$$

Equivalent heat $H = (\tfrac{1}{2}mv^2/J)$

$$= \frac{\tfrac{1}{2} \times (50 \times 10^{-3}) \times (420)^2}{4.18} \text{ calories}$$

$$= 1050 \text{ calories}$$

Heat liberated in cooling $= m\, s\, \Delta\, \theta$

$$= (50 \times 10^{-3}) \times (0.03 \times 10^3) \times 20 \text{ calories}$$

$$= 30 \text{ calories}$$

H′ = Total heat liberated = 1050 calories + 30 calories = 1080 calories
Mass of ice melted = H′/L

$$= \frac{1080}{80{,}000} \text{ kg} = 0.0135 \text{ kg}$$

Problem

Compute the approximate increase in temperature of water going over Niagara falls approximately 52m high.

4. First law of thermodynamics

This law states that for a system during cyclic process, the sum of the work transferred is equal to the sum of heat transferred (heat and work being measured in units, e.g. joules) and vice-versa.

Therefore, this law gives an equilibrium between heat and mechanical energy, i e. whenever work is done, an equivalent amount of heat is produced and whenever certain amount of heat disappears it appears in the form of work.

Now let there be a substance which takes some heat Q (in work units), performs W units of external work, whereby it passes from state 1 characterized by internal energy U_1 to state 2 characterized by internal energy U_2, so from conservation of energy we have

$$U_2-U_1 = Q-W \qquad \ldots(10)$$

This is the mathematical representation of first law. Thus, the first law is simply a statement of conservation of energy. *Energy can neither be created nor destroyed but it can be transformed from one form to another.* If now dU be the small increase in internal energy and dW be the amount of work done, due to absorption of a small quantity of heat dQ; we have

$$dU=dQ-dW$$

$$\text{or } dQ=dU+dW \qquad \ldots(11)$$

This is applicable to all states of matter

If work done dW is due to compression or expansion, then

$$dW = P\,dV \qquad \ldots(12)$$

$$(\text{ which implies } dQ = dU + P\,dV\,) \qquad \ldots(13)$$

The first law applies to any process whether it is irreversible or reversible (to be discussed shortly).

Question

An equivalent statement of first law is that *the energy of a system, including its surroundings, remains constant Elaborate on this statement.*

Let us now discuss some of the applications of first law.

1. Cyclic process and isothermal process

In a cyclic process, indicator diagram is a closed curve (Ref:

Fig. 3). Since internal energy is a single valued function of state, here change in internal energy dU must be zero.

Thus from first law $dQ = dU + PdV$

Since $dU = 0$

$$dQ = PdV \tag{14}$$

which shows that heat given to the system is equal to the external work done by it during a cyclic process.

An isothermal process (*iso* means same; *thermal* means temperature) is carried out in a manner that the temperature of the system remains constant, i.e. in an isothermal process $dT = 0$, which implies for a perfect gas $dU = 0$ (why) ? Therefore a perfect gas undergoing isothermal process will be an example of cyclic process. Hence in this case

$$dQ = dW = PdV \qquad \text{...(14a)}$$

If we are now interested in calculating work done during an isothermal process by an ideal gas (having n moles, say), we have

$$PV = n\,RT \qquad \text{...(15)}$$

Substituting from Eq. (15) in Eq. (14a)

$$dW = \frac{nRT}{V}dV$$

Let the volume changes from V_i to V_f ; then total work done by the system is

$$W = \int dW = \int_{V_i}^{V_f} nRT\frac{dV}{V} = nRT\int_{Vi}^{V_f}\frac{dV}{V}$$

$$= nRT\,(\log_e V_f - \log_e V_i)$$

$$W = nRT\log_e V_f/V_i \qquad \text{...(16)}$$

If initial pressure was P_i and final pressure was P_f : then one can also write Eq. (16) as

$$W = nRT\log_e P_i/P_f \qquad \text{(How ?)}$$

Problem

If the equation of state for a gas is given by PV^2=constant × T calculate the work done during isothermal process when volume charges from V_1 to V_2.

2. *Specific heat relation*

From the first law of thermodynamics we know

$$dQ = dU + PdV \qquad \text{...(18)}$$

The specific heat of a substance is defined as the heat per unit mass required to raise its temperature by one degree. The specific heat of one mole of a gas is called molar specific heat C and

$$C = \frac{\delta Q}{\delta T} \qquad \text{...(19)}$$

This will have different values according to varying external condition, e.g if volume V is constant, molar specific heat at constant volume $C_v = (\partial Q/\partial T)_v$ and if pressure P is kept constant molar specific heat at constant pressure $C_p = (\partial Q/\partial T)_p$, where subscripts v and p denote that volume V and pressure P have been held constant in cases of C_v and C_p respectively.

In some equations there is relationship between three or more variables. You can choose any pair of them to be the independent variables; the other one is then a dependent variable. Often we need to compute the rate of change of a dependent variable, holding the other constant This rate of change is called ***partial derivative.*** Thus $(\partial V/\partial T)p$ is the partial derivative of V with respect to T keeping P constant. Here V (dependent variable) has been expressed in terms of T and P (independent variables) before differentiation. If f is a function of x and y

$$df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$$

which gives total derivative df in terms of partial derivatives.

Now in Eq. (18) if differentiation with respect to temperature T is carried out holding volume V constant (which implies $dV = 0$), we have

$$\left(\frac{\partial Q}{\partial T}\right)_v = C_v = \left(\frac{\partial U}{\partial T}\right)_v \qquad \text{...(20)}$$

Let us suppose we have started with 1 mole of a perfect gas. In thermodynamics a perfect gas is defined in the following way :

(a) Equation of state for all values of P and T for n moles of perfect gas should be

$$PV = nRT$$

(b) The internal energy U should not be a function of volume, but only of temperature. (In general, U can be regarded as function of volume and temperature). Thus whether we hold volume constant or pressure constant, so far as change of internal energy due to change in temperature for a perfect gas is concerned, it does not matter. Hence

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v = \left(\frac{\partial U}{\partial T}\right)_p = dU/dT \qquad \text{...(21)}$$

Now differentiating Eq. (18) with respect to temperature holding pressure contant, we have

$$\left(\frac{\partial Q}{\partial T}\right)_p = C_p = \left(\frac{\partial U}{\partial T}\right)_p + P\left(\frac{\partial V}{\partial T}\right)_p \qquad \text{...(22)}$$

Substituting from Eq.(21) in Eq. (22), we have

$$C_p - C_v = P\left(\frac{\partial V}{\partial T}\right)_p \qquad \text{...(23)}$$

For 1 mole of a perfect gas

$$PV = RT$$

$$\left[\frac{\partial}{\partial T}(PV)\right]_p = P\left(\frac{\partial V}{\partial T}\right)_p + V\left(\frac{\partial P}{\partial T}\right)_p = P\left(\frac{\partial V}{\partial T}\right)_p = R \qquad \text{...(24)}$$

$$\left[\text{Since } \left(\frac{\partial P}{\partial T}\right)_P = 0; \text{ How ?}\right]$$

Substituting Eq (24) in Eq. (23), we have

$$C_p - C_v = R \qquad \text{...(25)}$$

Question

How would the Eq. (25) look like, if we would have proceeded with n moles of perfect gas ?

3. *Adiabatic process*

A process which takes place in such a way that no heat enters or leaves the system ($dQ=0$) is called an adiabatic process. This can be achieved either by surrounding the system with thick layer of insulating material or by performing the process quickly, e g bursting of a tyre.

Applying first law of thermodynamics we have

$$dU = -\, dW \text{ (since } dQ = 0) \qquad \ldots(26)$$

If the work done is negative, the internal energy of the system increases. If the work done is positive, the internal energy decreases. The increas or decrease in internal energy is usually accompanied by the rise or fall in temperature.

Question

How does the internal energy change when a system expands or contracts adiabatically ? Explain.

Let us now calculate the work done in an adiabatic process when temperature changes from T_1 to T_2 . We have seen in Eq. (21).

$$\frac{dU}{dT} = C_v$$

or

$$dU = C_v \; dT \qquad \ldots(27)$$

Substituting from Eq. (27) in Eq. (26), we have

$$dW = -\, C_v \; dT$$

If C_v is assumed to be independent of temperature, then required work done

$$W = -\, C_v \int_{T_1}^{T_2} dT = -\, C_v \,(T_2 - T_1)$$

Problem

Let the C_v of a substance be found to vary parabolically with temperature, that is $C_v = A + BT^2$; where A and B are constant. Determine the work done during an adiabatic process when temperature changes from T_1 to T_2.

4. *Boiling Process*

Consider the vaporization of a mass m of a liquid at constant temprature T and pressure P. Let V_i be the volume of the liquid and V_f be the volume of the vapour. The work done by the liquid in expanding will be given by

$$W = P(V_f - V_i) \qquad \text{...(29)}$$

Let the latent heat of vaporization be L. It represents heat needed per unit mass to change from liquid to vapour phase at constant temperature and pressure. Let the heat absorbed by the liquid be Q where

$$Q = mL \qquad \text{...(30)}$$

From first law of thermodynamics

$$Q = U_f - U_i + W$$

or, $$mL = U_f - U_i + P(V_f - V_i)$$

Hence, change in internal energy

$$U_f - U_i = mL - P(V_f - V_i) \qquad \text{...(31)}$$

Knowing m, L, P, V_f and V_i gain in internal energy can be easily calculated.

Example

1 g of water at 373 K is converted into steam at the same temperature. The volume of 1 cm^3 of water becomes 1671 cm^3 on boiling. Calculate the change in internal energy of the system if the heat of vaporization is 540 cal^{-1} g.

Solution

Here

$$mL = 1 \times 540\ cal = 540\ cal$$

$$P(V_f - V_i) = 10^6 \times (1671-1)\text{ ergs} = 1.67 \times 10^9\text{ ergs.}$$

(assuming atmospheric pressure $P = 10^6$ dynes cm^{-2})

Since $J = 4.2 \times 10^7$ ergs $^{-1}$ cal

$$P(V_f - V_i) = \frac{1.67 \times 10^9}{4.2 \times 10^7} \text{ cal} = \frac{167}{4.2} \text{ cal}$$

Thus change in internal energy

$$U_f - U_i = mL - P(V_f - V_i)$$

$$= 540 \text{ calories} - \frac{167}{4.2} \text{ calories}$$

$$= 499 \text{ calories}$$

Problem

By how much does the internal energy change when a block of copper of mass 50g is heated from 20°C to 50°C. Assume negligible volume change.

(Given specific heat capacity of Cu $= 0.1$ cal g^{-1} °C^{-1})

(Ans. 630 J)

5. *Reversible and irreversible processes*

Let there be a cylinder enclosing a perfect gas provided with a piston If we gradually place some weights upon the piston, the gas will be gradually compressed with no oscillations, and if the weights are gradually removed, then gas will be returning to its initial state without leaving behind any trace in its *universe* (system+surroundings). This is a reversible process. *Thus reversible process is one which is performed in such a way that at the end of the process, both the system and the surroundings constituting the universe may be fully restored to their initial states without producing any significant change anywhere in its universe. Or, in other words, all processes the effects of which can be completely annulled everywhere (at all parts of the universe) are called reversible.*

The effects can be completely eliminated, if the thermal and mechanical effects produced in the system and the surroundings at every stage of the direct process is exactly reversed in the corresponding stages in the reverse process, *i.e.*, the amount of heat received and work done in each step of the direct process should be the same at the corresponding step in the reverse process, but with opposite signs. Where work is done by the system in the direct process, an equal amount of work should be done upon the system in the reverse process Where heat is absorbed by the system in the direct process, it should be given out by the system in the reverse process.

These conditions demand that during the process the system should remain in mechanical equilibrium, *i e*, there should not be any unbalanced forces in the interior of the system or between the system and the surroundings. The system should always remain in thermodynamic equilibrium, i e. the temperature and pressure at every part of the system should be the same and are infinitesimally near the corresponding quantities of the surroundings A process satisfying the above two conditions is called a quasi (semi) static process and must invariably be performed infinitely slowly Thus for reversibility the process should be quasi static and should leave the system in chemical equilibrium, i e. it should not change its internal structure due to chemical reaction In addition the process should not be associated with any dissipative effects which may arise from electrical resistance, viscosity, friction, magnetic hysteresis, etc.

Even a cyclic process cannot be termed reversible if it produces such changes in the universe which cannot be abolished by suitable means In fact a reversible process can be traced in the opposite direction with all the thermal and mechanical effects remaining of same magnitude but of opposite sign to those in the direct process. Examples of reversible processes are a string undergoing extension in its length within elastic limit due to load suspended at one of its ends, ice undergoing melting on being exposed and subsequently changing into ice again on cooling the water gradually, and so on

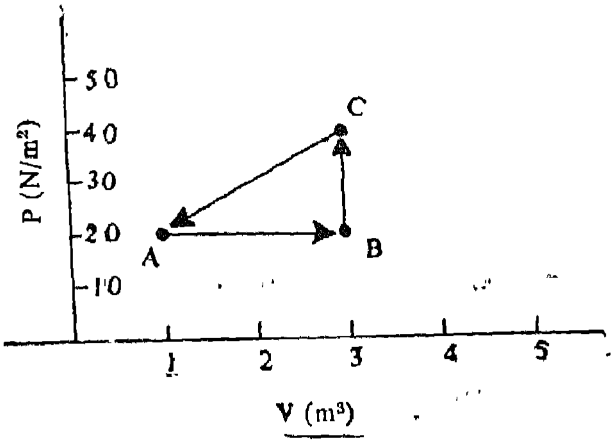
Now one can define irreversible process in the following manner. *If the whole system including the surrounding cannot be completely restored into its initial condition everywhere even with the help of all physical means, the process is said to be irreversible.* Obviously here the effects of the process cannot be completely obliterated from the universe. Irreversibility is normally due to dissipative effects (already listed) and finite temperature difference between system and its surroundings. The world of natural phenomena are dominated by irreversible processes. Examples are utilisation of heat for performing useful mechanical work, magnetising a magnetic specimen by sending current through it till the saturation magnetisation, and so on.

ASSIGNMENT

1. A thermodynamic system is taken from an initial state A to another state B and back again to A, via state C, as shown by the path ABCA in the P-V diagram of Fig. 7. (a) Complete the table by filling in appropriate + or — indications for the signs of the thermodynamic quantities associated with each process. (b) Calculate the numerical value of the work done by the system for the complete cycle ABCA.

Ans. 20 J

(a)



(b)

	Q	W	ΔU
A→B			+
B→C	+		
C→A			

Fig. 7

2. An iron ball is dropped on to a concrete floor from a height of 10 metres. On the first rebound it rises to a height of 0.50m. Assume that all the macroscopic energy lost in the collision with the floor goes into the ball. The specific heat of

iron is 0.12 cal. g^{-1} °C^{-1}. During the collision (a) Has heat been added to the ball? (b) Has work been done on it? (c) Has its internal energy changed? If so, by how much? (d) How much has the temperature of the ball risen after the first collision ? Ans. 0.18°C

3. A 50.0kg block of ice at 0°C slides along a horizontal surface, starting at a speed of 2.38ms^{-1} and finally coming to rest after travelling 28.3 m. Compute the mass of ice melted as a result of friction between block and the surface. List any assumptions you need to make in getting your answer.
   Ans. 0.42g

4. 20g of water at 29°C is converted into steam at 99°C (steam temperature.) If the net change in volume is from 19.9 cm^3 to 10,000 cm^3 calculate the net change in internal energy of the system
   Ans 5 × 10^4 J

   (Given, atmospheric pressure = 1.06 × 10^6 dynes cm^{-2} and latent heat of vaporization of water = 540 cal g^{-1}.)

5. A quantity of gas occupies an initial volume V_0 at a pressure P_0 and a temperature T_0. It expands to a volume V (a) at constant temperature (b) at constant pressure. In which case the gas does more work ? (c) Does a gas do any work when it expands adiabatically ? If so, what is the source of energy needed to do this work ?
   Ans. case (b) Yes, internal energy

SELF-ASSESSMENT

1. Distinguish between isothermal and adiabatic process (give five examples for each)
2. What is meant by internal energy of a system ? How do you measure change in internal energy for boiling processes ?
3. Show that the work done by a certain mass of a perfect gas under isothermal expansion is given by $W = n\,RT$ log $(V_f \,/\, V_i)$, where V_f and V_i represent the final and initial volume of the gas

4. Deduce $C_p - C_v = R$, from first law of thermodynamics.

5. (a) In a certain process 400 calories of heat are supplied to a system and at the same time 120 J of work is done on the system. Find the change in internal energy of the system.

   (b) One mole of an ideal gas is heated from 273 K to 546 K at constant pressure of 1 atmosphere. Compute (i) work done (ii) change in internal energy, and (iii) the heat required for the process.

6. Compute the velocity with which a hail storm at 0°C must strike the ground so that 7/15th of its kinetic energy when converted into heat will be enough to melt the given mass of hail storm completely into water at 0°C.

7. A process starting from A and ending at C may follow anyone of the two paths ABC or ADC Fig (8). Along ABC, Q = 120 cal and W = 75 cal. Find the value of W along ADC if Q along ADC is 100 cal (where Q and W have their usual meanings).

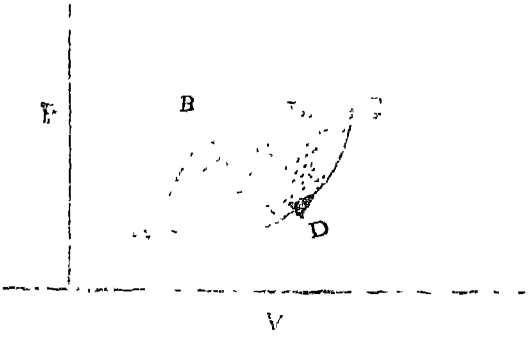


Fig. 8

Acc. No. F-15187(m)
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UNIT 13

Physics for Class XI

THERMODYNAMICS II

F-15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

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PREFACE

This unit is thirteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers Dr. K J George who is working as a Senior Project Fellow helped me in correcting the proofs

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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Reader
Department of Education in Science
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N.C E.R.T.

New Delhi,
20 October 1982

INTRODUCTION

You have seen that if some work is done in a room at temperature T, against friction, slowly enough, so that room temperature does not change much the lost work is converted into heat at the given temperature. What about the reverse possibility? Is it possible to convert the heat back into work at a given temperature? Second law of thermodynamics to be studied in the present unit asserts that it is not, although first law of thermodynamics (a consequence of the principle of conservation of energy) does not debar the occurrence of such events In other words, *if the whole world were at the same temperature, while the process of converting work into heat can take place at a given temperature, one cannot reverse it to get the work back again* This was the conclusion Sadi Carnot arrived at during the researches carried out by him to improve the efficiency of heat engines, also to be studied in the present unit Here you are also going to study about the refrigerator required for the purpose of artificial production of cold and which can be considered as a heat engine working in the reverse direction.

OBJECTIVES

To be able to

1. Derive the expression for efficiency of heat engine, and coefficient of performance of refrigerator.
2. State and explain the second law of thermodynamics in terms of thermal efficiency of the heat engine and the coefficient of performance of the refrigerator.
3. Describe the working of Carnot engine and compute its efficiency
4. Identify the Kelvin scale of temperature with that of the perfect gas scale

5 Solve problems based upon efficiency of the heat engine and coefficient of performance of the refrigerator.

SUGGESTED READING MATERIAL

1. Heat engine and refrigerator

You are familiar with different types of heat engines, such as external combustion engines (e.g. steam engine, where steam takes the heat from a boiler, part of it is used to do work and rest is rejected to the surrounding atmosphere) and internal combustion engines (e g. otto engine and diesel engine, where due to firing of a mixture of compressed gas and air, piston is thrown forward with great force). They are nothing but contrivances for converting heat into work. The following are the requirements for a heat engine :

(a) A reservoir of heat or a source at high temperature, where heat is supplied by the burning of coal, oil, wood, naptha, etc

(b) A working substance which can be a gas or vapour and can undergo expansion on heating. The working substance should be able to drive the different parts of heat engine.

(c) A set of machinery invariably necessary to drive an engine.

(d) A reservoir of a heat at low temperature called the sink. The magnitude of the source and the sink should be so large that their temperatures remain unchanged because of any heat supplied to them or removed from them.

The function of a heat engine, as summarised by Carnot, is to extract a certain amount of heat Q from the source, convert a part of it to work W and transfer the rest Q^1 to the sink as shown in Fig 1 and finally return back to the initial state. Thus in a heat engine, the system starting from an initial state is brought back to the same state after a series of processes essentially in the form of a closed curve. The whole process can thus be termed, *cyclic*. (Ref. Fig. 3, Thermodynamics I, Unit 12). The change in the internal energy of the system is zero Hence from first law of thermodynamics, we can equate the change in heat energy of the system

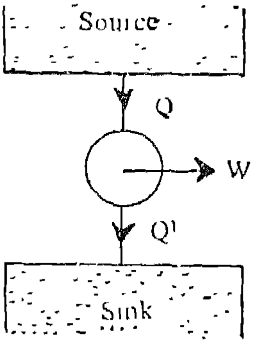


Fig 1 Heat engine

to the amount of work done by the system (which is + ve) i.e

$$Q - Q^1 = W \qquad ..(1)$$

As usual, the thermal efficiency of an engine, is defined as

$$\text{Efficiency } \eta = \frac{\text{Work output}}{\text{Energy input}}$$

$$\text{or} \qquad \eta = \frac{W}{Q} \qquad ..\ (2)$$

Substituting from Eq. (1) in Eq. (2), we get

$$\eta = \frac{Q - Q^1}{Q} = 1 - \frac{Q^1}{Q} \qquad ..\ (3)$$

The value of η is always found to be less than unity. The first heat engines constructed were, of course, inefficient (η much less than 1) devices. Only a small fraction of heat absorbed at the source could be converted to useful work. But even as engineering design improved, sizeable fraction of the absorbed heat was still discharged at the lower temperature exhaust of the engine, remaining unconverted to mechanical work. It was the cherished dream of engineers to devise an engine that could take heat from an abundant reservoir, like the ocean, and convert it completely into useful work, i e construct an engine with η equal to 1. But can it really be done? People had to wait for the answers of this, until the discovery of second law of thermodynamics.

Before discussing second law of thermodynamics, we want to discuss a bit about refrigerators. Take a heat engine (whose schematic diagram is shown in Fig 1) and let the processes take place in the reverse direction. What do you get? The extraction of certain amount of heat Q^1 from the sink by the working substance, with some work W being done upon it and finally transference of a large amount of heat Q to the source as shown in Fig. 2. Such a device which acts like a reversed heat engine is called refrigerator and its working substance is called the *refrigerant*. Since substance works in a cycle, there is no change in internal energy. Naturally from first law of thermodynamics

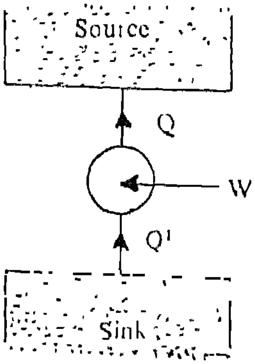


Fig. 2 Refrigerator

$$Q^1 - Q = -W \qquad ..(4)$$

(—ve sign with W means work is done upon the system)

or $$Q = Q^1 + W \qquad ...(5)$$

Let us now define a term called coefficient of performance (C O.P.) for a refrigerator.

$$\text{C.O.P.} = \frac{\text{Heat absorbed from the sink at lower temperature}}{\text{Work done upon the system}}$$

$$= \frac{Q^1}{W} \qquad ...(6)$$

Substituting from Eq. (5) in Eq. (6), we get

$$\text{C.O.P.} = \frac{Q^1}{Q - Q^1} \qquad \ldots(7)$$

The question arises, can C.O.P of a refrigerator ever be infinite? That is, such a refrigerator simply transfers heat from a cold body to a hot body, without requiring the expense of outside work. People got the answer to this query after the discovery of second law of thermodynamics. In household refrigerators the work is done by an electric motor. The working substance is *Freon* and it absorbs heat from food stuff kept inside and rejects it to surrounding air at higher temperature.

2. The second law of thermodynamics

The first law of themodynamics denies the possibility of creating or destroying energy. It only tells us there must be a balance between the energy before and after the occurrence of a process. We now consider some examples which do not violate the first law, but still they do not occur in nature.

Example 1

Consider a metal bar, hot at one end and cold at other. The heat will flow spontaneously from hot end to the cold end until the bar attains a uniform temperature. We never observe the reverse, i.e. a metal bar at uniform temperature spontaneously becomes hotter at one end, even though this would not violate the first law since we still have an energy balance before and after the change.

Example 2

Consider a tank of water containing a paddle (Fig 3) which will rotate

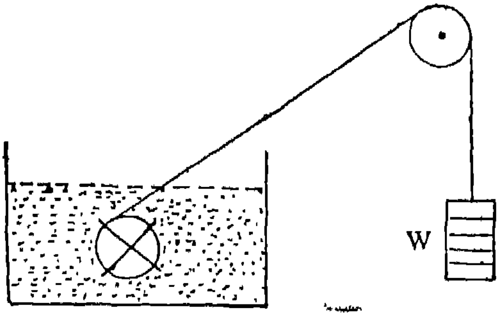


Fig. 3

when the weight W falls. The spontaneous fall of the weight decreases its potential energy with increase of the internal energy of the water. The reverse process, i.e. the cooling of water and the weight rising is never observed, even though, once again the first law would not be violated.

These examples clearly indicate that there should be a directional law which limits the way in which energy may be transferred or transformed, over and above the first law. Again, neither of the hopeful ambitions of constructing heat engine with efficiency as one and refrigerator with C.O.P. as infinity violate the first law of thermodynamics. Nevertheless neither of those ambitions have ever been achieved. All such experiences led to the formulation of second law of thermodynamics which can be stated in many ways, each emphasizing another facet of the law, but all can be shown to be equivalent to one another.

Kelvin (with Planck) stated the second law as

It is impossible to construct an engine operating in a cycle that will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work.

This statement, clearly, rules out the possibility that the efficiency of any engine can never be equal to one, for it implies that we cannot produce mechanical work by extracting heat from a single reservoir without returning any heat to a reservoir at lower temperature.

Clausius stated it as follows :

It is impossible for an engine operating in a cycle, unaided by an external agency, to transfer heat from one body to another at a higher temperature, i.e. heat will not pass spontaneously from one body to another at a higher temperature.

This statement clearly rules out the possibility of constructing a refrigerator with coefficient of performance infinity. We know that heat energy flows from hot body to cold body. Clausius' statement specifically rules out the reversal of the process of heat transfer from hot body to cold body (i.e. no heat flow can occur from cold to hot body) and thus determines the direction of transfer of heat. So metal bar of example 1 can never exhibit the reverse process.

Lord Kelvin separately stated the second law in the following form.

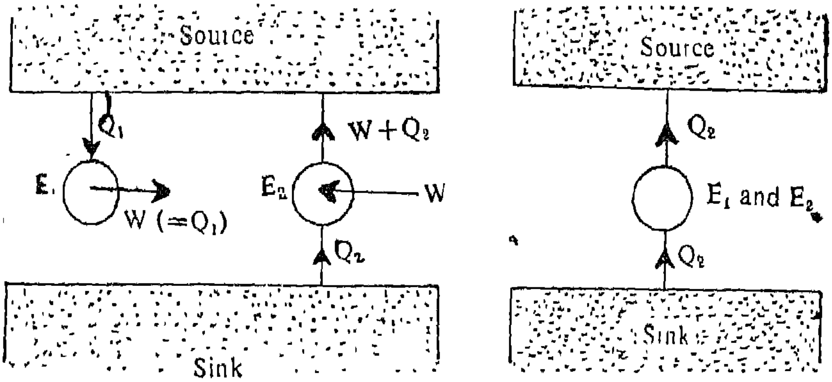
It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects

This statement can be illustrated by the example 2 just discussed, i.e. extracting internal energy of water by cooling it, we can never observe the mechanical effect such as raising the load Similarly it is impossible to run a ship by extracting heat from ocean, which obviously has an enormous amount of internal energy. *The extraction of heat from a piece of matter by cooling it and utilising it to do work is called perpetual motion of second kind and second law of thermodynamics completely denies the possibility of such motion.*

The Kelvin-Planck and the Clausius statements do not appear at first sight, to be equivalent But it can be shown that contradiction of either leads to a violation of other, hence they are equivalent

Let us show that contradiction of Kelvin-Planck statement leads to violation of Clausius statement.

Consider an ideal engine E_1 and, an engine E_2 operating as a refrigerator as shown in Fig (4a). E_1 contradicts the Kelvin-Planck statement, but E_2 does not violate Clausius statement. However when E_1 and E_2 are combined as shown in Fig. (4b), we get an engine which transfers heat Q_2 from sink to source without any aid from outside, thus violating Clausius statement.



(a) Fig 4 (b)

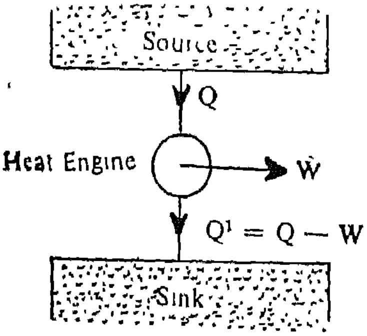
Question

1. Show that contradiction of Clausius statement leads to violation of Kelvin-Planck statement.

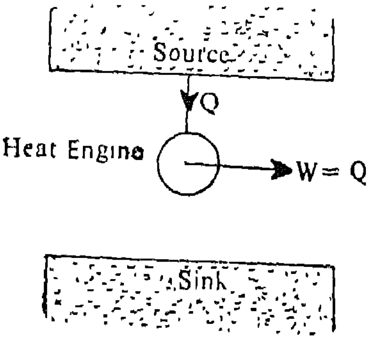
2. Can the internal energy of water be utilised to increase the potential energy of the loads in Joule's experiment?

The three different statements of the second law of thermodynamics can be diagramatically illustrated as follows :

Kelvin-Planck statement

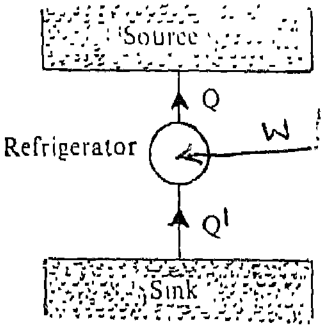


(Possible)



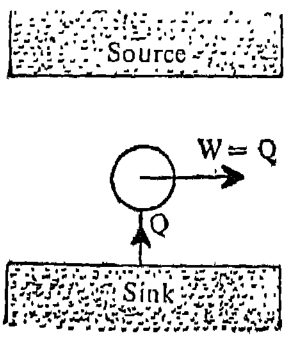
(Impossible)

Clausius statement



(Impossible)

Kelvin statement



(Impossible)

3. Carnot engine

From the second law of thermodynamics it is clear that efficiency of a heat engine should always be less than unity However, one can try to make it maximum. Let us suppose a heat engine with an ideal gas as working substance operates isothermally. When heat is absorbed from the source gas will expand and do some work. In order that work done is maximum one can make the engine operate in a cycle, i e. to bring back the working substance to the initial state by doing external work But then the indicator diagram for the complete cycle will be a line (Fig. 5) and consequently area enclosed will be zero and no work will be done in the process. On the other hand if we consider a heat engine working adiabatically, on compression the temperature of the gas will rise and the compressed gas will do some work.

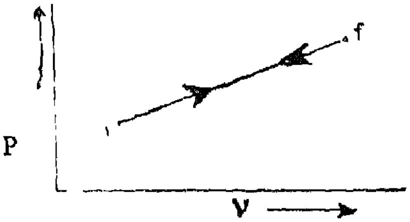


Fig. 5

If the gas is finally brought back to its original state by an adiabatic process, then we know $dQ = 0$ and since the gas is brought back to initial state $dU = 0$, from the first law of thermodynamics we get $dW = 0$, i.e. no net work is done in the process.

In order to get the maximum possible efficiency, Sadi Carnot, showed that if the indicator diagram of an engine is made of suitable combinations of isothermals and adiabatics, then more and more work can be done. The following set of operations constitute the Carnot cycle. Any engine which operates by performing these processes is called a Carnot engine The working can be described as follows·

Let S_1 and S_2 be source (at temperature θ) and sink (at temperature θ') respectively and C be the cylinder of the engine fitted with a piston and non-conducting cap P (Fig 6(a)). Let the working substance be an ideal gas. Let the following steps be followed

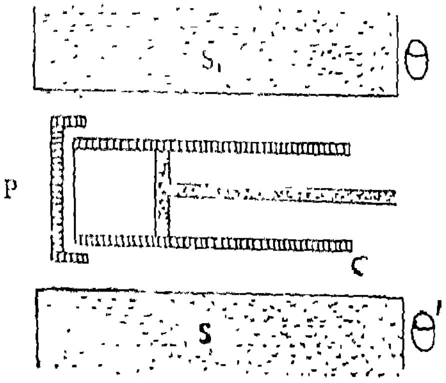


Fig 6 (a)

(a) *Isothermal expansion (at constant temperature)*

The gas is kept in thermal contact with the source at temperature θ. The initial pressure and volume of the gas are represented by the point A in the indicator diagram Fig. 6(b). Let the gas be allowed to expand

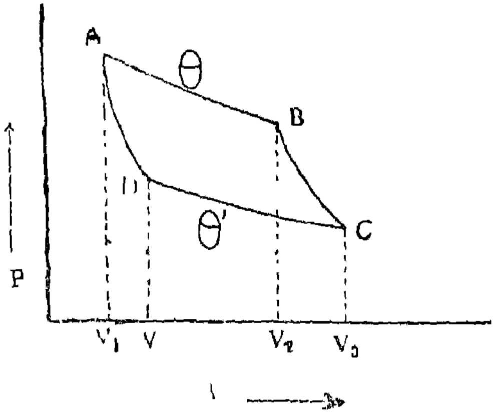


Fig. 6 (b)

reversibly and since during this process temperature falls, it will absorb heat Q from S_1 so that throughout the process of expansion the temperature of the gas will remain θ and during this *isothermal expansion*, suppose representative point specifying the state of the working substance, moves from A to B, thereby volume changes from V_1 to V_2. The work done during this process if we have started with *1* mole of perfect gas will be

$$W_1 = \int_{V_1}^{V_2} P\,dV = R\theta \int_{V_1}^{V_2} \frac{dV}{V} = R\theta \log_e (V_2/V_1) \qquad \ldots(8)$$

But since temperature is constant, change in internal energy is zero during the process. Thus from first law total heat absorbed Q will be

$$Q = W_1 = R\,\theta \log_e \frac{V_2}{V_1} \qquad \ldots(9)$$

where θ is the temperature in perfect gas scale.

(b) *Adiabatic expansion*

Let us put the insulating cap P on the cylinder such that there is no heat exchange between the gas and the surroundings, when representative point is at B. The piston will keep moving through some distance due to inertia but since we have made now the process of expansion adiabatic (cutting off the exchange of heat between gas and surrounding), the temperature will fall and let the temperature at the end when representative point is at C, be the temperature of the sink θ' where volume of the gas is V_3 and pressure is quite diminished.

(c) *Isothermal compression*

Since pressure at C is very low, in order to enable the gas to do more work, it is necessary to bring it back to original state A. Thus, at first P is removed and gas is kept in thermal contact with the sink at temperature θ' and it is compressed isothermally. Though during compression the temperature of the gas will tend to rise, it will, however, remain constant at θ' because heat developed Q^1 will be transferred to the sink. Suppose during isothermal compression representative point moves from C to D and thereby volume changes from V_3 to V_4. Here work done W_2 during the compression will be equal to the amount of heat rejected Q^1 to the sink, i.e.

$$W_2 = Q^1 = R\theta' \log_e (V_3/V_4) \qquad \ldots(10)$$

(Why $W_2 = R\theta' \log_e (V_3/V_4)$, why not $W_2 = R\theta' \log_e V_4/V_3$?)

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(d) *Adiabatic compression*

At this stage, cap P is replaced and the gas is further compressed in a reversible process. Since the process is adiabatic, the temperature of the gas will rise. The gas is compressed till the original state A is reached. Thus the cycle is completed.

From previous unit, we know that the total work done will be given by the area enclosed by the whole curve shown in Fig 6(b). The gas can be taken through the cycle again and again and more and more work can be done. *Obviously here heat absorption takes place during isothermal expansion and heat rejection takes place during isothermal compression.* Since system comes back to initial state, there is no change in internal energy; thus from first law the difference between heat supplied and heat rejected by the system will be equal to net work done by the gas during the cycle given by the area $ABCDA$, hence

$$W = Q - Q^1 \qquad \text{...(11)}$$

and Efficiency $\eta = \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{W}{Q} = \dfrac{Q-Q^1}{Q}$

or $$\eta = 1 - \frac{Q^1}{Q} \qquad \text{...(12)}$$

But from Eq. (9) and Eq. (10)

$$\frac{Q}{Q^1} = \frac{R\theta \log_e (V_2/V_1)}{R\theta' \log_e (V_3/V_4)} = \frac{\theta \log_e (V_2/V_1)}{\theta' \log_e (V_3/V_4)} \qquad \text{...(13)}$$

Applying PV = constant, for isothermals AB and CD, we have

$$P_1V_1 = P_2V_2 \qquad \text{..(14a)}$$

$$P_3V_3 = P_4V_4 \qquad \text{...(14b)}$$

where P_1, P_2, P_3 and P_4 are the pressures at A, B, C and D respectively.

Again applying PV^{γ} = constant (incidentally, for adiabatic processes PV^{γ} = constant, where $\gamma = C_p/C_v$) for adiabatics BC and DA, we get

$$P_2V_2^{\gamma} = P_3V_3^{\gamma} \qquad \text{...(15a)}$$

$$P_4V_4^{\gamma} = P_1V_1^{\gamma} \qquad \text{..(15b)}$$

From Eq. (14)

$$P_1P_3V_1V_3 = P_2P_4V_2V_4 \qquad \ldots(16)$$

From Eq. (15)

$$P_1P_3\,(V_1V_3)^{\gamma} = P_2P_4\,(V_2V_4)^{\gamma} \qquad \ldots(17)$$

Dividing Eq (17) by Eq. (16)

$$(V_1V_3)^{\gamma-1} = (V_2V_4)^{\gamma-1}$$

or,
$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \ldots(18)$$

Substituting from Eq. (18) in Eq (13) we get

$$\frac{Q}{Q^1} = \frac{\theta}{\theta'} \quad \text{or,} \quad \frac{Q^1}{Q} = \frac{\theta'}{\theta} \qquad \ldots(19)$$

where θ and θ' are temperatures on perfect gas scale. Now substituting from Eq (19) in Eq. (12), efficiency of Carnot engine can be written as

$$\eta = 1 - \frac{\theta'}{\theta} \qquad \ldots(20)$$

Hence the C.O.P. of Carnot refrigerator will be

$$\text{C.O.P} = \frac{Q^1}{W} = \frac{\theta'}{\theta - \theta'} \qquad \ldots(21)$$

Now having analysed the mode of operation of Carnot cycle, we now intend to show the following consequences:

(i) *Carnot cycle is reversible at each stage*, because instead of extracting an amount of heat Q from a source at θ and transferring a part Q^1 to a sink at θ' and utilising the rest to do work W, one can proceed in such a way that the machine extracts the heat Q^1 from the sink at θ' on performing work W upon the machine and the larger heat Q is transferred to the source at θ. This can be accomplished by proceeding along the reverse route ADCBA shown in Fig.7, i.e. first allowing gas to expand adiabatically from A to D followed by allowing it to expand isothermally from D to C in contact with the sink at θ'; the heat Q^1 is extracted in the process. Then we compress the gas adiabatically to reach B and further compress it isothermally to reach at A. Thus Carnot cycle is perfectly reversible. The larger amount of heat Q is thus transferred to source at θ (at higher tempe-

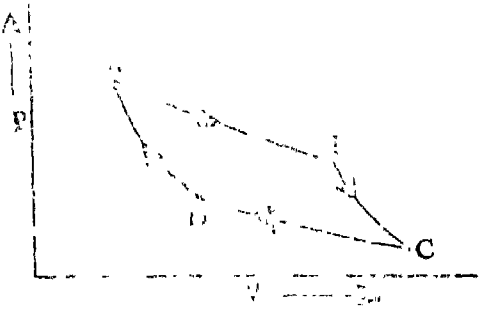

Fig. 7

rature) at the expense of the work given by the area ADCBA. The machine, therefore, instead of acting like a heat engine, acts like a refrigerator.

(ii) Although we have used the perfect gas as the working substance, it is clear from the expression for η that it is independent of the nature of the working substance and since,

$$\eta = 1 - \frac{\theta'}{\theta}$$

it only depends upon the temperatures of the source and the sink.

(iii) *It can be shown that no engine can be more efficient than reversible Carnot engine. The result is sometimes referred to as Carnot theorem.*

We do not intend to do this for you but, of course, give you some hint to work out on your own. Consider an irreversible engine, E_1 working in a cycle which can only extract heat Q from the source, perform work W and restore $Q-W$ to the sink. Now consider a reversible Carnot engine E_2 acting like refrigerator, which extracts $Q-W'$ amount of heat from sink upon doing work W' and restores to the source. Assume $\eta_{E_1} > \eta_{E_2}$. Then combining E_1 and E_2, you arrive at a situation where perpetual motion of second kind must occur if the assumption $\eta_{E_1} > \eta_{E_2}$ is correct. Since second law of thermodynamics debars the occurrence of perpetual motion of second kind, hence $\eta_{E_1} \ngtr \eta_{E_2}$, which implies $\eta_{E_2} > \eta_{E_1}$. All engines such as steam engine, diesel engine, etc. are irreversible and their efficiency is less than that of a Carnot engine

We have seen that no engine can be 100% efficient and we can't do better than Carnot engine. Therefore to get some useful work done some energy must be wasted which could result in thermal pollution. It is observed that GNP (Gross National Product) of a country is directly proportional to energy consumption. This also results in greater pollution. It may then appear that the way modern societies (which for comfort use *energy slaves* such as automobiles, washing machines, aeroplanes, vacuum cleaners, electric appliances such as electric tooth brush) are developing; greater affluence results in greater energy consumption and consequently greater pollution It is also feared that before too long we may run out of energy sources.

Question

What kind of society would you like to live in?

Questions

1. Couldn't we define $\eta = \dfrac{W}{Q^1}$ rather than defining it as $\eta = \dfrac{W}{Q}$

2. How is the efficiency of a reversible engine related to C.O P. of the reversible refrigerator obtained by running the engine backward?

3. Comment on the statement *To produce is to pollute.*

Example 1

A reversible Carnot engine utilises an ideal gas. The temperature of the source and the sink are 500 K and 375 K respectively. If the engine takes 600 k cal of energy per cycle from the source, compute

(a) the efficiency of the engine.

(b) the work done per cycle

(c) the heat rejected to the sink per cycle.

Solution

(a) Efficiency of Carnot engine $(\eta) = 1 - \frac{\theta'}{\theta}$

$$= 1 - \frac{375}{500} = 0.25$$

or $\eta = 25\%$

(b) $\eta = \frac{\text{Work done } (W)}{\text{Heat supplied } (Q)}$

Since heat supplied per cycle $(Q) = 600$ k cal

Work done per cycle $(W) = \eta \times Q$

$$= 0.25 \times 600 \text{ k cal} = 150 \text{ k cal}$$

$$= 6.27 \times 10^5 \text{ joules}$$

(c) If Q^1 be the heat rejected to the sink per cycle

$$Q^1 = Q - W$$

$$= (600 - 150) \text{ kcal} = 450 \text{ k cal}$$

$$= 1.88 \times 10^6 \text{ joules.}$$

Example 2

In a refrigerator, heat from inside at 277 K is transferred to room at 300 K. How many joules of heat will be delivered to the room for each joule of electrical energy consumed ideally. Also compute the coefficier of performance of this refrigerator.

Solution

For an ideal refrigerator

$$\frac{W}{Q} = \frac{Q - Q^1}{Q} = \frac{\theta - \theta'}{\theta}$$

Where the symbols have their usual meanings. Thus

$$Q = W\left(\frac{\theta}{\theta-\theta'}\right)$$

Here $\theta = 300$ K

$\theta' = 277$ K

$W = 1$ joule

Thus $Q = 1 \times \left(\frac{300}{300-277}\right)$ joules $= \frac{300}{23}$ joules $= 13$ joules

$$\text{C.O.P.} = \frac{Q^1}{W} = \frac{\theta'}{\theta-\theta'} = \frac{277}{300-277} = \frac{277}{23} = 12$$

4. Absolute scale of temperature

Efficiency of a heat engine is given by

$$\eta = 1 - \frac{Q^1}{Q} \qquad \text{...(22)}$$

where Q and Q^1 are the heat extracted from the source and heat rejected to the sink respectively. If the source and sink are at temperatures θ and θ' respectively on the perfect gas scale then we have shown for the reversible Carnot engine, using perfect gas as the working substances, that

$$\frac{Q^1}{Q} = \frac{\theta'}{\theta} \qquad \text{.. (23)}$$

The idea was utilised by Kelvin to define a new temperature scale called Kelvin scale or thermodynamic temperature scale through the relation

$$\frac{Q^1}{Q} = \frac{T'}{T} \qquad \text{.. (24)}$$

where T's are temperatures on Kelvin scale. Thus the *ratio of two temperatures on the Kelvin scale is defined as the ratio of heat rejected to heat absorbed by a Carnot engine working between two reservoirs at these temperatures.*

Kelvin scale (or thermodynamic scale) is based on the efficiency of Carnot engine (which is independent of the nature of the working substance), therefore it is independent of the nature of thermometric substance and hence also called the absolute scale of temperature.

Now efficiency η of a Carnot engine in Kelvin scale is

$$\eta = 1 - T'/T \qquad (25)$$

If we put $T'=0$, we get η equal to 1. This defines the absolute zero on the absolute (thermodynamic or Kelvin) scale

This also implies that negative temperature cannot exist on the thermodynamic scale (how ?).

Now since absolute zero is fixed on the thermodynamic scale, all that remains to be done is to fix the magnitude of a degree in the scale. For this, let us keep sink at the triple point (where water vapour, liquid water and ice are all in equilibrium) temperature (T_{tp} = 273.16 K) of water. Then from Eq. (23) the temperature of the source (θ) in perfect gas scale

$$\theta = \frac{Q}{Q^1} \times 273.16\text{K} \qquad \ldots(26)$$

And from Eq. (24), the temperature of the source (T) in the thermodynamic scale is

$$T = \frac{Q}{Q^1} \times 273.16\text{ K} \qquad \ldots(27)$$

From Eq. (26) and Eq (27) we find

$$T = \theta \qquad \ldots(28)$$

i e. thermodynamic scale (or absolute scale) of temperature is the same as that of a perfect gas scale.

Questions

1 Why do we call the thermodynamic scale of temperature as absolute scale ?

2. Define absolute zero in terms of efficiency of a reversible Carnot engine

ASSIGNMENT

1. (a) If we take a rubber band and pull it, it gets warm, and this warming is reversible; and a rubber band supporting a load contracts on being heated. Explain these facts.

   (Hint: Proceed with the idea that rubber consists of an enormous tangle of long chains of molecules and between the chains there are cross links)

   (b) Give a qualitative explanation of how frictional forces between moving surfaces produce heat energy. Does the reverse process (heat energy producing relative motion of these surfaces) occur ? Explain.

2. (a) We cause a real gas to go from an initial equilibrium state i

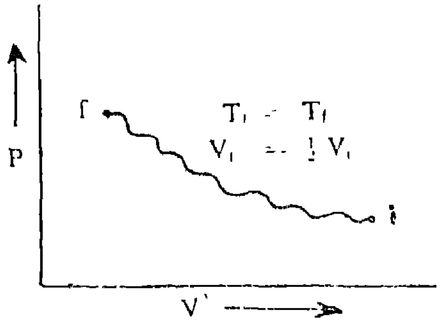

Fig. 8

(Fig 8) described by P_i, V_i, T_i to a final equilibrium state f described by P_f, V_f, $(=\frac{1}{2}V_i)$ and T_f $(=T_i)$. We carry out the process irreversibly, can we calculate the work done in terms of an area on a P-V diagram ? Is any work done ?

Ans. Yes

(b) To carry out a Carnot cycle we need not start at point A, but may equally well start at points B, C or D or indeed any intermediate point (Fig. 9). Explain.

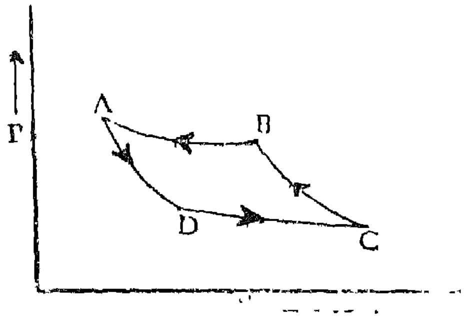

Fig 9

3 (a) An ideal gas heat engine operates in a Carnot cycle between 227°C and 127°C. It absorbs 6.0×10^4 kcal at the higher temperature. How much work per cycle is this engine capable of performing?

Ans 5.02×10^4 joules

(b) In a mechanical refrigerator the low temperature coils are at a temperature of —13C, and the compressed gas in the condenser has a temperature of 27°C. What is the theoretical coefficient of performance?

Ans. 6.5

4. A cylinder contains oxygen at a pressure of 2 atmospheres The volume is 3 litres and the temperature is 300 K The gas is carried through the following process. (i) heated at constant pressure to 500 K, (ii) cooled at constant volume to 250 K; (iii) cooled at constant pressure to 150 K; (iv) heated at constant volume to 300 K. Show these processes on a $P—V$ diagram. Calculate the net work done by the oxygen and the heat flowing into the oxygen.

Ans. 202 joules, 48 cal

5. What is the minimum power that must be supplied to a refrigerator that freezes 20 kg of water at 0°C into ice at 0°C in 5 minutes where the room temperature is 20°C ?

Ans. 1.6 kW

SELF-ASSESSMENT

1. (a) If a Carnot engine is independent of the working substance, then perhaps real engines should be similarly independent, to a certain extent. Why then, for real engines, are we so concerned to find suitable fuels such as coal, gasoline, or fissionable material? Why not use stones as a fuel?

   (b) What factors reduce the efficiency of a heat engine from its ideal value?

   (c) In order to increase the efficiency of a Carnot engine most effectively, would you increase T_1 keeping T_2 constant, or would you decrease T_2, keeping T_1 constant? Here T_1 and T_2 are the temperatures of the source and the sink respectively.

   (d) Can a kitchen be cooled by leaving the door of an electric refrigerator open? Explain.

   (e) Two samples of a gas initially at the same temperature and pressure are compressed from a volume V to a volume $(V/3)$, one isothermally and the other adiabatically In which sample is the final pressure greater?

2. A Carnot engine operates between 350°C and 0°C and it takes 5 k cal of heat from the source. Compute (a) its thermal efficiency (b) work done per cycle and (c) heat rejected per cycle

3. Assuming that a domestic refrigerator can be regarded as a reversible engine working between the temperature of melting ice and that of atmosphere at 17°C, calculate the energy required to freeze 1 kg of water at 0°C.

4. In a two-stage heat engine a quantity of heat Q_1 is absorbed at T_1, work W_1 is done, and a quantity of heat Q_2 is expelled at a lower temperature T_2 by the first stage. The second stage

absorbs the heat expelled by the first, does work W_2, and expels a quantity of heat Q_3 at a lower temperature T_3. Prove that the efficiency of the combination engine is $(T_1-T_3)/T_1$.

5. Using the equation of state of an ideal gas and the equation describing an adiabatic process for an ideal gas, show that the slope dP/dV, on a P—V diagram of an adiabatic curve can be written as $(-\gamma P/V)$ and of an isothermal can be written as P/V. From these results prove that adiabatics are steeper curves than isothermals.

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UNIT 14

Physics for Class XI

RADIATION

F-15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is fourteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr. K. J. George who is working as a Senior Project Fellow helped me in correcting the proofs.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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New Delhi
20 October 1982

INTRODUCTION

From the first law of thermodynamics you have seen that heat energy flows from warmer bodies to cooler bodies. The flow of heat occurs in three ways, viz. conduction, convection, and radiation. The phenomenon of heat conduction is believed to take place by molecular collisions in solids. In convection heat is transferred by the bodily motion of the molecules in a fluid. In contrast to these two methods of heat transfer, radiation does not require any material medium for the transfer. For example, light, heat, etc. are transferred to the earth from the sun by radiation because space is mostly empty. The term radiation refers to continuous emission of energy from the surface of all bodies This energy is called radiant energy and is in the form of electromagnetic waves in the infra-red region. Laws of radiation which you will come across in this unit are based upon thermodynamics. However, their applications will be mainly in astrophysics—to estimate the temperature of the sun and the distant stars, to identify the elements in solar atmosphere, and so on.

OBJECTIVES

To be able to

1. State the important facts which one can deduce from the radiant energy spectrum of a heated body.
2. Define emissive power and absorptive power of a body.
3. Apply Wien's law to estimate the temperature of heavenly bodies,
4. Deduce Kirchoff's law and explain its two applications, viz. to study the atmosphere of stars and to design a perfectly black body for experimental purposes.

5. Apply Stefan's law to estimate the temperatures of hot bodies, solar constants at various planets of the solar system, and also to deduce Newton's law of cooling.

6. Explain the principle behind the temperature measurement of hot bodies by a radiation pyrometer.

7. Solve problems based on objectives (3), (4) and (5).

SUGGESTED READING MATERIAL

1. Spectrum of the radiant energy

All bodies emit radiation with wavelengths that are characteristic of the body's temperature, e.g. the sun, at 6000K, emits energy mainly in the visible spectrum and the earth at an ideal radiation temperature of 255K, radiates energy mainly in the far infra-red (heat) region of electro-magnetic spectrum. The human body also radiates energy in the infra-red region.

The term radiation refers to the continual emission of energy from the surface of all bodies. This energy is called radiant energy and is in the form of electromagnetic waves. These waves travel with the velocity of light and are transmitted through vacuum as well as through air. Upon absorption the radiant energy transforms to energy of molecular motion (heat).

Let us now perform a simple experiment Take a piece of blackened platinum wire in a dark room and pass through it electrical current which serves to heat the wire. The magnitude of the current is gradually increased. After sometime the wire becomes warm and emits radiant energy. When a slightly stronger current is passed the wire begins to glow with dull red light. This shows that the wire is just emitting red radiation of sufficient intensity to affect the human eye. Accurate observation has shown that this takes place at nearly 525°C. With increasing temperature, the colour of the emitted radiation changes from dull red to cherry red (at nearly 900°C) to orange (at nearly 1100°C), to yellow (at nearly 1250°C), until at about 1600°C, it becomes white. What do you infer from this? Firstly, the temperature of a luminous body can be estimated from its colour Secondly, *with the increase in temperature, more waves of shorter wavelengths (since red light is of longer wavelength than orange, yellow, etc) are emitted by a heated body in sufficient intensity.* Vice-versa one may argue that when the temperature of the wire is below 525°C, it emits waves longer than red, but these waves can be detected only by their heating effect.

Now at any temperature the radiant energy emitted by a body is a mixture of waves of different wavelengths, but most intense of these waves will have a particular wavelength (say λ_m) e g., at 400°C the λ_m will be about 5×10^{-4} cm for a copper block, for wavelengths either greater or less than this value the intensity decreases as shown in Fig 1 where rate of emission for Cu block per unit range of wavelength e_λ has been plotted against wavelength λ The corresponding distribution of energy at higher temperatures is also shown.

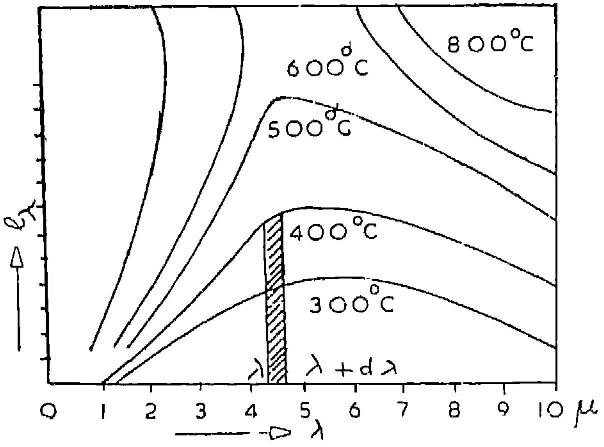

Fig. 1

Evidently area between each curve and the horizontal axis represents the total rate of radiation at that temperature (How?). A study of the curves shown in Fig. 1 clearly indicates two facts, viz.

(1) Total rate of radiation at a particular temperature (represented by the area between each curve and the horizontal axis) increases rapidly with temperature.

(2) Each curve has a definite energy maximum e_m and corresponding to that a wavelength λ_m (i e wavelength of the most intense wave). The λ_m shifts towards left, or towards shorter wavelengths with increasing temperature.

The second fact which one has inferred from the curves shown in Fig. 1 can be expressed quantitatively by what is known as *Wien's displacement law* The law was actually deduced from thermodynamic considerations. It simply states that λ_m *shifts towards shorter wavelengths as the temperature of a body is increased.* This law is, strictly valid only for black bodies (about which we are going to discuss shortly). Mathematically

$$\lambda_m \propto \frac{1}{T}$$

$$\text{or,} \quad \lambda_m T = \text{constant} \qquad \ldots(1)$$

The constant in Eq (1) is found to have a value of 2884 micron-degrees. This furnishes us with a simple method of determining the temperature of all radiating bodies including those in the heavens. Thus for the moon, $\lambda_m = 14$ microns, and hence temperature of the moon $T = 2884/14 = 206$ K.

Questions

1. Can you apply Wien's displacement law for estimating the temperatures of heavenly bodies in the strict sense?
2. It is given that $(e_\lambda / e_{\lambda'}) = (T/T')^5$. How will you obtain the distribution of energy at a second temperature T' if the distribution at any temperature T is given ?

Problem

1. (a) If the temperature of the sun is 6050 K, calculate the wavelength of the maximum emission ? What does it correspond to, in the spectrum of electromagnetic waves ?

   (b) The ideal radiation temperature of the earth is 255 K, estimate the wavelength corresponding to the most intense wave.

From the curves shown in Fig. 1, it is clear that *at a particular temperature and for a particular wavelength range* λ *and* $\lambda + d\lambda$, *the rate of emission (i.e. amount of radiant energy emitted per square metre area of a surface, per second), per unit wavelength range* e_λ *is constant. Hence at a given temperature and wavelength, the energy emitted by a surface can be best expressed in terms of* e_λ. *We will, hereafter, refer to* e_λ *as the emissive power of the surface at a given temperature and wavelength.* For your convenience, let us put e_λ in the form of a derivative.

Let dE_λ = Amount of energy emitted by 1 m^2 area of a surface in one second within the wavelength range λ and $\lambda + d\lambda$ (of course, at a given temperature).

and, $d\lambda$ = Wavelength interval.

then $$e_\lambda = \frac{dE_\lambda}{d\lambda} \qquad \text{...(2)}$$

When radiation is incident over a surface, a part of it gets reflected, a part of it gets transmitted and rest are absorbed. Of course, for different surfaces, degrees of reflection, transmission and absorption will be different, e g we know from our experiences in everyday life that a bright polished surface reflects most of the radiation incident upon it, whereas a rough black surface absorbs most of the radiation falling on it. We define *absorptive power* a_λ of a surface, as

$$a_\lambda \text{ (at a given temperature and wavelengh } \lambda\text{)}$$

$$= \frac{\text{Total amount of radiation absorbed between } \lambda \text{ and } \lambda + d\lambda}{\text{Total amount of radiation incident between } \lambda \text{ and } \lambda + d\lambda} \qquad \text{.. (3)}$$

Questions

1. Why is heat loss by radiation so critically dependent on the temperature of a body ?

2. Why do we wear light coloured clothing during summer ?

3 Are the emissive power and absorptive power of a surface constants ? Explain.

2. Black body radiation and Kirchoff's law

As we have already pointed out, when radiation falls on matter, it may be partly reflected, partly absorbed and partly transmitted.

If for a particular wavelength λ, and for a given surface

r_λ = fraction of total incident energy reflected.

a_λ = fraction of total incident energy absorbed (absorptive power of the surface).

t_λ = fraction of total incident energy transmitted.

then obviously

$$1 = r_\lambda + a_\lambda + t_\lambda \qquad \ldots(4)$$

For a perfectly black body, $r_\lambda = t_\lambda = 0$ *and* $a_\lambda = 1$.

Thus, radiations incident over black bodies will be totally absorbed. Lamp black is the nearest approach to such a body. Apparently, it neither reflects nor transmits the light which falls on it, absorbs all, and hence appears black. But the perfectly black body does not exist in reality, for even the lamp black is found to transmit light of long wavelength. It absorbs about 96% of visible light, platinum black absorbs about 98%.

A perfectly white body, in contrast, is defined as a body with $a_\lambda = 0$, $t_\lambda = 0$ and $r_\lambda = 1$ and surface is such that reflection is diffused. A piece of white chalk approximates, as far as visible radiation is concerned, to a perfectly white body.

We now want to discuss about what is known as *Kirchoff's law* which is true only for *temperature radiation*. The term *temperature radiation* needs a little elaboration. Material substances at all temperatures are found to radiate, at low temperatures radiation of long wavelengths and if raised to higher temperature radiation of smaller wavelengths. Such kind of radiation is *temperature radiation*. Other methods of making matter to emit radiation such as by passing electric discharge through its gaseous state, or by chemical actions as in flames, do not come under this category.

In order to deduce *Kirchoff's law* we take an enclosed space and let its walls be opaque to radiation of all wavelengths. The walls are first imparted a uniform temperature and then thermally insulated from the surroundings. Imagine the enclosure to be filled with radiation being emitted by the walls having wavelength lying between λ and $\lambda + d\lambda$ and let a body A be placed inside it It is not difficult to prove that whatever the initial temperature of A might have been, it will ultimately acquire the temperature of the walls, i e , A will come in thermal equilibrium with its immediate surroundings. Now since the temperature is constant, the rate of emission of A must be equal to its rate of absorption of energy. For the given temperature of the body if e_λ is the emissive power of the surface then the energy emitted per square metre, per second, between λ and $\lambda + d\lambda$ will be $e_\lambda \, d\lambda$ [Ref Eq. (2)] Let the energy falling on unit area per second within the same wavelength range and temperature be dQ. If for the given temperature and wavelength, absorptive power is a_λ, then the total energy absorbed per unit area per second will be $a_\lambda \, dQ$ [Ref. Eq (3)].

Since body is in thermal equilibrium energy emitted by the body can be equated, to the energy absorbed and we have

$$e_\lambda \, d_\lambda = a_\lambda \, dQ$$

or

$$\frac{e_\lambda}{a_\lambda} = \frac{dQ}{d\lambda} \qquad \text{...(5)}$$

We know that for any given temperature, dQ is a constant. So for a fixed range of wavelength $d\lambda$, dQ will be constant. This implies that e_λ / a_λ is a constant for any surface. In case of a perfectly black body, let, E_λ be the emissive power and a_λ be the absorptive power, where of course, $a_\lambda = 1$. Using Eq. (5) for such a body

$$E_\lambda \; 1 \, d\lambda = 1 \, dQ \qquad \text{...(6)}$$

$$\frac{dQ}{d\lambda}=E_\lambda \qquad \ldots(7)$$

Eq. (5) and Eq. (7) give

$$\frac{e_\lambda}{a_\lambda}=E_\lambda \qquad \ldots(8)$$

Eq. (8) is known as Kirchoff's law which can be stated as follows :

At any temperature the ratio of emissive power to the absorptive power of a substance is constant and equal to the emissive power of a perfectly black body

We have proved the law here for bodies inside the enclosure. A little consideration will show that, since the emissive and absorptive power depend only upon the physical nature of the body and not upon its surroundings, the law will hold for all bodies under all conditions for pure temperature radiation.

This law clearly implies that bodies with high emissive powers (good emitters) will also have very high absorptive powers (good absorbers, since the ratio of these is constant for a given temperature and wavelength range. But since each body must either absorb or reflect the radiant energy reaching it, a *good absorber must be a poor reflector* (*or good emitter*).

3. Applications of Kirchoff's law

(1) All elements on being heated emit their characteristic wavelengths. A given element will be a good emitter of its characteristic wavelength and naturally from Kirchoff's law it will also be a good absorber of its characteristic wavelength, e.g, when a flame tinged with NaCl is observed in the spectrometer two sodium yellow lines of wavelengths 5890 A° and 5896 A° are seen. If white light is viewed, through a sodium vapour using a spectrometer of very high resolving power in the laboratory two dark lines (due to absorption by sodium) are observed in the continuous spectrum corresponding to wavelengths 5890 A° and 5896 A°.

Let us take the case of sun consisting of glowing mass emitting a continuous spectrum without any dark lines. When this light passes through sun's atmosphere which is at a relatively lower temperature many elements present in it absorb their characteristic wavelengths and dark lines are seen in those regions of the spectrum. Thus by studying the emission spectra of the sun different elements present can be identified.

The dark lines observed in the spectrum of sunlight are called Fraunhofer lines and are denoted by the letters A,B,C,D...... Similar procedure can also be adopted with other stars.

(2) Kirchoff's law also enables us to design a perfectly black body for experimental purposes. We go back to an enclosure at constant temperature containing radiations between wavelength range λ and $\lambda + d\lambda$. Now let us make a small hole in the enclosure and examine the radiation escaping out of it. This radiation is made up of single, double, triple, etc. reflection from the walls. Thus if the reflecting power of the surface of the wall is r, while the emissive power is e, the total radiation escaping out is

$$= e + er + er^2 + er^3 + \ldots\ldots$$

$$= e(1 + r + r^2 + r^3 + \ldots\ldots$$

$$= \frac{e}{1-r} \qquad \ldots(9)$$

But from Kirchoff's law $\frac{e}{a} = E$.

$$\text{or, } e = Ea \qquad \ldots(10)$$

where E is the emission from a black body. If now walls are assumed to be opaque (i.e. $t = 0$) from Eq. (4).

$$a = 1 - r \qquad \ldots(11)$$

Substituting from Eq. (11) in Eq. (10), we get

$$e = E(1-r)$$

$$\text{or, } \frac{e}{1-r} = E \qquad \ldots(12)$$

Comparing Eq. (9) and Eq. (12) it is clear that *the diffuse radiation emerging out of the hole will be nearly identical with radiation from a perfectly black emissive surface.* Smaller the hole, the more completely black the emitted radiation is. So we see that the uniformally heated enclosure with a small cavity behaves as a black body towards emission.

Again such an enclosure behaves as a perfectly black body towards incident radiation also. For any ray passing into the hole will be reflected internally within the enclosure and will be unable to escape outside. This may be further improved by blackening the inside. Hence the enclosure is a perfect absorber and behaves as a perfectly black body.

In Fig 2, such a black body due to Fery has been shown. There is a cavity in the form of a hollow sphere with its inside coated with black material and has a small conical opening O. Note the conical projection P opposite the hole O. This is to avoid direct radiation from the surface opposite the hole which would otherwise make the body not perfectly black

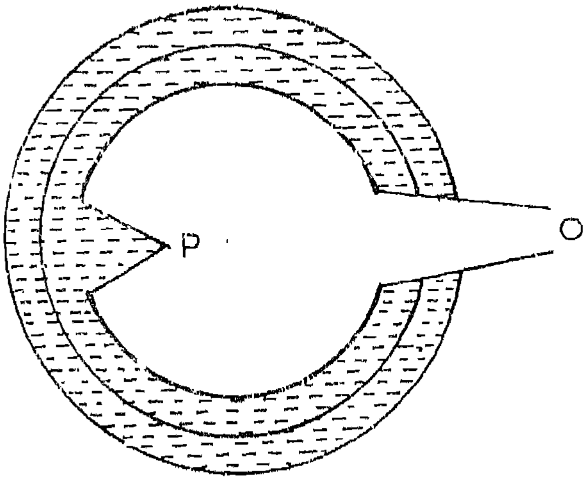

Fig. 2

The black body commonly used (due to Wien) in practice consists of a metallic chamber made of brass or platinum and blackened inside.

It is electrically heated using a heating element The radiation coming out of the small hole in the chamber is used for experimental purposes.

Questions

1. Is the blackening of the walls of the enclosure to make black body for experimental purposes, really necessary ?
2. A hole in the wall of a cavity radiator is sometimes called a black body, why ?
3. It is stated that if we look into a cavity whose walls are maintained at constant temperature no details of the interior is visible. Does this seem reasonable ?
4. A piece of decorated china is heated in a furnace to about 1000°C and then taken out suddenly in a dark room, the decorations appear much brighter than the white china, why?
5. When a piece of green glass is heated in a furnace and then taken out, it glows with a red light, why? What will happen, if instead, you take a red glass?

   [Hint : We know that green glass looks green because it absorbs red light strongly and reflects or transmits the green (red and green being complimentary colours)].

4. Stefan's law

Experimental measurement of the rate of emission of radiant energy from the surface of a body were made by John Tyndall and on the basis of these Joseph Stefan and Boltzmann concluded that the energy emitted per square metre of the surface per second could be expressed by the relation

$$E = e \sigma T^4 \quad \ldots(13)$$

where σ is a constant, called Stefan's constant, and has the value 5.672 × 10^{-8} $J/m^2s\ deg^4$. T is the Kelvin temperature of the surface and e is a quantity, sometimes called the emissivity or relative emittance, *which depends upon the nature of the surface and temperature.* The values of e lies between 0 and 1 (of course, even for the same material if temperature ranges are different, e will be different), being small for polished metals and almost 1 for black, rough materials. The emissivity e of an ideal black surface is equal to unity, hence Stefan's law (Eq. 13) for ideal black surface will be

$$E_B = \sigma T^4 \qquad \dots(14)$$

Where E_B -- The energy emitted per square metre of an ideal black surface per second.

One may tend to think from Eq. (13) that if the surfaces of all bodies are continually emitting radiant energy, why don't they eventually radiate away all their internal energy and cool down to a temperature of absolute zero [where $E = 0$ from Eq. (13)]. The answer is that they would do so if energy were not supplied to them in some way. In fact, all the objects are both radiating and absorbing radiant energy simultaneously. If a body is at the same temperature as its surroundings, the rate of emission is same as rate of absorption, there is no net gain or loss of energy, and no change in temperature. However, if a body is at lower temperature than its surroundings, the rate of absorption will be greater than the rate of emission and temperature will rise till its temperature is equal to room temperature. Similarly if a body is at higher temperature, the rate of emission will be greater than rate of absorption and hence there will be a net energy loss. Hence for such a body at a temperature T_1, with surroundings at a temperature T_2, the amount of net energy loss (if $T_1 > T_2$) per second per square metre is

$$E_{net} = e\,\sigma\,(T_1^4 - T_2^4),\ for\ T_1 > T_2 \qquad \dots(15)$$

Question

What will be the expression for net energy gained per second per square metre if $T_2 > T_1$.

One can deduce *Newton's law of cooling* from Stefan's law. Newton's law of cooling states that the *rate of loss of heat (cooling) of a hot body is directly proportional to the mean excess of temperature of the hot body over that of its surrounding provided the difference of temperature is very small.*

Let a body at temperature TK be surrounded by another body at T_0K. The rate at which heat is lost per unit area per second by the hot body is

$$E = e\,\sigma\,(T^4 - T_0^4)$$

$$= e\,\sigma\,(T - T_0)(T^3 + T^2T_0 + TT_0^2 + T_0^3)$$

If $(T\text{-}T_0)$ is very small, $T \sim T_0$.

$$T^3 \sim T_0^3,\ T^2T_0 \sim TT_0^2 \sim T_0^3$$

$$E = e\sigma\,(T\text{-}T_0)\,4\,T_0^3.$$

$$E = k\,(T\text{-}T_0) \text{ where } k - 4e\,\sigma\,T_0^3.$$

or

$$E \propto (T\text{-}T_0) \qquad ..(16)$$

Which is the Newton's law of cooling.

Example I

Determine the surface area of the filament of a 100 W incandescant lamp at 3000 K. Given $\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4}, and emissivity e of the filament = 0.3.

Solution

According to Stefan-Boltzmann law

$$E = eA\,\sigma\,T^4$$

Where

E = Rate of energy emitted = 100 W

A = Surface area

T = Temperature of the surface = 3000 K

Hence $$A = \frac{E}{e\,\sigma\,T^4}$$

$$= \frac{100}{0.3 \times 5.7 \times 10^{-8} \times (3000)^4}\ \text{m}^2$$

$$\therefore A = 7.27 \times 10^{-5}\ \text{m}^2$$

Example 2

A pan filled with hot food cools from 365 K to 361 K in 2 minutes, when the room temperature is 293 K. How long will it take to cool from 344 K to 342 K ?

Solution

Average of 365 K and 361 K is 363 K According to Newton's law of cooling

$$\frac{\text{Change in temperature}}{\text{Time}} \propto (T - T_0); \text{ (How ?)} \qquad \ldots(16a)$$

Change in temperature = (365 − 361) K = 4 K

Time = 2 Minutes

$$(T - T_0) = (363 - 293) \text{ K} = 70 \text{ K}.$$

Hence from Eq. 16 (a)

$$\frac{4}{2} \text{ K min}^{-1} = \text{Constant} \times 70\text{K} \qquad .\ (16b)$$

Again average of 344 K and 342 K is 343 K. Thus

$$\text{Change in temperature} = (344 - 342) \text{ K}$$

$$= 2 \text{ K}$$

$$(T - T_0) = (343 - 293) \text{ K} = 50 \text{ K}$$

$$\text{Hence } \frac{2}{\text{Time}} \text{ K min}^{-1} = \text{Constant} \times 50 \text{ K} \qquad \ldots(16c)$$

from Eq. 16(b) and Eq. 16(c)

$$\text{Time} = \frac{70}{50} \text{ min}$$

$$= 1.4 \text{ min}$$

Question

Write down the factors upon which radiation *emitted by a body depends.*

Problems

1. The filament of a tungsten lamp radiates energy at the rate of 200 watts when operating at a temperature of 2500K. A second lamp operating at a temperature 3000 K radiates four times as much energy; compare the areas of the filaments.

(Ans. 1.93 : 1)

2. A copper block at a temperature of 373 K radiates about 0.03 watt from each square centimetre of its surface. At a temperature of 773 K how many watts will be radiated from one square cm area of it ? How many watts at a temperature of 1273 K ?

(Ans 0.54 watt/cm^2, 4 watt/cm^2)

3. The energy radiated by two black bodies are in the ratio 1:2. If the temperature of first body is 2000 K, compute the temperature of the second body.

An application of Stefan's law

Stefan's law can be used to determine the solar constants at different planets of the solar system. Let the radius of the sun be r, then total amount of energy e emitted by the sun in one second (assuming sun to be a black body) is

$$e = (4\pi r^2) \times (\sigma T^4) \qquad \ldots(17)$$

Where T = Temperature of the sun

σ = Stefan's constant

Then the amount of energy E received per unit surface of the earth in one second is

$$E = \frac{e}{4\pi R^2} = \left(\frac{r}{R}\right)^2 \sigma T^4 \qquad \ldots(18)$$

Where R = Distance between the sun and the earth.

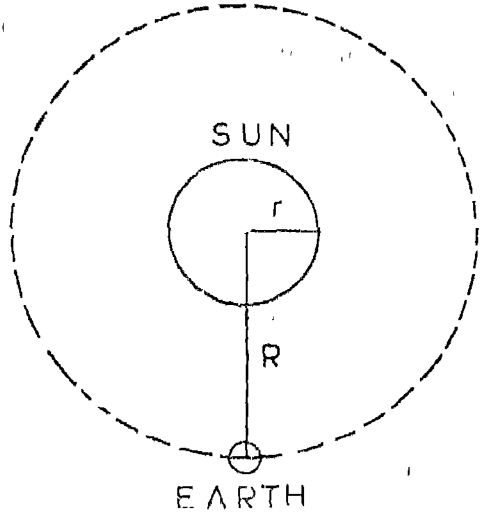

Fig. 3

Eq. (18) gives the solar constant at the earth. If solar constant at any other planet, distant R′ from the sun, is E', then

$$E' = \left(\frac{r}{R'}\right)^2 \sigma T^4 \qquad \text{...(19)}$$

Hence

$$\frac{E'}{E} = \left(\frac{R}{R'}\right)^2 \qquad \text{...(20)}$$

From Eq. (20) it is clear that if E and (R/R′) are known E' can be calculated. The solar constant for the earth is found to be 1.36×10^3 $Jm^{-2}s^{-1}$. The solar constant at mars whose distance from the sun is 1.52 times that of the earth will be approximately 6×10^2 $Jm^{-2}s^{-1}$ [Using Eq. (20)].

5. Radiation pyrometers

Stefan's law, Wien's displacement law, etc. can be used to measure the temperature of hot bodies. But you must have realised that these laws are true only for black bodies and actual hot bodies are seldom black.

The instrument developed to measure high temperatures using emitted radiations is called a radiation pyrometer. A pyrometer differs from conventional devices for measuring temperature like thermometer and thermocouples from the following facts :

(a) A pyrometer need not be kept in contact with the hot body whose temperature is to be measured.

(b) With a pyrometer, although lower practical limit of temperature measurement is about 900 K, there is no higher limit.

However, a radiation pyrometer suffers from a serious draw back. It can measure accurately the temperature of black bodies only

In Fig. 3, a typical radiation pyrometer due to Fery is shown. The beam of radiation from the hot body enters the instrument through the aperture AA. The concave mirror

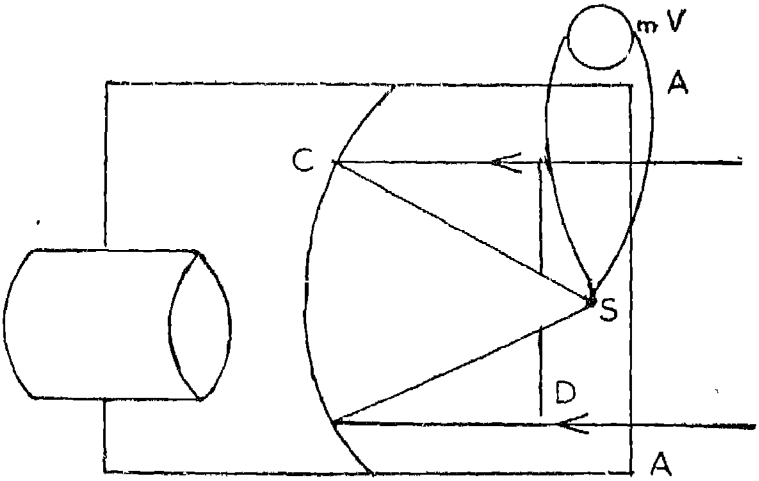

Fig. 4

C converges the beam. The focussed beam is made to pass through a limiting diaphram D before it is incident on the thermocouple S. The *emf* of the thermocouple is measured by a milli-voltmeter.

Generally pyrometer is used to measure the temperature of non-black bodies. If, however, pyrometer is first sighted on a black body at temperature T_1 and E_1 is the thermo *emf* produced (which is propoitional to temperature difference between hot and cold junction of the thermo-couple), from Stefan's law,

$$E_1 \quad a\,(T_1^4 .. T_0^4) \qquad ...(21)$$

Where T_0 is the temperature of the pyrometer surroundings. The constant a depends upon Stefan's constant and the absorptive power of of the thermo-junction, etc.

Now pyrometer is sighted on a non-black body at temperature T_2 (which is to be measured). Let the thermo *emf* thus produced be E_2 and e_c and r' be the emissivity and reflectivity of the non-black body respectively. Then again from Stefan's law

$$E_2 = a(e_c T_2^4 + r' T_0^4 - T_0^4) \qquad ...(22)$$

(The second term of Eq. (22) shows that radiations at temperature T_0 is being reflected black by the non-black body).

If we assume that the non-black body transmits very little, then $r' \sim (1 - e_c)$, which gives.

$$E_2 = a\,[e_c\,(T_2^4 - T_0^4)\,] \qquad ...(23)$$

Hence from Eq. (21) and Eq. (23)

$$\frac{E_2}{E_1} = e_c \frac{(T_2^4 - T_0^4)}{(T_1^4 - T_0^4)} \qquad ...(24)$$

If e_c of the non-black body is known, from Eq. (24), its temperature T_2 can be calculated.

Problem

When a Fery total radiation pyrometer, in a room with surroundings at 300 K, is sighted on a black body at 500 K the deflection observed in the galvanometer is 5 divisions. When sighted on a non-black body (emissivity $e_c = 0.3$) the deflection is found to be 400 divisions, calculate the temperature of the latter.

ASSIGNMENT

1. The operating temperature of a tungsten filament in an incandescent lamp is 2450 K and its emissivity is 0.30. Find the surface area of the filament of a 25 W lamp.

   Ans. 4×10^{-5} m^2

2. A solid cylindrical copper rod 10 cm long has one end maintained at a temperature of 20 K. The other end is blackened and exposed to thermal radiation from a body at 300 K, no energy is lost or gained elsewhere When equilibrium is reached, what is the temperature of the blackened end? Assume thermal conductivity equal to 0.92 cal/cms K.

   Ans. 20 12 K

3. At what wavelength does a cavity radiator at 300 K radiate most ? Explain in brief the law involved.

   Ans. 9.61μ

4. A cavity whose walls are held at 6000 K has a circular aperture 5.0 mm in diameter. At what rate energy will be escaping from the aperture. Ans. 1.4×10^3 J

5. (a) What is a pyrometer? How is it superior to other temperature measuring instruments used for measuring high temperatures.

   (b) 'Pockets' formed by the coals in a coal fire are seen brighter than coal themselves. Is the temperature in such pockets appreciably higher than the surface temperature of an exposed glowing coal?

SELF-ASSESSMENT

1. (a) Wavelength corresponding to E_{max} of a star is 4000 A, compute the temperature of the star.

   (b) The temperature of tungsten filament in an incandescent lamp is 3000K, find the surface area of the filament of a 200 W lamp. The emissivity of the filament is 0.4.

   Given $\sigma = 5.7 \times 10^{-8}\ \mathrm{Jm^{-2}s^{-1}K^{-4}}$,

2. A blackened solid copper sphere of radius 2 cm is placed in an evacuated enclosure whose walls are kept at 100°C. At what rate must energy be supplied to the sphere to keep its temperature constant at 127 C

3. A cylindrical metal can 10 cm high and 5 cm in diameter contains liquid helium at 4K, at what temperature its latent heat of vaporisation is 5 cal/g. Completely surrounding the helium can are walls maintained at the temperature of liquid nitrogen (80 K), intervening space being evacuated How much helium is lost per hour ? Assume the emissivity to be 0.2.

4. (a) Do all incandescent solids obey the fourth power law (Stefan's law) of temperature ? Explain.

   (b) State Kirchoff's law. Discuss some applications of this law.

5. (a) State the important facts, which one can obtain from the experimental study of the spectrum of black body radiation,

   (b) The relation $E = \sigma T^4$ is exact for truely black bodies and holds for all temperatures. Can we use this relation as the basis of a definition of temperature?

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SURFACE TENSION

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PREFACE

This unit is fifteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr. K. J. George who is working as a Senior Project Fellow helped me in correcting the proofs

At this stage this unit is an experimental edition I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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New Delhi
10 November 1982

INTRODUCTION

You must have observed water drops dripping from a wet cloth. You may also have observed the behaviour of mercury globules. In these cases one can find that liquid drops assume spherical shape. Naturally one might ask, why such tendency is there? There are a few other questions like how does water surface behave when a needle is placed over it gently. In the present unit our endeavour would be to satiate such curiosities You will come to know how surface tension can explain above mentioned tendencies. Also we will understand the nature of intermolecular forces which are different from gravitational forces We call them as Van der Waals' forces after the name Van der Waals who predicted their existence making some modifications in the perfect gas model about which you studied in previous units. Besides, laws of thermodynamics will be applied to the liquid films.

OBJECTIVES

To be able to

1. Describe the nature of intermolecular forces.
2. Explain the formation of concave and convex miniscus of different liquids under the action of cohesive and adhesive forces.
3. Explain the origin of surface energy and surface tension.
4. Enunciate the dependence of angle of contact in terms of surface energy.
5. Derive the relation for excess pressure for a surface film or liquid drop.

6. Explain the phenomenon of capillarity and derive expression for capillary ascent and decent.

7 Solve problems based on surface energy, excess pressure, and capillarity, etc.

SUGGESTED READING MATERIAL

1. General ideas about intermolecular interactions

In unit 10, you studied about the perfect gas model Following are the basic assumptions for such a model :

(a) The molecules of a gas are mass points occupying no volume or negligible volume. In other words, the diameters of the molecules are negligible as compared to the intermolecular distance.

(b) There is no force of interaction between the molecules of a gas.

The equation of state for n moles of such a gas is

$$P V = n R T \qquad ...(1)$$

where the symbols have their usual meanings. As you know, Eq. (1) is, strictly speaking, not true for real gases. The equation of state for the real gases was put forward first by Van der Waals on the basis of the following assumptions which are antithetic to the assumptions written above (regarding a perfect gas model).

(a) The molecules of a real gas are hard spheres occupying very small but finite volumes.

(b) A weak intermolecular force of interaction exists between the gas molecules which depends only upon the intermolecular distance and not upon the orientation of molecules, the force being practically vanishing at distances several times the molecular diameter.

This intermolecular interaction, (which is also called Van der Waals' force) exists not only in gases but also in liquids and even in some solids. It arises due to the interaction between permanent and induced dipoles (A dipole is the combination of two opposite charges of same magnitude separated by a small distance as shown in Fig. 1). One pertinent question is how do we get dipole out of a neutral molecule? The answer, however, lies on the fact that the atoms in the molecule are normally so arranged that the centre of mass of the positive charges does not fall exactly on the centre of the mass of the negative charges. In other words, it is due to uneven distribution of charges on a molecule (We are not elaborating it further because you must have studied about it in chemistry.) The non-coincidence of two centres of charges gives rise to dipole formation. The dipole moment (a vector, directed from positive towards negative charge) Q for a dipole shown in Fig. 1 is defined as

$$Q = q \Delta l \qquad (2)$$

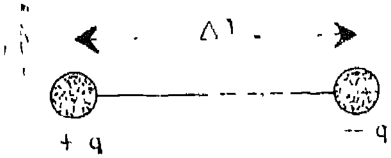


Fig. 1

Some molecules (like those of water) have permanent dipole moments, where as molecules of CO_2, N_2, O_2, etc. have induced dipoles (i.e., only when placed in an electric field, dipole formation will occur). The dipole—dipole (permanent) interaction is much stronger than interaction between induced dipoles. Hence molecules with permanent dipoles are normally gases (why ?). As already mentioned, Van der Waals' interaction is due to interaction between dipoles (permanent and induced), hence the interaction is essentially electrical in origin.

Now, given an assembly of molecules interacting via Van der Waals' force, the interaction should be such that it is attractive at large distances and repulsive at short distances otherwise the whole assembly will collapse to a point. Since attractive force should decrease very sharply with distance of separation between molecules, it is convenient to consider a sphere of action around a molecule, and any molecule whose centre is lying inside the sphere of action will exert some force over the molecule concerned. If the force of interaction happens to be attractive, in taking away a molecule from the assembly, some work must be done which will be negative and this work will be stored up in the form of negative potential energy. Similarly if force is repulsive, given a slight initial push, molecule will get moved on its own; hence work done or potential energy will be positive. This inter molecular potential energy V is a function of intermolecular separation r, i e,

$$V = V(r) \qquad \text{.. (3)}$$

Thus, the intermolecular force F defined by the relation $F = -\frac{dV}{dr}$ could also be a function of r. In Fig. 2 we have shown the variation of $V(r)$ with r. The minimum potential energy correspond to a

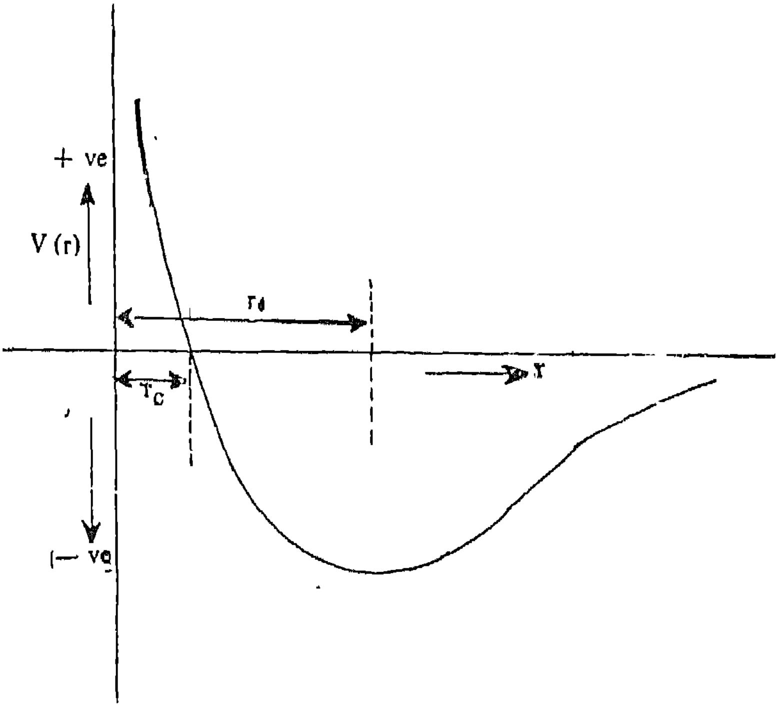


Fig. 3

particular intermolecular separation r_0, where r_0 is found to be of the order of 10^{-9}m. We can also regard r_0 ($\sim 10^{-9}$m) to be the radius of sphere of molecular action. At this particular value r_o, we get the most stable configuration. Of course, there exists another critical intermolecular separation r_c, if $r > r_c$, $V(r) = +ve$, which implies that in this region intermolecular force is repulsive. *If* $r > r_c$, $V(r) = -ve$ which corresponds to attractive intermolecular force. In case of attractive intermolecular interaction $F(r) \propto 1/r^7$. Whereas in the repulsive case $F(r) \propto 1/r^9$ which shows in this region $F(r)$ decreases still more rapidly.

The combination of the intermolecular forces and the thermal motion gives rise to three different states of matter. In case of solids attractive forces are strong and thermal agitations cannot break the bonds, between the molecules; whereas in case of fluids attractive forces are weak, and consequently molecules are more free.

Questions

1. How does the intermolecular potential energy vary with inter molecular separation r for $r < r_c$ and $r > t_c$?

2. If the curve $V(r)$ *vs* r is symmetric about r_o as shown in Fig. 3, what would be the consequences ? Explain.

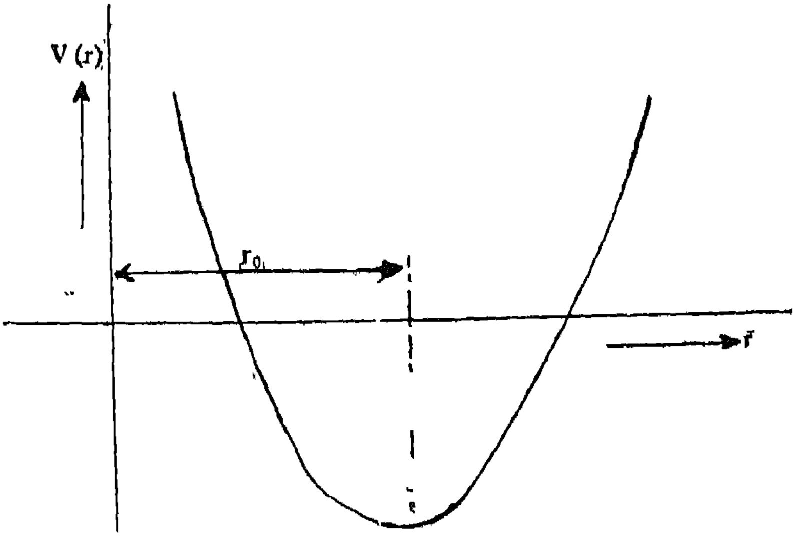


Fig. 3

2 Cohesive and adhesive forces

The intermolecular force of attraction between identical molecules is called *cohesion*, whereas attraction between dissimilar molecules is called *adhesion*, e.g sticking of water into glass If the force of cohesion is greater than force of adhesion (e g , mercury) free surface of the liquid is convex. Whereas, if force of cohesion is less than force of adhesion (e.g., water), free surface of the liquid is concave. In case of liquids, it is the force of cohesion due to which liquids can maintain definite volume, whereas in case of gas or vapour cohesion is practically nil. This is why liquid molecules on being heated, get sufficient kinetic energy to overcome the force of cohesion (or intermolecular binding energy) and gets transformed into vapour Thus knowing the latent heat of vaporization one can calculate the intermolecular binding energy. Given below is a solved example to illustrate this fact.

Example 1

Calculate the intermolecular binding energy of liquid helium at 4 K whose latent heat of vaporization at that temperature is 5 cal/g (Given $N = 6 \times 10^{23}$)

Solution

Molecular mass of He = 4

Number of molecules in 1 g of liquid He

$$n = \frac{6 \times 10^{23}}{4}$$

$$= 3/2 \times 10^{23}$$

Energy required to unbound n molecules of He

$$= 5 \text{ cal} = 5 \times 4.2 \text{ J} = 21 \text{ J}$$

Thus intermolecular binding energy is

$$\frac{21}{3/2 \times 10^{23}} \text{ J} = 14 \times 10^{-23} \text{ J}$$

$$= 1.4 \times 10^{-22} \text{ J}$$

Problem

Calculate the intermolecular binding energy of water molecules, where latent heat of vaporization is 540 cal/g (Given $N = 6 \times 10^{23}$)

3. Surface tension

All liquids tend to maintain a characteristic spherical shape Why so? A needle placed gently over the surface of a liquid floats although density of the material of the needle is much higher than the density of the liquid. Or, how does a bug float on water? In order to give satisfactory answers to above questions we must understand the origin of a force called *surface tension*. A concise discussion is given below·

Consider a molecule in the interior of the liquid and also the sphere of molecular activity around it If centres of other molecules lie within the sphere of action, they will exert some force of interaction over the molecule under consideration. But since forces are acting in all directions as shown in the Fig. 4, they will nullify each other.

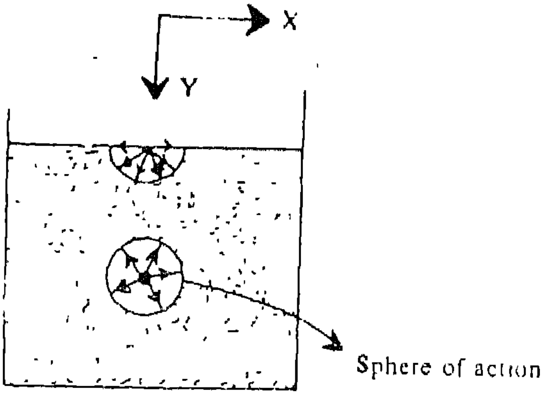


Fig 4

Now consider a molecule at the surface and a hemisphere of molecular activity around it. The force acting upon the molecule can be resolved along X and Y axes. The components along +*ve* and — *ve* directions of X axis will counter balance each other, but components along Y axis will remain unbalanced. Hence in order to carry a molecule from the interior to the surface, some work is to be done against these unbalanced components. This work will be stored up in the liquid surface in the form of surface potential energy (S P.E). Since from the fundamental principles of physics, we know that potential energy tends to acquire a minimum value hence surface potential energy will try to acquire a minimum value. Due to this reason, liquid will try to maintain minimum surface area. *This tendency of the liquids to maintain minimum surface area can be explained by assuming the existence of a force acting on unit length of the liquid boundary tangential to the liquid surface. This is known as force of surface tension.* Since it is force acting on unit length, the dimensions are $ML^{0}T^{-2}$ and unit is Nm^{-1} (SI Unit).

Now we are in a position to answer the various questions we raised earlier. All liquids show a tendency to assume spherical shape because the geometrical shape that has the least surface area for a given volume is a sphere, so the liquids come as close as they can to assume the form of a spherical drop. The departure from the spherical shape is due to weight of the drop and the force of adhesion, if any.

A thin coin placed gently over a liquid surface keeps on floating because it is in equilibrium under the action of three forces, viz. weight of the coin *W*, surface tension σ and the up-thrust of the liquid displaced by the coin *B*. In the Fig. 5 we have given a front view of the situation.

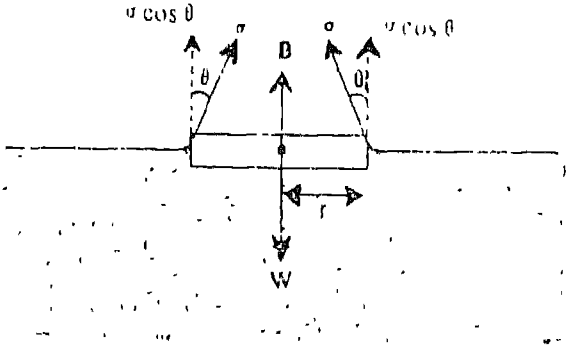


Fig. 6

If θ is the angle between the direction of the force of surface tension and the vertical and r is the radius of the coin, then force due to surface tension

$$F = 2\pi r \sigma \cos\theta \qquad ...(4)$$

For the equilibrium, of course

$$W' = B + 2\pi r \sigma \cos\theta \qquad ...(5)$$

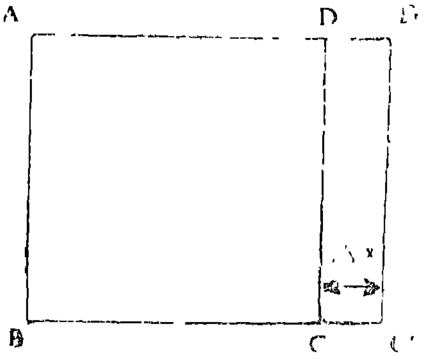
Questions

1. How does a bug float on water? Explain qualitatively.
2. A person can float over the water surface of *DEAD SEA* by keeping himself flat, why?

   (Hint : the water of Dead Sea is extremely saline)

4. Surface energy

As you have seen in the preceding section, work has to be done to bring a molecule from the interior to the surface. Thus, if we want to create a surface, work has to be done for increasing the number of molecules on the surface. The energy required to create a liquid surface like this, is called the *surface energy*. Let us calculate an expression for the surface energy. Let us take a rectangular frame of wire ABCD (Fig. 6).



Fig, 6

The arm CD of length l can slide. Now the frame is carefully dipped into a soap solution, so that a rectangular film ABCD is formed on the frame. Let σ be the surface tension of the liquid, then force acting on the arm CD is

$$F = \sigma\, 2\, l \qquad ...(6)$$

(A numerical factor 2 is introduced because we have two surfaces, viz. front and rear.) If CD slides to new position C'D' by a distance Δx, then increase in the surface area of the film will be

$$\Delta A = 2\, l\, \Delta x \qquad \text{...(7)}$$

The work done in creating the new surface ΔA is given by

$$\Delta W = F\, \Delta x$$

$$= \sigma\, 2\, l\, \Delta x$$

$$= \sigma\, \Delta A \qquad \text{...(8)}$$

This is the expression for work done (surface energy) in creating a new surface Eq. (8) gives us yet another definition of surface tension. *One can define surface tension σ of a liquid also as the surface energy per unit area* Although the dimensions of σ defined in the fashion will still be $ML^{0}T^{-2}$, but unit will be J/m^2.

Question

How will you measure the force acting on the moveable arm CD? Explain.

On the basis of the concept of surface energy we will try to show why do two or more liquid drops (say of Hg) when brought in contact, form a single drop. As you know configuration with minimum surface energy is the most stable configuration Hence in this case, if we can show that the union of liquid drops results in decrease in surface energy, our problem will be solved. Let us consider two drops of radii r_1 and r_2 of a liquid of surface tension σ. Volumes of the drops are $(4/3)\,\pi r_1^3$ and $(4/3)\,\pi r_2^3$ respectively. Their surface areas are $4\,\pi r_1^2$ and $4\pi\, r_2^2$ respectively The total surface energy before union is, obviously, $(4\pi\, r_1^2 \sigma + 4\pi\, r_2^2\, \sigma)$. Let us suppose after the union, radius of bigger drop is R. Then

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\, (r_1^3 + r_2^3)$$

or

$$R = (r_1^3 + r_2^3)^{1/3} \qquad \text{...(9)}$$

Surface energy of a single large drop after union is

$$E = 4\pi R^2 \sigma$$

$$= 4 \pi \sigma (r_1^3 + r_2^3)^{2/3} \qquad \ldots(10)$$

whereas, total surface energy before union is

$$E' = 4 \pi (r_1^2 + r_2^2) \sigma \qquad \ldots(11)$$

Let us now take numerical values, e.g.

$$r_1 = 1\ cm,\ r_2 = 2\ cm$$

$$E = 4 \pi \sigma (1^3 + 2^3)^{2/3}$$

$$= 4 \pi \sigma\, 9^{2/3}$$

$$= 4 \pi \sigma \times 4.3$$

$$E' = 4 \pi \sigma (1^2 + 2^3)$$

$$= 4 \pi \sigma \times 5$$

You can clearly see that $E' > E$; which we wanted to show.

Example 2

If n small drops of water, of equal radii R, coalesce to form a single big drop of radius R_1, show that rise in temperature is given by

$$\Delta \theta = \frac{3\sigma}{J\rho}\left(\frac{1}{R} - \frac{1}{R_1}\right)$$

Where ρ = Density of water

σ = Surface tension of water

J = Mechanical equivalent of heat

Solution

Due to composition of n drops to form a single drop decrease in surface area

$$= 4 \pi R^2 n - 4 \pi R_1^2$$

$$= 4 \pi (nR^2 - R_1^3)$$

Energy released $= 4 \pi \sigma (nR^2 - R_1^2)$

Heat produced $= M \times S \times \Delta \theta$

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where M = Mass of the single big water drop $= \frac{4}{3}\pi R_1^3 \rho$

S = Specific heat capacity of water = 1 cal/g°C.

θ = Rise in temperature

Thus $\left(\frac{4}{3}\pi R_1^3 \rho\right) \times 1 \times \theta = \frac{4\pi\sigma}{J}(nR^2 - R_1^2)$

$$\therefore \theta = \frac{3\sigma}{J\rho}\left(\frac{nR^2}{R_1^3} - \frac{1}{R_1}\right)$$

But since

$$\frac{4}{3}\pi n R^3 = \frac{4}{3}\pi R_1^3$$

Hence $\quad nR^3 = R_1^3$

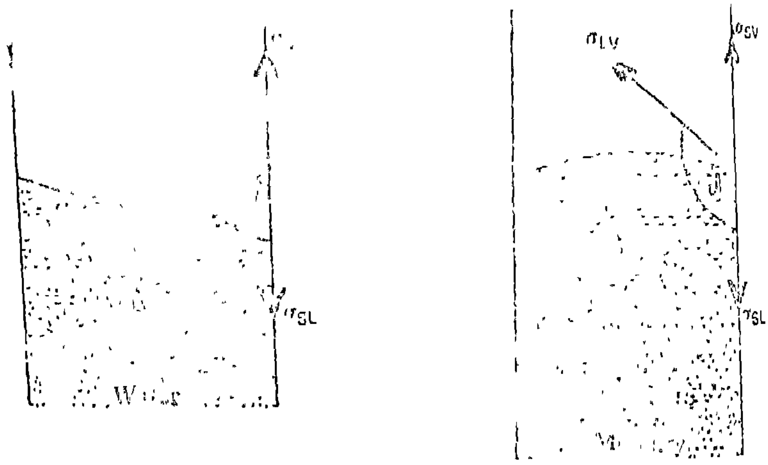
$$\therefore \theta = \frac{3\sigma}{J\rho}\left(\frac{1}{R} - \frac{1}{R_1}\right)$$

Problem

A liquid drop of diameter D breaks into 27 into drops. Find the resultant change in energy. Ans : $2\pi D^2\sigma$

5. Angle of contact

Consider a liquid kept in a container. Near the liquid miniscus on the walls (Fig. 7) three surface films are formed, viz. liquid-vapour, solid-vapour, and solid-liquid.



(a) (b)

Fig. 7 Three surface films along with angle of contact θ are shown for (a) concave miniscus and (b) convex miniscus liquids

Let σ_{SV} = surface tension for solid-vapour film; σ_{SL} = surface tension for solid-liquid film, and σ_{LV} = surface tension for liquid-vapour film.

We define the angle of contact as the angle between σ_{LV} and σ_{SL},

or

It is the angle between the tangent planes to the liquid surface and the containing wall surface at the point of contact.

Now consider a system of a solid S and liquid L. Let σ_{SL} be the surface tension of solid-liquid film (i e., equal to S.P.E. per unit area for S and L). If now S and L are separated and there is a thin layer of vapour V in between as shown in Fig. 8 some amount of work has to be done. If W_{SL} is the work done per unit area, then from the principle of conservation of energy the initial energy of the S-L system plus the work done must be equal to final energy of the S-L-V system. Thus

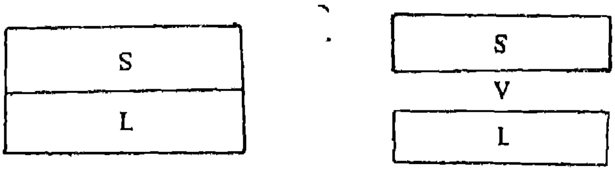


Fig 8

$$\sigma_{SL} + W_{SL} = \sigma_{SV} + \sigma_{LV} \qquad \text{...(12)}$$

We isolate a small portion of L-V, S-V and S-L films, at their junction as shown in Fig. 9. The isolated portion will be in equilibrium under the action of four forces. Three of which are surface tensions of the three films, the fourth one F_a is the adhesive force between isolated portion of the film and the wall. Thus we have

$$\sigma_{LV} \sin\theta = F_a \qquad \text{...(13)}$$

$$\sigma_{SL} + \sigma_{LV} \cos\theta = \sigma_{SV}$$

which gives

$$\sigma_{LV} \cos\theta = \sigma_{SV} - \sigma_{SL} \qquad \text{...(14)}$$

From Eq. (12)

$$W_{SL} - \sigma_{LV} = \sigma_{SV} - \sigma_{SL} \qquad \text{...(15)}$$

Equating L.H.S. of Eq (14) and Eq. (15) wet get

$$W_{SL} = \sigma_{LV} - \sigma_{L}^{V}\cos\theta$$

$$W_{SL} = \sigma_{LV}(1 \cos\theta)$$

or $$W_{SL}/\sigma_{LV} = (1 + \cos\theta) \qquad ...(16)$$

Case I

If $W_{SL} > \sigma_{LV}$; $(1 + \cos\theta) > 1$

$$\cos\theta > 0$$

$$\cos\theta > \cos 90^\circ$$

or, $$\theta < 90^\circ$$

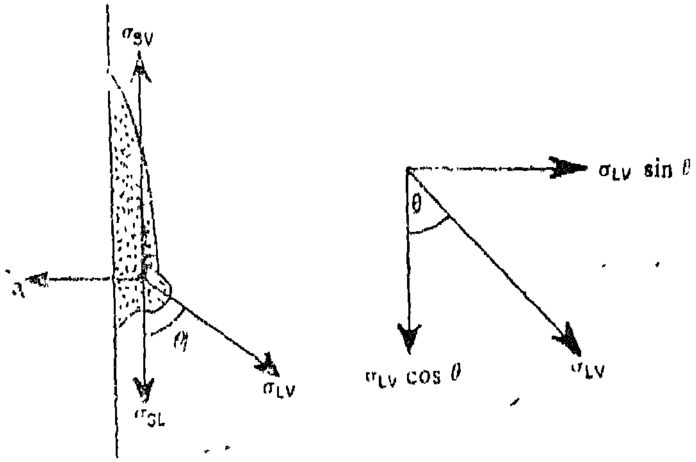


Fig. 9

Thus in this case angle of contact is acute.

In case of water, since force of adhesion is greater than two forces of cohesion, the work done in removing unit area of water surface from the glass surface W_{SL} is more than the surface tension of water-vapour film σ_{LV}. Thus angle of contact will be acute and miniscus of water will be concave as shown in Fig. 7(a).

Case II

$$\text{If } W_{SL} < \sigma_{LV}\ ;\ 1 + \cos\theta < 1$$

$$\cos\theta < 0$$

$$\cos\theta < \cos 90^\circ$$

or, $$\theta > 90^\circ$$

Angle of contact is obtuse.

In case of mercury, since force of cohesion is greater than force of adhesion, W_{SV} will be smaller than σ_{LV}. Hence angle of contact will be obtuse and miniscus will be convex as shown in Fig 7(b).

6. Effect of detergents on surface tension and angle of contact

Let us now discuss the effect of detergents on the surface tension and angle of contact. By now you are in a position to understand that low value of surface tension of a liquid implies low surface energy which in turn allows liquid to penetrate into narrow spaces or pores. The surface tension of water is about 75.8×10^{-3} N/m at 0°C and decreases by 0.152×10^{-3} N/m for each degree rise of temperature (which is relatively higher because even mercury has S.T. equal to about 547×10^{-3} N/m at 17.5°C). *If, however, detergent is added to it, the surface tension is lowered, simply because addition of detergents results into liquid spreading out as a thin film, which means surface energy as well as surface tension is lowered*

That is how aqueous solutions of detergents ensure cleaning property because solutions can seep into fine pores where water could not.

It is clear from Eq. (13) that when detergent is added σ_{LV} is lowered, consequently sin θ must increase in order that sticking force F_a is increased. *This implies that angle of contact θ must increase due to addition of detergents to water.* Same will be the case for liquids like mercury

Question

Why is it that a coin may float on clear water but will sink when some detergent is added to water ?

Answer

Why does a coin float on clear water has already been explained

(Ref. Eq. (5)). From Eq. (5)

$$W' = B + (2\pi r \cos\theta)\sigma$$

We have seen addition of detergents results in increase in θ, and decrease in $\cos\theta$, hence $(B + 2\pi r\sigma\cos\theta) < W$ when detergent is added. Thus the coin sinks.

Questions

1. Explain why $\theta < 90°$ for water and $\theta > 90°$ for mercury, in terms of σ_{SV} and σ_{SL} (Hint : Use Eq (14)).
2. Explain clearly why should angle of contact for Hg increase due to addition of foreign elements into it.

7. A simple method for the determination of angle of contact

A glass plate is dipped in the experimental liquid as shown in Fig. 10. A beam of light is allowed to fall over the glass plate; the incident ray being vertical. Now the inclination of glass plate is so adjusted that reflected ray is also vertical The liquid surface is horizontal at the point of reflection as shown in Fig. 10(a) and Fig. 10(b). The angle of contact is the inclination of the glass plate with the horizontal.

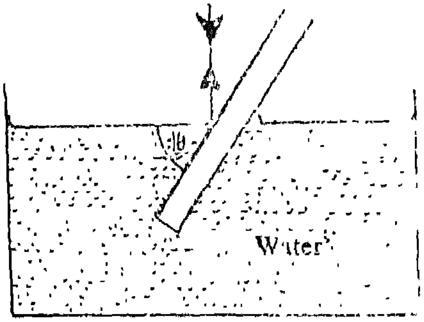


Fig. 10 (a)

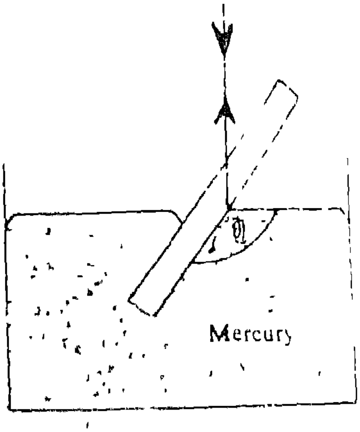


Fig. 10 (b)

Questions

1. A large air bubble is trapped inside a liquid as shown in

Fig. 11 (a). Identify the angle of contact. Should the angle of contact be acute or obtuse ?

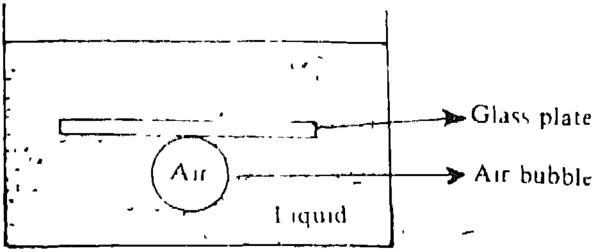


Fig. 11 (a)

2. A liquid drop is placed over a glass plate as shown in Fig. 11 (b). Is the angle of contact acute or obtuse ? Justify your answer.

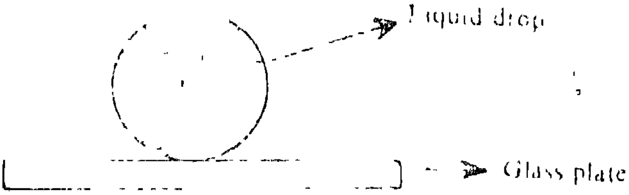


Fig. 11 (b)

8. Pressure difference across a surface film

Whenever the surface of a liquid or surface film is curved there exists a difference of pressure inside and outside the surface. If the inside pressure P_{in} is greater than the outside pressure P_{out}, the liquid surface is convex as shown in Fig. 12(a). If the P_{in} is less than P_{out}, the liquid

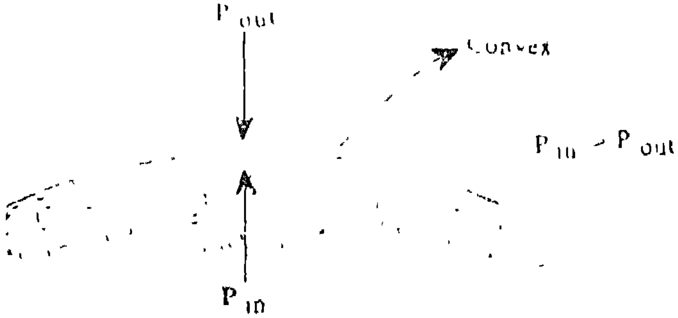


Fig. 12 (a)

surface is concave as shown in Fig. 12(b) In here, we want to calculate the difference of pressure ($P = P_{in} - P_{out}$) Let us consider a spherical drop of a liquid of radius R. Due to the spherical shape there will exist

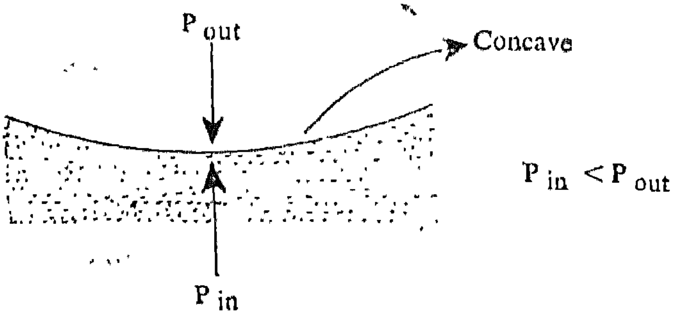


Fig 12 (b)

excess pressure inside the liquid as compared to outside atmospheric pressure. Let the excess pressure be P. Let us enlarge the liquid drop from radius R to $R + dR$. The surface area of the drop increases from S to $S + dS$.

Since

$$S = 4\pi R^2 \qquad \ldots(17)$$

$$\text{and } S + dS = 4\pi (R + dR)^2 \qquad \ldots(18)$$

$$dS = 4\pi (R + dR)^2 - 4\pi R^2$$

$$= 4\pi (R^2 + (dR)^2 + 2\,RdR) - 4\pi R^2 \qquad \ldots(19)$$

since dR is very small, $(dR)^2$ can be neglected.

$$\therefore\ dS = 8\pi R\, dR \qquad \ldots(20)$$

From the definition of surface tension, the work done W in increasing this surface area will be

$$W = \sigma\, dS \quad 8\pi R \sigma\, dR \qquad \ldots(21)$$

This work done W can also be calculated by multplying force with distance But force F is pressure times area, i.e.

$$F = P \times 4\pi R^2 \qquad \ldots(22)$$

Hence work done

$$W = F \times dR$$

$$W = 4\pi PR^2 \, dR \qquad \text{...(23)}$$

Equating R.H.S. of Eq. (21) and Eq. (23) we have

$$8\pi R \sigma \, dR = 4\pi PR^2 \, dR$$

Hence, $P = \frac{2\sigma}{R}$ ...(24)

It follows from Eq. (24) that if σ is a constant, the pressure difference is very large for small radius R of the drop. Due to this large internal pressure, tiny fog drops have rigidity properties like those of solids.

N.B. For single spherical surface, i.e., spherical air bubble inside the liquid or a spherical liquid drop, P is given by Eq. (24), however, for double spherical surface, viz. front and rear surface, (e.g., for a spherical soap bubble) P is given by

$$P = 4\sigma/R \qquad \text{...(25)}$$

Question

How do the ice skaters slide over the surface of smooth ice ?

Example 3

What would be the gauge pressure inside an air bubble of 0.2 mm radius situated just below the surface of water ? Surface tension of water is 0.07 N/m. Atmospheric pressure = 1.06 × 10^6 dynes/cm^2.

Solution

Let, gauge pressure = P_{gauge} ; Atmospheric pressure = P_{atm}

Pressure difference $P = P_{gauge} - P_{atm}$

$$= P_{gauge} - 1.06 \times 10^5 \text{ N/m}^2$$

Also $P = 2\sigma/R$

$$= \frac{2 \times 0.07}{0.2 \times 10^{-3}} \text{ N/m}^2 = \frac{14 \times 10^{-2}}{2 \times 10^{-4}} \text{ N/m}^2$$

$$= 7 \times 10^2 \text{ N/m}^2$$

$$(P_{gauge}) - 1.06 \times 10^5\ \text{N/m}^2 = 7 \times 10^2\ \text{N/m}^2$$

$$P_{gauge} = (1.06 \times 10^5 + 7 \times 10^2)\ \text{N/m}^2$$

$$= 1.067 \times 10^5\ \text{N/m}^2$$

Example 4

A U-tube (Fig. 13) is such that the diameter of one limb is 0.4 mm and that of the other is 0.8 mm. Find the difference in the level of water in the limbs. (Given $\sigma = 0.07\ \text{N/m}^2$, $g = 10\ \text{m/s}^2$)

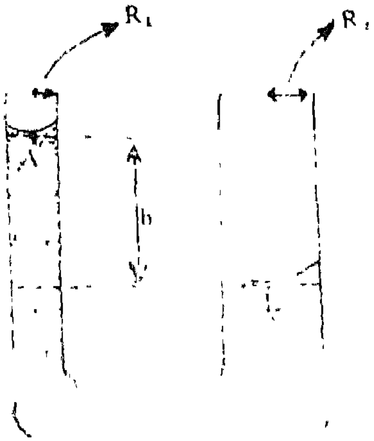


Fig. 13

Solution

Pressure at A,

$$P_A = \frac{2\sigma}{R_1}$$

$$= \frac{2 \times 0.07}{2 \times 10^{-4}}\ \text{N/m}^2$$

$$= 7 \times 10^2\ \text{N/m}^2$$

Pressure at B, $P_B = \frac{2\sigma}{R_2} = \frac{2 \times 0.07}{4 \times 10^{-4}}\ \text{N/m}^2 = 3.5 \times 10^2\ \text{N/m}^2$

$P_A - P_B = h \rho g$; h = Difference in the level

$$7 \times 10^2\ \text{N/m}^2 - 3.5 \times 10^2\ \text{N/m}^2 = h \times 10^3\ \text{kg/m}^3 \times 10\ \text{m/s}^2$$

$$3.5 \times 10^{2}\,\text{m} = h \times 10^{1}$$

$$h = \frac{3.5 \times 10^{2}}{10^{4}}\,\text{m} = 3.5 \times 10^{-2}\,\text{m}$$

$$= 3.5 \times 10^{-4}\,\text{m}$$

Example 5

Inside a liquid, an air bubble is formed as shown in Fig. 14. Set up a relationship between gauge pressure P_{gauge}, radius of the air bubble r, surface tension σ of the liquid, density of the liquid ρ and depth of the air bubble x.

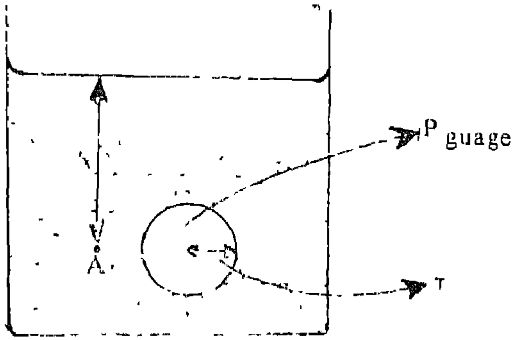


Fig. 14

Solution

$$\text{Excess pressure} = P_{gauge} - P_A = \frac{2\sigma}{r}$$

$$\text{But } P_A = x \rho g + P_{atm}.$$

(Where ρ = Density of the liquid)

P_{atm} = Atmospheric pressure

Hence

$$P_{gauge} = (x \rho g + P_{atm}) + \frac{2\sigma}{r}$$

This was the relation to be set up.

Problem

A glass tube of 1 mm bore is dipped vertically in a container of mercury with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end? Given, σ of Hg = 0.5 N/m, density ρ of Hg = 13.6 g/cc., $P_{atm} = 1.06 \times 10^5$ N m^{-2}, and $g = 10$ m s^{-2}.

9. Capillarity

We have seen that near the walls of a container a liquid like water rises and a liquid like mercury falls. If a capillary tube is dipped inside a liquid like water it rises (Fig. 15(a)) inside the capillary tube, whereas for a liquid like mercury the liquid falls (Fig. 15 (b)). This phenomenon is called capillarity (Capillary ascent or decent).

Let us consider the case of capillary ascent as shown in Fig. 15(a). We want to set up an expression for the surface tension σ of the liquid in terms of rise of the liquid h, radius of the capillary tube r, angle of contact θ and density of the liquid ρ.

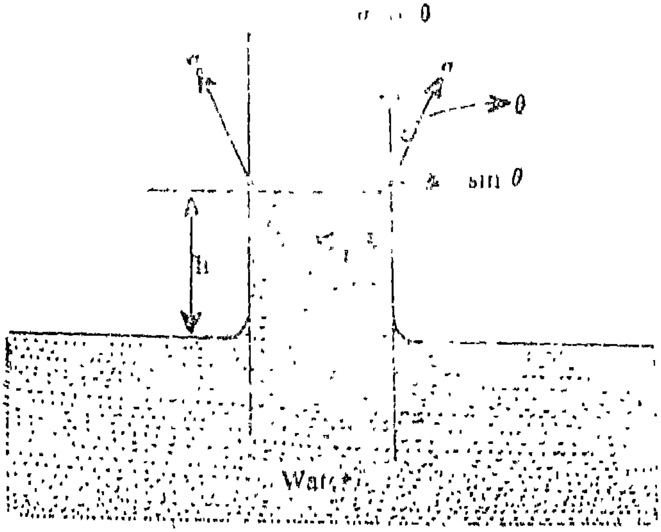


Fig. 15 (a)

The liquid column of height h is in equilibrium, hence upward force must counter balance downward force. The force acting in the vertically upward direction is the component of the force due to surface tension, i.e. ($2\pi r \sigma \cos \theta$). The force acting in the vertically downward direction is the weight of the liquid column

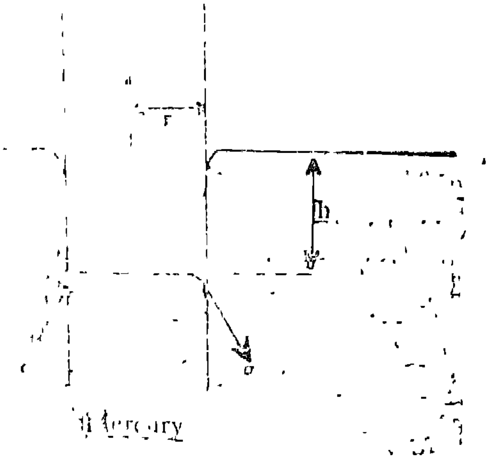


Fig. 15 (b)

For the liquid column of height h, weight would be ($\pi r^2 h \rho g$). But since height is measured from lowest point of the concave miniscus, we have to consider the weight of the liquid in the shaded part of the Fig. 16. Volume of this shaded portion is approximately equal to the volume of a cylinder of height r minus volume of a hemisphere of radius r. And its weight will be $(\pi r^2 r - (2/3)\pi r^3)\rho g = (1/3)\pi r^3 \rho g$

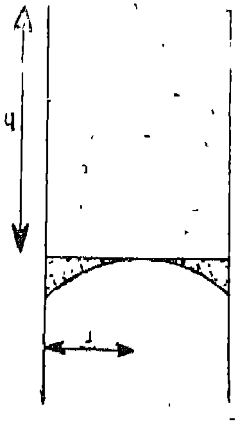


Fig. 16

Thus, total force in the vertically downward direction

$$= \pi r^2 h \rho g + \frac{1}{3} \pi r^3 \rho g$$

$$= \pi r^2 (h + \frac{1}{3} r) \rho g \qquad \ldots(26)$$

Total force in the vertically upward direction

$$= 2 \pi r \sigma \cos \theta$$

Equating R.H.S. of Eq. (26) and Eq. (27) we get ...(27)

$$2 \pi r \sigma \cos \theta = \pi r^2 (h + \frac{1}{3} r) \rho g$$

$$\sigma = \frac{r \rho g (h + \frac{1}{3} r)}{2 \cos \theta} \qquad \ldots(28)$$

For distilled (pure) water angle of contact $\theta \to 0$ and since radius of the capillary tube $r << h$, one can write Eq (28) as

$$\sigma = \frac{1}{2} h r \rho g \qquad \ldots(29)$$

Eq. (28), gives us a method to determine the surface tension of a liquid if angle of contact is known. The rise of liquid in capillary and the radius of capillary are measured with the help of a travelling microscope. Before performing the experiment the tube must be thoroughly cleaned. It is due to the phenomenon of capillarity only that oil rises in a wick lamp, ink is absorbed by a blotting paper and so on.

Question

1. If one wants to determine the surface tension of a liquid by the method of capillary rise, one should clean the capillary tube thoroughly before performing the experiment Why ?
2. If the height of a capillary tube is less than the height given by Eq. (28), will the water come out in the form of a fountain through the other end of the capillary ? Explain.

Example 6

Derive Eq (29) making use of the formula for excess pressure given by Eq. (24).

Solution

As shown in Fig. 17(a), pressure at A, P_A = pressure at C, P_C

= Pressure at D, P_D

= Atmospheric pressure.

$P_A - P_B$ = (Pressure at the concave side)

— (Pressure at the convex side)

= Excess pressure

$= 2\,\sigma/R$ ...(i)

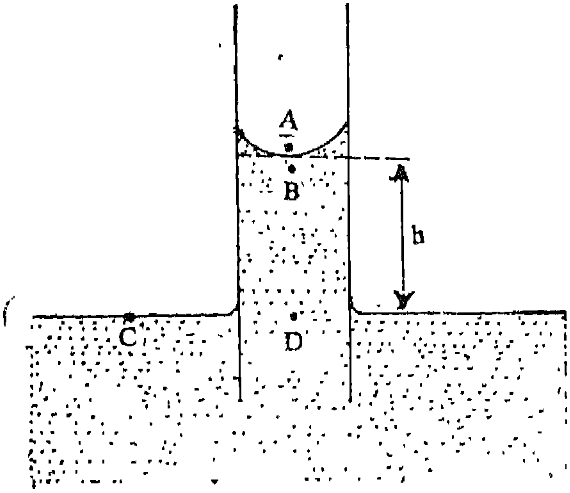


Fig. 17 (a)

Where R is the radius of the surface which is assumed to be spherical and in Fig. 17(b) it is AB. Let r be the radius of the capillary tube which is OA in Fig. 17(b), since in $\triangle\ OAB$

$$\frac{OA}{AB} = \cos\theta$$

$$OA = AB \cos\theta$$

$$r = R \cos\theta \qquad \text{...(ii)}$$

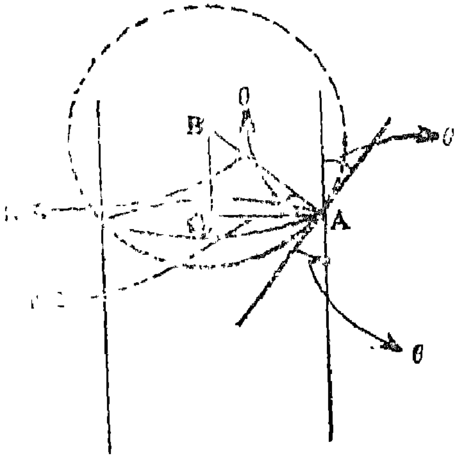


Fig. 17 (b)

Now $P_A - P_B = P_D - P_B$ ...(iii

But $P_D - P_B = h \rho g$ ...(iv)

ρ being the density of the liquid. Substituting (i) and (iv) in (iii) we get

$$\frac{2\sigma}{R} = h \rho g$$

$$\sigma = \tfrac{1}{2} h R \rho g$$

But from (ii)

$$R = r/\cos\theta$$

Hence

$$\sigma = \tfrac{1}{2} \frac{h r \rho g}{\cos\theta}$$

For water as $\theta \to 0$, $\cos\theta \to 1$ .. (v)

$$\sigma = \tfrac{1}{2} h r \rho g \quad \text{...(iv)}$$

Example 7

A U-tube consists of a horizontal limb and two vertical limbs. The diameters of vertical limbs are 1 mm and 2 mm respectively. Find the difference of levels in the liquids if

Surface tension σ = 500 dynes/cm

Angle of contact θ = 140°

Density of the liquid ρ = 13.6 g/cc

Solution

Let R_1 and R_2 be the radii of the spherical liquid surface in the narrow and broad limbs respectively (Fig. 18).

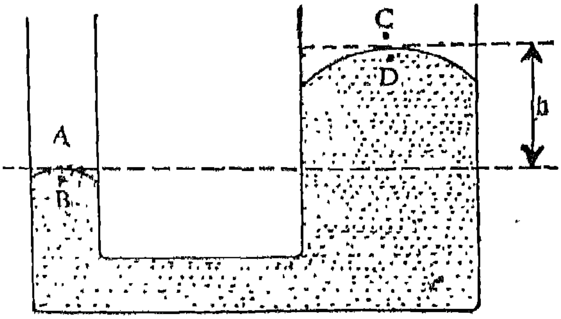


Fig. 18

If P_A, P_B, P_C and P_D are pressures at points A, B, C, and D respectively

$$P_B - P_A = \text{Excess pressure} = 2\,\sigma/R_1$$

$$P_D - P_C = \text{Excess pressure} = 2\,\sigma/R_2$$

$$(P_B - P_A) - (P_D - P_C) = 2\,\sigma\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$(P_B - P_D) + (P_C - P_A) = 2\,\sigma\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

But $P_A = P_C$

$$P_B - P_D = h\rho g$$

Hence $h\rho g = 2\sigma\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

r_1 = Radius of the narrow limb

$\theta' = 180 - \theta$ (angle of contact)

Thus as shown in Fig: 19

$$r_1 = R_1 \cos\theta'$$

or

$$\frac{1}{R_1} = \frac{\cos\theta'}{r_1}$$

Similarly, let r_2 = Radius of the broad limb

$$\frac{1}{R_2} = \frac{\cos\theta'}{r_2}$$

Hence $h\rho g = 2\sigma\left(\dfrac{\cos\theta'}{r_1} - \dfrac{\cos\theta'}{r_2}\right)$

$$h = \frac{2\sigma\cos\theta'}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

But $\theta' = 180^\circ - \theta = 180^\circ - 140^\circ = 40^\circ$

Hence, $h = \dfrac{2\times 500\times\cos 40^\circ}{13.6\times 981}\left(\dfrac{1}{0.05} - \dfrac{1}{0.1}\right)$cm

$h = 0.57$ cm

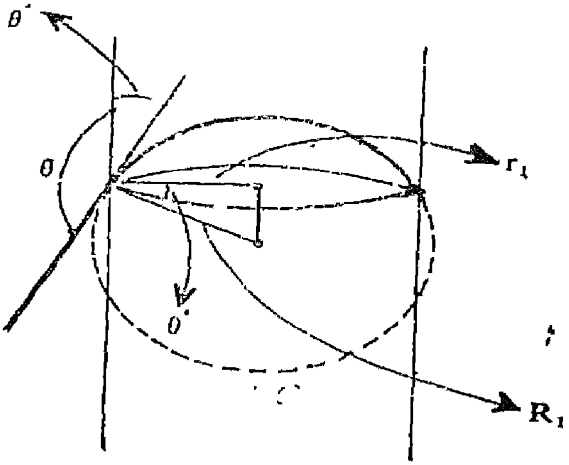


Fig. 19

ASSIGNMENT

1. (a) Why does a diver dive down the water surface keeping himself vertically downward ?

   (b) What will happen if detergent is added to water on which needle is supported ?

   (c) Why do two drops when brought in contact, form a single drop.

2. If the variation of surface tension with temperature t is given by the formula

   $\sigma(t) = \sigma_0 (1 - \alpha t)$ (which shows S.T. decreases with temperature)

   α = coefficient of S. T. for the liquid.

   σ_0 = S.T. at 0°C of the liquid.

Then show that total energy/unit area of the liquid film (S. P. E. + Energy absorbed/unit area during isothermal expansion) is independent of the temperature over a wide range.

(Hint : A Carnot cycle should be operated upon and work done during isothermal expansion equated to surface tension times change in area.)

3. (a) By means of a sprayer a water drop of radius 1 mm is broken into 1 million drops of same size. If the σ of water is 75×10^{-3} N/m, calculate the amount of work done.

   Ans. 9.4×10^{-5} J

   (b) Find the pressure difference between inside and outside of a soap bubble of diameter 4 mm. (σ for soap solution = 1.6 N/m).

   Ans. 3.2×10^3 Nm^{-1}.

4. (a) A U–tube consists of a horizontal limb and two vertical limbs. The diameter of the vertical limbs are 1mm and 2mm respectively. Find the difference in levels of the liquid in the U—tube. Given surface tension of the liquid = 50

UNIT 16

Physics for Class XI

FLUID DYNAMICS

F-15187

विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

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PREFACE

This unit is sixteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in Class XI physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Dr. K.J. George who is working as a Senior Project Fellow on this project.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
N.C.E.R.T.

New Delhi
10 November 1982

INTRODUCTION

You might have watched the flow of water through a drain characterized by local circular currents, a rather unsteady type of flow. This type of disorderly and irregular flow usually occurs when the rate of flow is rather high. However, if the rate of flow is small you may have noticed that flow becomes orderly and steady. In the present unit you are going to study about these two types of flow, viz, orderly and disorderly flow, in brief about the dynamics of fluid. In the preceding unit you studied about a property of liquid (only), namely surface tension The frictional resistance to the flow of a fluid moving in layers is yet another important property which causes disorderly flow. More about frictional resistance (viscosity) is to be studied in this unit. Here you will also learn the principle involved in the lift of an aircraft wing and many other phenomena such as the operation of a device whereby squeezing the bulb you spray scent, and deviation of spinning cricket ball from the usual trajectory of a spin free projectile.

OBJECTIVES

To be able to

1. Differentiate between streamline and turbulent motion of fluids, and determine the factors upon which critical velocity depends.
2. State and prove Bernoulli's equation
3. Explain some practical applications of Bernoulli's equation and solve related problems.
4. Explain the origin of viscosity and state Newton's law of viscous flow.
5. Deduce, and apply Stokes law.

SUGGESTED READING MATERIAL

1. Streamline and turbulent motion

Streamline motion

Let us consider a tube (Fig 1) through which fluid particles of velocity less than 0.01 m/s flow. Consider a point A. No matter whichever particle passes through the point A, if it has the same velocity v_A

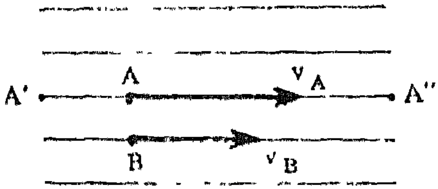


Fig. 1

then the flow is *steady*. Thus in this case when one particle leaves A with velocity v_A, it is followed by another particle which also passes through A with velocity v_A. Consider any other point B The particles passing through B will always have velocity v_B and so on. It is not necessary for these two velocities v_A and v_B to be equal. The paths, e.g, A'AA'' followed by liquid particles in this case are called streamline. Obviously a streamline is parallel to the velocity of fluid particles Now we are in a position to define *steady* (*or streamline*) motion. *When a fluid is allowed to flow through a narrow tube such that each particle of the fluid moves along the same path as its preceding particle the motion is said to be orderly or streamline motion.*

Question

Can streamlines cross each other ? Explain.

Turbulent motion

The streamline motion occurs for low rates of flow, i.e. when the velocity of flow is below a certain limiting value called the *critical velocity* v_C. When the velocity of liquid reaches beyond the critical value, the motion becomes unsteady; the liquid no longer flows in a regular manner but flows in a sinuous course from side to side of the tube. This type of motion is called turbulent motion. In this case random, irregular, local circular currents called *vortices*, develop throughout the fluid. The resistance to flow increases tremendously.

An expression for the critical velocity v_c has been obtained experimentally

$$v_c = N\left(\frac{\eta}{\rho D}\right) \qquad \ldots(1)$$

where ρ is the density of the fluid flowing through a pipe of diameter D and η is the coefficient of viscosity of the liquid (about which you are going to study). N is a constant called *Reynold's number.* The flow of fluid is said to be streamlined if $0 \leqslant N \leqslant 2000$, for $2000 \leqslant N \leqslant 3000$ the flow is unstable and may switch over from one type to another. For $N > 3000$, flow is turbulent. Since N is a pure number its value is same in any consistent set of units.

Let us deduce Eq (1) by dimensional method

Let the critical velocity $v_c \propto \eta^a$

$$\propto \rho^b$$

$$\propto D^c$$

Thus $\qquad v_c = \text{constant} \times \eta^a \ \rho^b \ D^c$

Dimension of $\eta = ML^{-1}\ T^{-1}$

hence $M^{0}L\,T^{-1} = \text{constant} \times [ML^{-1}\ T^{-1}]^a \times [ML^{-3}]^b \times [L]^c$

$$M^{0}L\,T^{-1} = \text{constant} \times [M]^{a+b}\ [L]^{-a-3b+c}[T]^{-a}$$

Equating the powers of M, L and T we get

$$a+b=0; \quad -a-3b+c=1; -1=-a$$

or $\qquad a=1; \ b=-1; \ c=-1$

Thus

$$v_c = N \times (\eta/\rho D)$$

where the constant N is the Reynold's number.

Question

Estimate the critical velocity v_c for $N=2000$, of a liquid passing through a pipe of diameter 2.5 cm. Take $\rho=0.85$ g/cm^3, $\eta=0.0005$ N m^{-2}s. Ans: 0.47 cm/s

2. Bernoulli's equation

Present in every fluid are frictional forces which offer resistance to its flow. But many physical phenomena can be explained qualitatively and to a good approximation, when we make our task simpler by neglecting the effect of frictional resistance. A fluid free from frictional resistance will be regarded as an *ideal fluid*. It is analogous to perfect gas model where we ignore the small but finite forces of interaction between the molecules. Consider the flow of an *ideal fluid* through a pipe (Fig. 2).

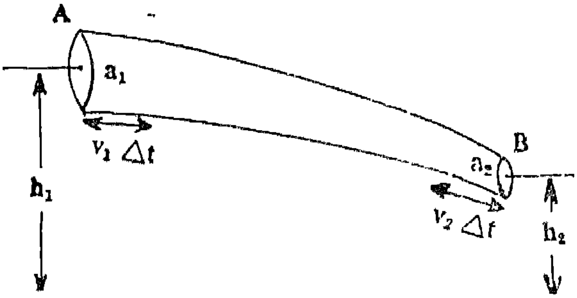


Fig. 2

Let heights, areas of cross-sections, pressures and fluid velocities at ends A and B be represented by h_1, a_1, p_1, v_1, and h_2, a_2, p_2, v_2 respectively. During a small time Δt, fluids in A and B are pushed through distance $v_1 \Delta t$ and $v_2 \Delta t$ respectively.

At A, work done on the system is

$$W_1 = \text{Force} \times \text{Displacement}$$

$$= p_1 a_1 \times v_1 \Delta t \qquad \text{...(2)}$$

At B, work done by the system is

$$W_2 = \text{Force} \times \text{Displacement}$$

$$= p_1 a_1 \times v_1 \Delta t \qquad \text{...(3)}$$

Hence, the net work done by the fluid pressure is

$$W = W_1 - W_2 \quad p_2\, a_2\, v_2\, \Delta t - p_2\, a_2\, v_2\, \Delta t \qquad \ldots(4)$$

Since there is no accumulation of liquid anywhere inside the pipe the amount of liquid entering into the tube in a given time Δt must be the same as the amount of liquid leaving the pipe during the same time.

Total volume V of the liquid entering or leaving is

$$V = a_1\, v_1\, \Delta t = a_2\, v_2\, \Delta t \qquad \ldots(5)$$

Eq. (5) is called the *equation of continuity.*

The mass of this fluid of volume V and density ρ is ρV.

Substituting from Eq. (5) in Eq. (4) we get

$$W = (p_1 - p_2)\, V \qquad \ldots(6)$$

Now net change in gravitational potential energy, when the fluid of mass ρV moves from A to B is

$$\rho\, Vg\, (h_1 - h_2) \qquad \ldots(7)$$

We know that when work is done on an element of mass $\rho\, V$, its kinetic energy changes. The principle of conservation of energy gives

Change in kinetic energy = Change in gravitational potential energy
+ Work done by the fluid pressure.

$$\tfrac{1}{2}\, \rho V\, (v_2^2 - v_1^2) = \rho\, V\, g\, (h_1 - h_2) + (p_1 - p_2)\, V \qquad \ldots(8)$$

Dividing throughout by $\rho\, V$, we get

$$\tfrac{1}{2}\, (v_2^2 - v_1^2) = g\, (h_1 - h_2) + (p_1 - p_2)/\rho$$

On rearranging the terms

$$\frac{p_1}{\rho} + \tfrac{1}{2}\, v_1^2 + gh_1 = \frac{p_2}{\rho} + \tfrac{1}{2}\, v_2^2 + gh_2. \qquad \ldots(9)$$

We define p/ρ as pressure energy per unit mass, $\frac{1}{2} v^2$ as kinetic energy per unit mass, and gh as gravitational potential energy per unit mass. Eq. (9) clearly shows that the sum of these three quantities is constant This is *Bernoulli's equation which states that for an ideal, incompressible fluid flowing in the form of streamlines the sum total of kinetic energy per unit mass, potential energy per unit mass, and pressure energy per unit mass is constant.* This is merely a consequence of principle of the conservation of energy.

If Eq. (9) is divided throughout by g we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \qquad \text{...(10)}$$

It is easy to check that all three terms present in the L.H.S. of Eq. (10) have the dimension of length: that is why they are called *heads*; $p/\rho g$ is the *pressure head*, $v^2/2g$ is the *velocity head*, h is the *gravitational head.*

Thus from Eq. (10), one can state Bernoulli's equation in a slightly different way as follows:

For an ideal, incompressible fluid, flowing in the form of streamlines, the *sum of the pressure head, the velocity head and the gravitational head is constant.*

Looking at Eq. (9) or Eq. (10) one can easily infer that *where the velocity is high, the pressure is low and vice versa*

Example 1

The velocities of a fluid at the entrance and exit of a horizontal pipe of uniform cross section are 0.1 m/s and 0.2 m/s (Fig. 3) respectively.

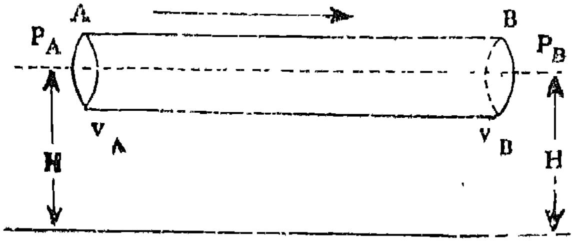


Fig. 3.

D; ermine the pressure difference in terms of height of the mercury column Take $g = 10\ m/s^2$ (Given density of fluid $= 0.8\ g/cm^3$;

density of mercury $= 13.6\ g/cm^3$)

Since the ends A and B are at the same height Bernoulli's equation gives

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 = \frac{P_B}{\rho} + \frac{1}{2}v_B^2$$

$$P_A - P_B = \frac{1}{2}\rho\,(v_B^2 - v_A^2) \qquad ...(11)$$

If h = Height of the Hg column corresponding to pressure difference ($P_A - P_B$) and d = Density of Hg

then

$$P_A - P_B = h\,d\,g$$

Using this in Eq. (11) we get

$$h = \frac{\rho}{2dg}(v_B^2 - v_A^2)$$

$$= \frac{0.8\ (0.04 - 0.01)}{2 \times 13.6 \times 10}\,m$$

$$= 9 \times 10^{-5}\,m$$

Example 2

At what speed will the velocity head of a stream of water be equal to 40 cm of Hg ?

Solution

$$\text{Velocity head} = \frac{v^2}{2g} = 0.4\ m$$

Take $g = 10\ m/s^2$

$$v^2 = 0.8 \times 10\ m^2/s^2$$

$$v = \sqrt{8}\ m/s = 2.8\ m/s$$

Question

If a small table tennis ball is placed in a vertical jet of air or water (Fig. 4) it will rise to a given height above the nozzle and stay at that level. Explain

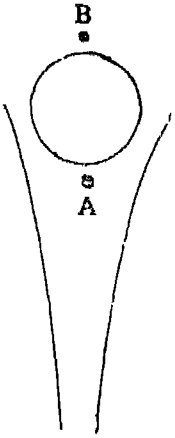


Fig. 4

3. Applications of Bernoulli's equation

The pitot tube

It is a device to measure the velocity of an aircraft. It is mounted over the aircraft as shown in Fig. 5. At A, velocity of the air coming in is zero; whereas at B velocity of air be the same as the velocity of aircraft v relative to air.

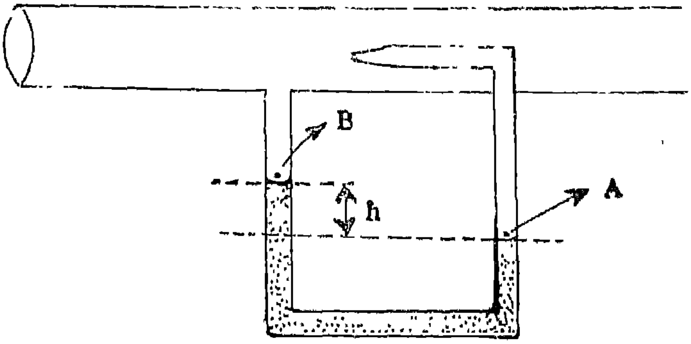


Fig 5

Problem 1

Apply Bernoulli's equation at A and B to show that the velocity of the aeroplane is

$$v=\sqrt{\frac{2\,hdg}{\rho}}$$

where d is density of mercury and ρ is density of air.

Problem 2

A pitot tube is mounted on an aeroplane wing to measure the speed of the plane (Ref. Fig. 5). The tube contains alcohol and shows a level difference of 40 cm, What is the speed of the plane relative to air (specific gravity of alcohol = 0.8, and density of air = 1 kg/m^3, g = 10 m/s^2)?

Ans: 80 m/s

Atomizer

In an atomizer, by squeezing the bulb B we spray oil or scent from the nozzle as shown in Fig. 6. In fact when the bulb is squeezed the air blows out through the tube T creating a partial vacuum, thereby pressure at T, p_1 is lowered down to a great extent. The atmospheric

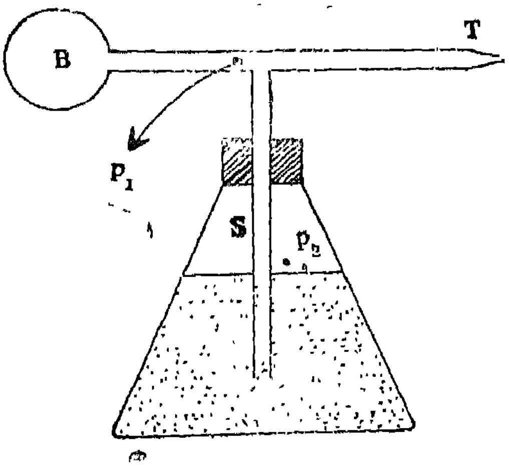


Fig. 6

pressure p_2 on the liquid, being much higher, pushes the liquid up the stem S which is then blown out with air stream through the nozzle.

Lift on an aircraft wing

Aeroplane wings are designed in such a way that total distance travelled by the air over the wing is longer than that of the air flowing under it. Consequently velocity above is higher than velocity below which gives rise to a pressure difference, with pressure at the top being smaller than pressure below. This unbalanced pressure causes a force F to act on the wing, which can be resolved into two components. The vertical component (Fig. 7) is responsible for the lift L and the horizontal component provides the drag D. However, if the streamline flow, above and below, breaks down due to greater angle of attack, Bernoulli's equation will not be applicable.

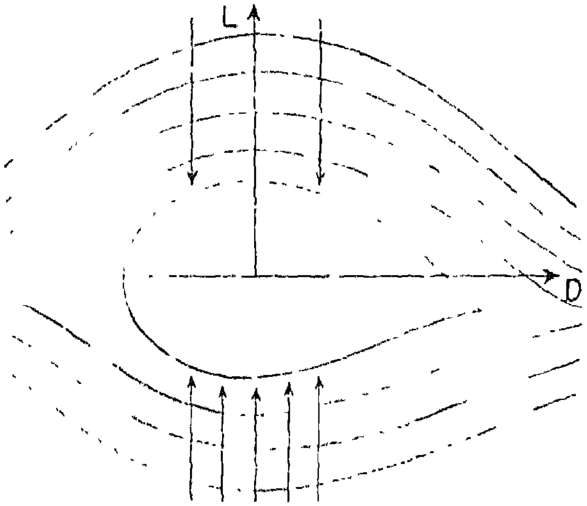
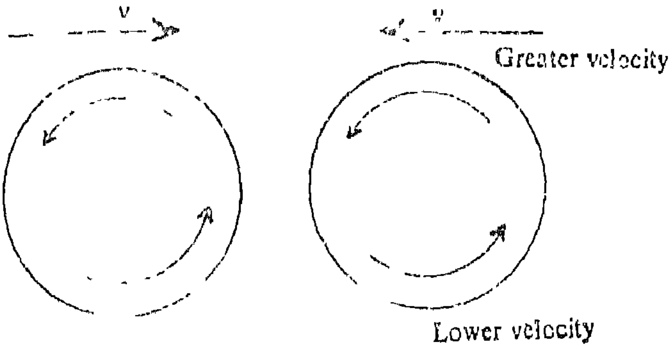


Fig 7

Curved path of a spinning ball

Consider a cricket ball moving to the right with a velocity v, be given a spin as shown in Fig. 8 (a). Since the ball is moving forward (as well as spinning in the air), wind will be drifted in the direction opposite to the ball with a velocity same as the velocity of the ball as shown in Fig. 8 (b). Thus just above the ball the wind has a velocity greater than v as the velocity component due to spin adds up to v, and just below the ball since velocity component due to spin gets subtracted from v, resultant velocity will be lower. Consequently, the pressure above the ball will be less than pressure below (from Bernoulli's equation). The pressure difference thus created produces an additional unbalanced force due to



(a) Fig. 8 (b)

which ball gets deviated from the path which otherwise it would have persued had there been no spin In this case, obviously ball curves upward.

The Venturi meter

In a Venturi meter (Fig. 9) the portion AB is wider (cross-sectional area $=a_1$) than portion CD (cross sectional area $=a_2$). Tube A B C D E F is horizontal. The rates of flow v_1 and v_2 of liquid in A B and C D (i.e. through α and β can be determined.

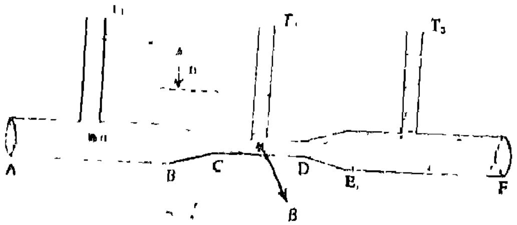


Fig. 9

Problem 3

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(a) Show that for Venturimeter equation of continuity gives

(b) Bernoulli's equation applied at α and β gives

$$2\,hg = v_2^2 - v_1^2$$

(c) The Venturi meter can be used to determine v_1 if v_2 is given or v_2 if v_1 is given. Show that

$$v_2 = \sqrt{\frac{2\,hg}{1-(a_2/a_1)^2}}$$

Questions

1. During a wind storm or cyclone the roofs of houses (particularly one with thatched roofs) are blown off without damaging other parts of the houses. Explain why ?

2. If the spin given to a cricket ball is opposite in the sense to what has been shown in Fig. 8 how does the ball curve? Explain.

4. Viscosity

Consider a fluid flowing over a fixed horizontal surface M N (Fig. 10). If the motion is streamline the fluid in contact with the fixed surface will be at rest and every other layer will move parallel to the fixed surface, the velocity of flow increasing in the upward direction to a maximum value. Now consider an horizontal plane in the fluid. The molecules of the fluid immediately above this plane tend to accelerate the motion of the molecules lying in the plane, while the molecules of the fluid lying

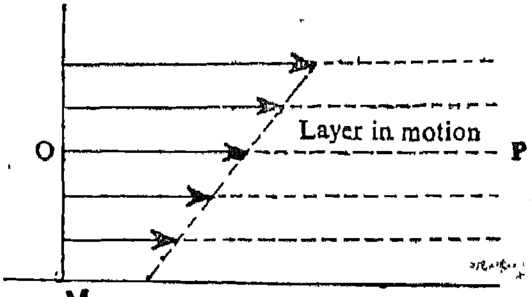

Fig. 10

immediately below the plane tend to retard the motion of the molecules in it. Each layer of the fluid thus tends to exert on the layer next to it a tangential backward force which tends to destroy the relative motion between the two layers. Consequently if a relative velocity is to be maintained between the parallel layers of the fluid an external force is required to overcome this backward drag. In the absence of this external force the relative motion between different layers ceases after some time. Now we are in a position to define viscosity. *The property by virtue of which a liquid opposes relative motion between its different layers is called internal friction or viscosity.* The viscous force arises due to force of cohesion between the molecules of the fluid flowing in layers and also due to force of adhesion between the fluid molecules and the molecules of the tube.

From Newton's law of viscous flow for streamline motion the tangential force F between parallel layers A B and C D (Fig. 11) each of area A, at distance dx apart and moving with relative velocity dv is proportional to the area A and the velocity gradient dv/dx. The results were obtained experimentally. Thus

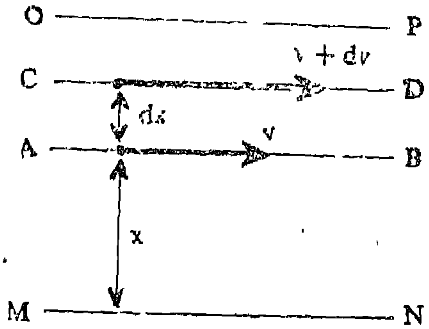

Fig 11

$$F \propto A\, dv/dx \qquad ...(12)$$

$$F = \eta A\, dv/dx \qquad ...(12a)$$

Where η is a constant called the *coefficient of viscosity* of the fluid and depends upon the nature and physical condition of the fluid. In the equation if

$A = 1$ and $dv/dx = 1$, we get

$$F = \eta$$

Therefore, *the coefficient of viscosity may be defined as the tangential backward force acting on unit area of fluid layer moving in a region of unit velocity gradient.* The dimensions of η can be easily calculated, $\eta=(F/A)/(dv/dx)$, $|\eta| = ML^{-1}T^{-1}$

$$|\eta|_{SI} = Nm^{-2}s$$

$$= kg/ms$$

In C.G.S system, units of η are dyne cm^{-2}s which is called a *poise.*

The coefficient of viscosity of liquids decreases very rapidly with the rise in temperature and increases with the rise in pressure.

5. Stokes law

We have studied the flow of liquid through a solid pipe. Let us now study the motion of a solid through a stationary liquid For simplicity let the solid be a sphere of radius r and density ρ dropped in a liquid of density ρ' and coefficient of viscosity η as shown in Fig. 12. The forces acting on the sphere are

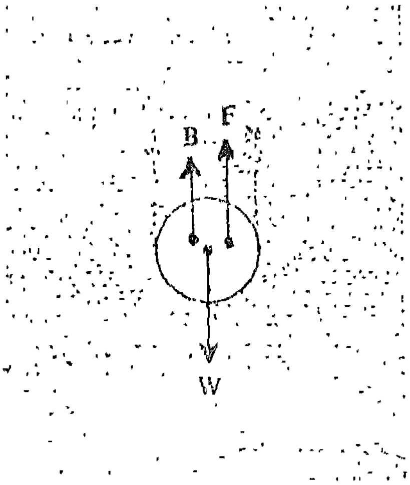

Fig. 12

(a) Its weight, $w = mg = (4/3)\pi r^3 \rho g$

(b) Force of buoyancy acting upward is equal to the weight of the liquid of volume equal to the volume of the sphere, $B=(4/3)\pi r^3\rho' g$

(c) The viscous drag F, acting upwards

From Newton's second law

$$ma = mg - (B+F)$$

$$= (mg-B)-F \qquad .. \; 13$$

where m is the mass of the solid sphere and a is its acceleration in the liquid.

The expression for F can be obtained using dimensional analysis, assuming

$$F \propto r^x \eta^y v^z$$

$$|F| = \text{constant} \; |r|^x \; |\eta|^y \; |v|^z$$

$$MLT^{-2} = \text{constant} \; |L|^x \; |ML^{-1}T^{-1}|^y \; |LT^{-1}|^z$$

Equating powers of M, L and T

$$y = 1, \; x-y+z = 1, \; -y-z = -2$$

which gives $x = y = z = 1$

$$\text{Thus } F = (\text{constant}) \; \eta \; r \; v \qquad ...(14)$$

Detailed calculations have shown that the constant is 6π

$$\text{Thus } F = 6\pi \; \eta \; r \; v \qquad ...(15)$$

This is known as the Stokes law. The viscous drag F increases with velocity v of the sphere and as long as it is less than other forces, the velocity of the sphere increases as it falls down (Eq 13). Finally when the viscous drag equals the *effective weight* $(mg\text{-}B)$, there is no resultant force acting on the sphere and consequently it acquires a constant velocity ($a=0$, Eq. 13) called the *terminal velocity* v_0. Using Eq. (13) and Eq. (15) the terminal velocity is given by

$$F = mg - B$$

$$6\pi\eta r \, v_0 = (4/3)\pi r^3 \, g \, (\rho-\rho')$$

$$v_0 = \frac{2}{9} \, \frac{r^2 (\rho-\rho')g}{\eta} \qquad ...(16)$$

Questions

1. How will you determine η experimentally using Eq. (16)?
2. Why do very small objects such as dust particles and tiny rain drops appear almost suspended in air? (Hint : $v_0 \propto r^2$.)
3. Why does a paratrooper hurt himself as he strikes the ground if he delays the opening of his parachute long enough?

Example 3

With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 Nm^{-2}s and specific gravity 0.901? What is the terminal velocity of the same air bubble in water? Take g=10m/s^2, density of air =1 kg/m^3, density of water 1001 kg/m^3, and η of water= 0.001 Nm^{-2}s.

Solution

(a) In this case the force of buoyancy B is greater than the weight of the air bubble, therefore the bubble rises up. In the given liquid

$$v_0 = \frac{2}{9}\,\frac{r^2(\rho-\rho')\,g}{\eta}$$

$$= \frac{2}{9}\,\frac{(4 \times 10^{-4}\text{m})^2(1-901)\ \text{kg/m}^3 \times 10\ \text{m/s}^2}{0.15\ \text{Nm}^{-2}\text{s}}$$

$$= -\,2.1 \times 10^{-3}\ \text{m/s}$$

the negative sign indicates that air bubble will rise up.

(b) The terminal velocity in water is given by

$$v_0 = \frac{r^2\,(\rho-\rho')\,g}{\eta}$$

$$= \frac{2}{9}\,\frac{(4 \times 10^{-4}\text{m})\,(1-1001)\ \text{kg/m}^3 \times 10\text{m/s}^2}{0.001\ \text{Nm}^{-2}\text{s}}$$

$$= -\,0.35\ \text{m/s}$$

which is higher in maguitude than in case (a).

ASSIGNMENT

1. Compute the maximum average velocity of water in a tube of diameter 3 cm so that the flow is streamlined ?
(Reynold's number $N = 2000$;
coefficient of viscosity $\eta = 0.0001\ \mathrm{Nm^{-2}s}$) Ans 6.7m/s

2. A horizontally placed pipe is running full of water. The pipe tapers gradually from one end where the diameter is 50 cm to the other end where the diameter is 20 cm. If the difference of pressures at the two ends is 10 cm of Hg, calculate the rates of flow of water at both the ends.
(Given density of Hg = 13.6 g/cc) Ans. 16.4 lt/s

3. Find the velocity of efflux of a liquid of density 200 kg/m^3 from a tank in which the pressure of the liquid at the orifice (constricted opening) is 1.2 atmosphere. Ans. 14.2 m/s

4. (a) Explain what do you understand by streamline and turbulent motion. What is Reynold's number and what is its significance ?

   (b) On the basis of Bernoulli's equation show that the velocity of efflux from the jet, as shown in Fig. 13 is

$$v = \sqrt{2gh}$$

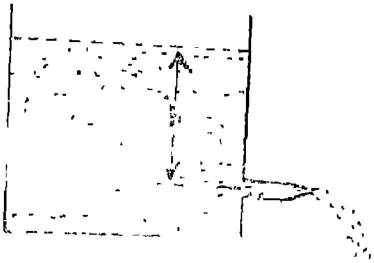

Fig. 13

5. Determine the terminal velocity of a steel ball 4 mm in diameter falling through glycerine. Specific gravity of glycerine is 1.3, specific gravity of steel is 9, and coefficient of viscosity of glycerine is 8.4 poise. Ans. 8cm/s

6 Calculate the viscous drag between two parallel layers of water each of area 0.1 m^2, when the velocity gradient is $0.01 s^{-1}$, η of water is 0.001 $Nm^{-2}s$ Ans: 10^{-6}N

SELF ASSESSMENT

1. (a) State and explain the equation of continuity.

   (b) Why can t the two streamlines intersect each other? Explain

   (c) Do you find anything else analogous to streamlines in physics?

   (d) What is the central point of Bernoulli's equation ?

   (e) What is an ideal fluid ?

2. "Bernoulli's equation is nothing but the principle of conservation of energy applied to an ideal, incompressible fluid". Elaborate this statement.

3. (a) Prove that $v_c = N\eta/\rho D$ by dimensional method, where symbols have their usual meanings.

   (b) Calculate the maximum average velocity of water in a pipe of radius 10 cm, so that the flow is streamlined. The coefficient of viscosity of water $\eta = 10^{-3}$ $Nm^{-2}s$, and Reynold's number $N = 200$.

4. Water is flowing through a Venturi meter at the rate of 10 m^3/min. The pressure in the wider portion is 2 atmospheres and the diameter is 16 cm, determine the pressure in the narrow portion if its diameter is 6 cm.

5. (a) At what speed will the velocity head of a stream of water be equal to 76 cm of Hg ?

(b) Explain why do spinnig cricket balls curve either upward or downward.

6. (a) How does viscous drag differ from friction ?

(b) If the viscous drag between two parallel layers of glycerine each of area 0. 5m^2 is 5N, calculate the velocity gradient, if coefficient of viscosity of glycerine is 1 3 poise.

7. (a) State the conditions under which a spherical body attains terminal velocity.

(b) Determine the terminal velocity of a silver ball of diameter 0.6 mm and specific gravity 10, falling through a liquid of density 800 kg/m^3 and η = 10 poise.

8. Deduce Stokes law using dimensional analysis.

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UNIT 17

Physics for Class XI

ELECTRICITY I

F 15/87

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
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PREFACE

This unit is nineteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr K. J George, Senior Project Fellow, helped in correcting the proofs.

At this stage this unit is an experimental edition I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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22 November 1982

INTRODUCTION

In preceding units you have studied about motion, work, power, etc. In the present unit you will apply those concepts of mechanics to electrical charges in motion. Early electrical knowledge, dating back many centuries, was concerned with charges obtained by rubbing things. Electric circuits, with current to run lamps and motors, could not be developed until batteries were invented. Then the growth was so rapid that it produced not just a part of physics but a new electrical civilization, in less than a century, making modern man inevitably dependent on it for the functions of his everyday life. Since it is of immense importance to the mankind, you must know about this wonder and its various effects in details. One of such effects called 'heating effects of current' will be discussed in the present unit. Other effects such as thermoelectric effect, chemical effect, and magnetic effect will be discussed in subsequent units.

OBJECTIVES

To be able to

1. Define electric current, potential, potential difference, resistance, work and power.
2. Derive the equation of drift velocity and deduce Ohm's law
3. State and derive Joule's law
4. Solve problems based on above objectives.

SUGGESTED READING MATERIAL

1. Current and resistance

It was known to the ancient Greeks that amber rubbed with wool acquired the property of attracting light objects. But it was after the discovery of electrons, we could understand that there are two types of materials, viz., conductors (possessing a lot of semi-free electrons which do not belong to any individual atom but belong to the bulk of the material and can't escape from the surface due to strong surface barrier), and insulators (possessing practically no semi-free electrons) Between conductors and insulators, is a class of material called semiconductors *If a body is negatively charged, it means semi-free electrons are in surplus, whereas if a body is positively charged, there is a deficit of such electrons.* Thus electrical charge of a conductor can be defined as *deficit or surplus of semi-free electrons.*

The force of interaction between electrical charges is governed by Coulomb's laws (which you have studied in class X). If you still remember it, answer the following question and skip rest of the discussion on Coulomb's law. Otherwise a brief discussion is given below for recapitulation.

Question

Two charges Q and q ($|Q| > |q|$) are separated by a distance r. Determine the point(s) at which force due to charge Q on a unit positive charge will nullify the force due to q, (a) if Q and q both are positive charges, (b) Q is $+$ ve but q is $-$ve.

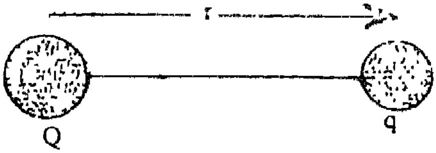


Fig. 1. The Coulomb's force on charge q is given by $F_q = \frac{Qq}{r^3}\,\mathbf{r}$ where the direction of $\mathbf{r}$ is from Q to q.

Coulomb's laws

Coulomb's laws are empirical but agreement with experiment is quite remarkable.

Law 1

The magnitude of the force of interaction between two electrical charges is directly proportional to the product of magnitude of charges. (Ref. Fig. 1.)

$$F \propto |Q|\ |q|$$

Law 2

The magnitude of the force of interaction between charges is inversely proportional to the square of distance of separation between them.

$$F \propto 1/r^2$$

Thus $$F \propto \frac{|Q|\ |q|}{r^2}$$

or $$F = k\frac{|Q|\ |q|}{r^2}$$

If the intervening space is free space (a space free from electrical charges, such as, vacuum), constant k is given by

$$k = \frac{1}{4\pi \in_o} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

where $\in_o$ is the permitivity of free space (By permitivity we mean power of conduction of electrical lines of force) and its value is

$$\in_o = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

In vector form Coulomb's laws can be expressed as

$$\boldsymbol{F} = \left[\frac{1}{4\pi\in_o}\frac{Q\,q}{r^3}\right]\boldsymbol{r}\ ,\ \boldsymbol{r} = r\,\hat{\mathbf{r}} \qquad ..\ (1)$$

where $\hat{r}$ is a unit vector along the line joining the two charges. When both Q and q are like charges, $\boldsymbol{F}$ has same direction as $\boldsymbol{r}$ (from source to field point) which implies that the force is repulsive. But when Q and q are unlike charges, $\boldsymbol{F}$ is negative implying attraction.

The electric field

Let us imagine an electric charge Q located at a point A in space as shown in Fig. 2. The space surrounding Q is affected by the presence of this charge, in that, if we bring another electric charge q at a point B, it will experience an attractive or repulsive force. We can as well say that the charge Q has set up an electric field at the point B and this electric field is exerting a force on charge q placed at B.

The electric (electric field strength) intensity at a point is defined as the force experienced by a unit positive charge placed at that point The electric intensity at a point distant r from charge Q will be given by

$$E = F/q. \qquad \text{...(2)}$$

E is a vector whose direction is the direction of vector F. E for point charge can be easily calculated from Coulomb's law.

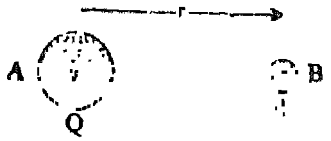


Fig. 2

$$E = F/q = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,\hat{r} \qquad \text{...(3)}$$

Example 1

Compute the electric field created by one proton at a distance of one *Fermi*. Given, 1 Fermi $= 10^{-15}$ m, charge of proton $= 1.6 \times 10^{-19}$C, $k = 9 \times 10^9$ Nm^2/C^2

Solution

$$\text{Electric field } E = \frac{kQ}{r^2}$$

$$= \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (1.6 \times 10^{-19} \text{ C})}{(10^{-15} \text{ m})^2}$$

$$= 1.44 \times 10^{21} \text{ N/C.}$$

Lines of force

The lines of force are imaginary lines used to visualize electric field patterns. The line of force is so drawn that the tangent to it at any point gives the direction of electric field at that point The number of lines of force per unit cross sectional area is proportional to the magnitude of E. Large E is indicated by closer lines of force, and weak E is represented by lines of force drawn far apart. Fig 3 shows lines of force due to a positive charge. The number and direction of lines of force depends on the charge distribution of the source.

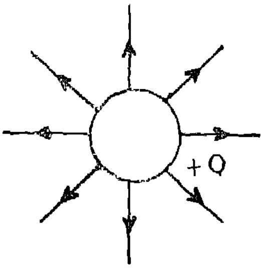


Fig. 3

Question

Draw lines of force due to a negatively charged sphere.

Electric potential

We know that the mechanical work is given by

$$W = \int F . dr$$

We have just now shown that force acting on charge q placed at a point A distant r from a charge Q, producing electric field E at that point, is given by $F = q E$.

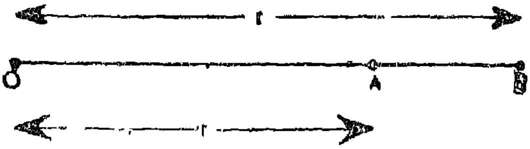


Fig. 4

Hence work done to displace q through a small distance dr will be

$$dW = q\, E.\, dr$$

Hence total work in displacing q from A to B (Fig 4) will be

$$W = q \int_{r}^{r'} E.\, d\, r$$

But we have shown

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}\, \hat{\mathbf{r}}$$

$$\text{Hence } W = \frac{Q\, q}{4\pi\epsilon_0} \int_{r}^{r'} \frac{dr}{r^2}$$

$$= \frac{Q\, q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{r'} \right]$$

If B is a point situated at infinity ($r' = \infty$), then work done is

$$W = \frac{Q\, q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{\infty} \right) = \frac{Q\, q}{4\pi\epsilon_0} \cdot \frac{1}{r} \qquad ...(4)$$

The above work W is the work done in carrying the positive charge q from A to ∞. If we do it the other way round i.e., we carry q from ∞ to A, the total work done is called as the *electrical potential energy* at A, simply because the work done will be stored up at A in the form of a potential energy. This potential energy if divided by q gives the work done in bringing a unit positive charge from ∞ to the point A; which is called *electrical potential at* A.

$$\text{or} \qquad V = \frac{W}{q} \qquad ...(5)$$

Since work is a scalar, potential, too, is a scalar. If charge Q creating electrical field is positive, work has to be done by someone in carrying unit positive charge, if on the other hand Q happens to be negative, work will be done by unit positive charge itself, (why?). *Thus potential at a point is the work done in carrying a unit positive charge from infinity to that point.* The unit of potential is *volt* If the work done in bringing one coulomb of charge from ∞ to a certain point is 1 *joule*, we say that the potential at that point is 1 *volt*. If we consider any two points in an electric field, their potentials will be different. The potential difference between them will be given by the work done in carrying a unit positive charge from one point to another, e.g. let us consider two points A & B with potentials V_A and V_B respectively. Work done W in carrying an electron from B to A (Fig 5).

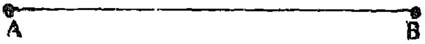


Fig 5

$$W = (-e)\,(V_B - V_A) = e\,(V_A - V_B). \qquad ...(6)$$

Therefore electrical work in displacing a charge from one point to another is equal to charge multiplied by potential difference.

Questions

1. How will you distinguish between *emf* and potential difference ?
2. If a proton and a neutron, separately, is carried from B to A, then how much is the work done in each case ?

You know fluids flow from higher pressure to lower pressure till the pressures become equal. Heat flows from higher temperature to lower temperature till the temperatures become equal. Similarly, the flow of charge from one conductor to another depends on the charge contents of the conductors, which governs their potentials. Thus as temperature is interpreted as thermal pressure, potential of a conductor can be interpreted as electrical pressure. Thus flow of +ve charge will occur from a body at higher potential to a body at lower potential till the potentials become equal. Now positive charge in motion from one point to another due to difference in potential between them constitutes conventional current. Thus the direction of flow of electrons is always opposite to the direction of *conventional current*.

Question

1. How does a fluid flow from lower pressure to higher pressure ?
2. In which case does charge flow from lower to higher potential and how ?

Electric current

Quantitatively electric current can be defined as rate of flow of charge. In the language of differential calculus, if at an instant t, charge passing through a conductor in time dt is dQ, the instantaneous current is

$$I = \frac{dQ}{dt} \qquad \cdots (7)$$

Electric current can be of various types. If the current is constant in magnitude and direction, it is called *direct current* (d.c.) However, if current changes its magnitude and direction, it is called *alternating current* (a c.) Currents can also be such whose magnitude changes but direction is constant, such currents are called *varying currents*. In the Fig. 6 given below, curve (a) represents d.c. while curve (b) represents a.c. and curve (c) represents a varying current.

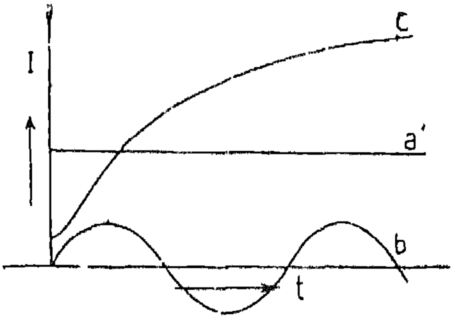


Fig. 6

From Eq (7) $Q = \int dQ = \int I\,dt$. If current I is a constant $Q = It$.

Electric current can propagate through metals (all metals are more or less good conductors), through electrolytes (e.g. acidified water, aqueous solution of NaCl, etc.), vacuum tubes (e.g. diodes, triodes, etc.) and semiconductors. For metallic conduction, specifically, an important law 'Ohm's law' is always obeyed. Ohm's law states that

"At constant temperature, current through a conductor is directly proportional to the potential difference across its two ends."

Putting it mathematically, if current is I through a conductor with potential across its two ends V_A and V_B; then from Ohm's law, at constant temperature, (Fig. 7)

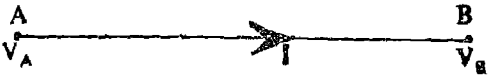


Fig 7

$$(V_A - V_B) \propto I$$

Let the potential difference be $V = V_A - V_B$, then

$$V \propto I$$

or $$V = I R \qquad .(8)$$

Where R is not merely a constant of proportionality, it is the measure of resistance of the conductor and its unit is 1 Ohm (Ω). *Resistance of a conductor is a property by which conductor puts obstruction to the flow of electrons through it.* Resistance of a *conductor* depends on temperature. In fact, with the rise in temperature resistance increases approximately according to the formula

$$R_t = R_0 (1 + a t)$$

where R_0 = Resistance of the conductor at 0°C,

R_t = Resistance of the conductor at t°C,

a = Temperature coefficient of resistance whose value is different for different conductors.

For a conductor of length l and cross sectional area A the resistance of the conductor R at constant temperature is related to l and A in the following fashion (Fig. 8)

$$R \propto l, \text{ if } A \text{ is constant}$$

$$R \propto 1/A; \text{ if } l \text{ is constant.}$$

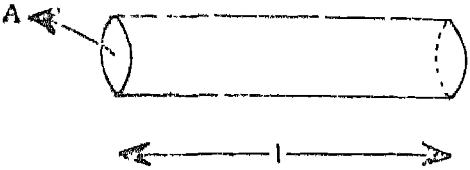


Fig. 8

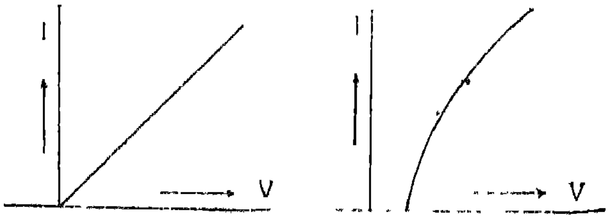
These are known as laws of resistance which give

$$R = \rho \frac{l}{A}$$

where ρ is called the specific resistance of the conductor. Reciprocal of ρ is called specific electrical conductivity σ or simply electrical conductivity.

$$\sigma = 1/\rho.$$

It is clear from Ohm's law (which is valid for metallic conductor only), that a graph of V against I will be a straight line passing through the origin (Fig. 9a) For electrolytic conduction, vacuum tube conduction and conduction through semi-conductors, V-I graphs are as shown in Figs. (9a), (9c) and (9d) respectively.



a

b

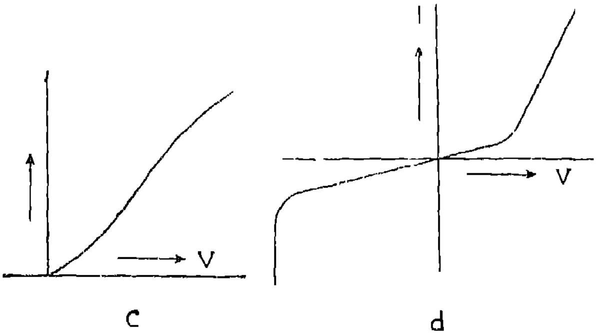


Fig. 9

It is clear from the graphs that none of them, except what is shown in Fig. 9a, are straight lines passing through the origin as being demanded by Ohm's law.

Questions

1. Given a piece of metal, where do semi-free electrons reside ? Why are they called semi-free ?
2. Why is the direction of flow of conventional current opposite to the direction of flow of electrons ?
3. What do you think are the important reasons for the non-obedience of Ohm's law for electrolytic and vacuum tube conduction ?
4. Is the current a scalar or vector ?

Example 2

How many electrons pass through a lamp in one minute if the current is 300 mA ?

Solution

Current $I = 300$ mA

$= 3 \times 10^{-1}$ A

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Time $t = 1$ minute

$= 60$ s

If n is the number of electrons and e is the charge on each electron, $Q = n\,e$ is the total charge. Hence

$$n\,e = I\,t$$

$$n = \frac{I\,t}{e}$$

$$= \frac{3 \times 10^{-1}\ \text{A} \times 60\ \text{s}}{1.6 \times 10^{-19}\ \text{C}}$$

$$= 10^{20}$$

Example 3

What is the number of semi-free electrons in a piece of silver of area of cross section 1.0×10^{-4} m² and length 1 m. Atomic weight of silver $= 108$, density of silver $= 105 \times 10^{2}$ kg/m³. Assume there is one semi-free electron per atom

Solution

Cross sectional area $A = 1.0 \times 10^{-4}$ m²

Length $l = 1$ m

Volume $V = l\,A$

$$= 1.0 \times 10^{-4}\ \text{m}^3$$

Mass of the silver piece $M = V\,d$

where $d =$ Density of silver

$$M = 1.0 \times 10^{-4} \times 105 \times 10^{2}\ \text{kg}$$

$$= 105 \times 10^{-2}\ \text{kg}$$

$$= 1.05\ \text{kg}$$

Silver is monoatomic, hence $\frac{108}{1000}$ kg of silver must have 6.023×10^{23}

atoms or same number of semi-free electrons. Hence 1 05 kg of silver will have $\left(\frac{6\ 023 \times 10^{23}}{0{\cdot}108} \times 1.05\right) = 5.83 \times 10^{24}$ semi-free electrons.

Problems

1. The electron in the hydrogen atom circles around the proton with a speed of 2.18×10^6 m/s in an orbit of radius 5.3×10^{-11} m. Calculate the equivalent current.

2. In a copper wire PQ, a current of one ampere flows in the direction P to Q. Calculate the number of electrons transferred from Q to P in 16 seconds (charge on the electrons $= 1.6 \times 10^{-19}$ C).

2. Drift velocity and derivation of Ohm's law

Metals are crystalline where molecules are arranged in regular periodic arrays called crystal lattices. They also possess numerous semi-free electrons leaving the atoms at lattice sites as charged ions Since heat is identified with kinetic energy of molecules, the semi-free electrons possess thermal energy and are in a state of chaotic motion much similar to the assembly of gas molecules. They in fact move with velocity of the order of 10^5 m/s at room temperature and these *velocities are randomly distributed* (*i e. in all possible directions*), hence one can say very safely that average velocity of the assembly of N electrons is

$$u_{AV} = \frac{u_1 + u_2 + \dots \dots + u_N}{N} = 0 \qquad \dots(9)$$

here u_1, u_2, ... u_N, etc. are the velocities of individual electrons. But their average speed is not zero. (Why ?)

Since their average velocity is zero, there is no mass motion of electrons in any particular direction. But the moment, a potential difference V is applied giving rise to an electric field E (where $E = \frac{-dV}{dr}$ i.e. E is the rate of variation of V with displacement r, with a negative sign), electrons start drifting in the direction of positive plate (since electric field is applied by two plates connected to two terminals of an electricity generator).

Since by definition E is the force experienced by a unit positive charge, electrical force on a single electron is negative (where 'e' is the electronic charge). If m is the mass of an electron, its acceleration a due to application of electric field will be

$$a = \frac{F}{m} = eE/m \text{ (From Newton's second law)} \qquad ...(10)$$

But when we consider the assembly of electrons in the metallic conductor, where is no net acceleration because the electrons keep colliding with the ions at lattice sites. The total effect of these collisions results into conversion of kinetic energy of electrons into vibrational energy of lattices. This conversion is total and continual and hence electrons acquire a constant average drift speed V_d in the direction $-E$. The drift speed V_d of electrons can be computed in the following fashion.

Fig. 10 shows the conduction electrons in a wire, moving to the right with constant drift speed V_d If A and l are cross-sectional area and length of the wire, the volume of the wire is Al. If n is the number of conduction

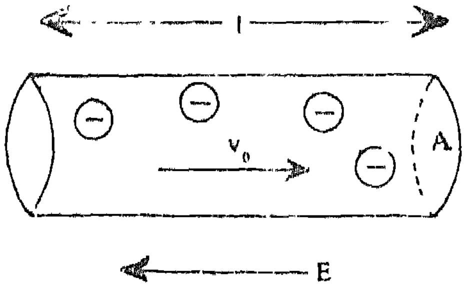

Fig. 10

electrons in unit volume, then total number of conduction electrons is nAl If electronic charge is e, total charge $nAle$ is present in the wire. If this charge takes time t to pass out of the wire, through its right end, then current is

$$I = \frac{nAle}{t} \qquad ...(11)$$

But electrons are drifting with a constant speed V_d given by

$$V_d = l/t \qquad \ldots(12)$$

Substituting from Eq. (12) in Eq. (11), we get

$$I = n\, e\, A\, V_d \qquad (13)$$

We define a quantity called current density j

$$j = I/A \qquad \ldots(14)$$

Hence from Eq. (13) and Eq. (14)

$$V_d = \frac{j}{n\, e}. \qquad \ldots(15)$$

Consider a single electron that has just collided with an ion core. The collision will destroy the tendency to drift and electron will have a random motion after the collision If λ is the average distance described by an electron between successive collisions (called *mean free path*) and since average thermal speed as assumed earlier is ν, then mean time between collisions is (λ/ν). At its next collision, electron velocity on the average, has changed by $[a\ (\lambda/\nu)\ ]$. We call this as drift speed V_d or the average velocity acquired by the electron since it suffered its last collision.

$$V_d = a\left(\frac{\lambda}{\nu}\right) = -\frac{e\,E}{m}\left(\frac{\lambda}{\nu}\right) \qquad \ldots(16)$$

It is clear from Eq. (16) that drift velocity is dependent on electric field E. For very weak field (say 1 volt/cm), thermal velocity of the electrons will be more dominant, whereas for very high electric field, drift velocity will be much more important than thermal velocity. Eq. (15) and as well as Eq. (16) are called drift velocity equations, from which Ohm's law can be derived in a very natural fashion. Let us equate Eq. (15) and Eq. (16)

$$\frac{I/A}{n\, e} = \frac{e\,E}{m}\left(\frac{\lambda}{y}\right)$$

Hence $I = neA\left(\frac{eE}{m}\right)\left(\frac{\lambda}{v}\right) = \left(\frac{ne^2A}{m}\frac{\lambda}{v}\right)E$ ...(17)

In Fig 10, $V = El$, where V is the potential difference applied across the two ends of the conductor of length l, and $\lambda/v = \tau$ is the mean time between collisions. Hence Eq. (17) can be written as

$$I = \left(\frac{ne^2A\tau}{ml}\right)V \qquad ...(18)$$

Since $(ne^2A\tau/ml)$ is a constant for a given conductor, let us put

$$R = \frac{ml}{ne^2A\tau} \qquad ...(19)$$

which is the resistance of the conductor.

Hence from Eq. (18) and Eq. (19)

$$V = IR \qquad ...(20)$$

which is the *Ohm's law* From Eq (19) we get the law of resistance.

$$R = \rho\, l/A$$

where $\rho = \frac{m}{ne^2\tau}$ $\qquad \sigma = \frac{ne^2\tau}{m}$

Questions

1. In Eq. (16) take mean time between collision (λ/v) as 10^{-14}s and put E equal to 1 V/m and hence determine the approximate value of V_d
2. In question (1), we will find the value of V_d to be extremely low. How does then an electric bulb light almost immediately when it is switched on ? Explain.
3. Can you think of a way to measure the drift speed of electrons by timing their travel along a conductor ?

Example 4

When a 20 cm long wire is connected between terminals of a 10 volt battery, the drift velocity is 1 cm/s What is the average time between collisions ?

Solution

Voltage applied $V = 10$ volts

Length of the wire $l = 20$ cm

Drift velocity $V_d = 1$ cm/s

$= 0.01$ m/s

Average time between collision $\tau = ?$

Electric field applied $E = \dfrac{V}{l}$

$= 10/20 = \frac{1}{2}$ volt/cm

$= 50$ volt/m

Drift velocity $V_d = e\,E\,\tau/m$

$$\text{Hence } \tau = \frac{m}{e\,E}\,V_d$$

where $m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C

$$\text{Hence } \tau = \frac{9.1 \times 10^{-31} \times 0.01}{1.6 \times 10^{-19} \times 50}\ \text{s}$$

$$= 1.14 \times 10^{-15}\ \text{s}$$

Example 5

A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm

(a) What is the resistance measured between two square ends ?

(b) Between two opposing rectangular faces ? The specific resistance ρ of carbon at the room temperature is 3.5×10^{-5} ohm m.

Solution

(a) The area of a square end is 1.0 cm^2 or $1.0 \times 10^{-4} \text{ m}^2$. Thus from the expression $R = \rho\, l/A$

$$R = \frac{(3.5 \times 10^{-5} \text{ ohm m})\ (0.50 \text{ m})}{1.0 \times 10^{-4} \text{ m}^2} = 0.18 \text{ ohm}$$

(b) For the resistance between opposing rectangular faces (area $= 5.0 \times 10^{-3} \text{ m}^2$). We have $R = \rho\, l/A$

$$= \frac{(3.5 \times 10^{-5} \text{ ohm m})\ (10^{-2} \text{ m})}{5.0 \times 10^{-3} \text{ m}^2} = 7.0 \times 10^{-5} \text{ ohm}$$

Thus we find that a given conductor can have a number of resistances, depending on how potential difference is applied to it.

Problem

Calculate the drift velocity of electrons in a metal conductor of area of cross section 10^{-5} m^2 for a current of 20 A. Assume that there are 8.4×10^{28} electrons per cubic metre.

(Ans : 1.5×10^{-4} m/s)

3. Heating effect of current

It is a matter of common observation that when electric current flows through a conductor, heat is produced. This is known as heating effect of current. The question is why is the heat produced ? Reasons are simple to understand if you are aware of the fact that whenever some mechanical work is done, an equivalent amount of heat is produced. (Ref : First law of Thermodynamics, $W = JQ$).

Now electrons driven through a conductor under the effect of potential difference, require work to be done against the resistance of the conductor, hence the heat—it is as simple as that,

Consider a conductor AB (Fig. 11) through which current I flows from A to B Evidently potential at A (V_A) is greater than potential at B (V_B). But electrons are flowing from B to A. Hence work done in carrying a single electron from B to A will be

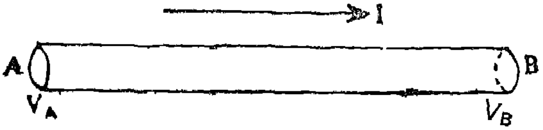

Fig 11

$$dW = (-e\, V_B) - (-e\, V_A)$$

$$= e\,(V_A - V_B) \qquad \text{...(21)}$$

(Electrons being negatively charged, their potential energies at B and A are negative.)

If in time dt, n number of electrons are moving from B to A, then total charge passing in time dt is $n\,e$. This constitutes a current

$$I = \frac{n\,e}{dt}$$

$$\text{Hence} \quad n = \frac{I\,dt}{e} \qquad \text{...(22)}$$

Now in Eq. (21), we have calculated the work done in carrying a single electron from B to A, but for n electrons the work done

$$dW = n\,e\,(V_A - V_B) \qquad \text{...(22a)}$$

Substituting from Eq. (22) in Eq. (22a) we find

$$dW = I\,dt\,(V_A - V_B)$$

Let $V_A - V_B = V$

Hence $dW = V\,I\,dt$

Thus the rate at which electric energy is consumed is power

$$P = \frac{dW}{dt} = V\,I \qquad \ldots(23)$$

If R is the resistance of the conductor, from Ohm's law we know

$$V = I\,R$$

Hence $\quad P = I^2 R = V^2/R$

If we want to calculate average heat Q produced during a time t, then

$$Q = I^2 R\, t \text{ joules} \qquad \ldots(24)$$

Joule is the SI unit of heat energy.

If we want to convert joules into calorie then, since we know 1 calorie is equivalent to 4.2 joules, we divided Eq. (24) by 4.2. Thus

$$H = \frac{I^2 R\, t}{4.2} \text{ calorie} \qquad \ldots(25)$$

This is known as *Joule's law*.

For calculating the total consumption of electricity in practical life, *joule* being very small unit, we take 1 kilowatt hour (1 kWh) where

1 kWh = 1000 watt-hour (Wh)

(Since if power P is in watts and time t in hours, then energy consumed is $W = P\,t$ watt-hour.)

and also, 1 kWh = 3.6×10^6 joules

1 kWh is also called as board of trade unit (B. O. U) and for computing monthly bill of electricity, for 1 B. O. U. say, fifty paise is charged.

Questions

1. The equation $P = I^2R$ seems to suggest that the rate of joule heating in a resistor is reduced if the resistance is made less; equ-

ation $P = V^2/R$ seems to suggest just the opposite How do you reconcile to this apparent paradox ?

2. Is the filament resistance lower or higher in a 5000 watt bulb than a 100 watt bulb ? Both bulbs are designed to operate on 110 volts.

Example 6

A heater is marked 500 watt, 220 volt. Find

(i) The current it draws.
(ii) Its resistance.
(iii) The cost of using it for four hours at 15 paise per unit.

Solution

Power $P = 500$ W

Voltage $V = 220$ V

Current $I = ?$

Resistance $R = ?$

We know $P = VI$

Thus $500 \text{ W} = (220 \times I)\, V$

$$I = \frac{500}{220} \text{ A}$$

$$= 2\ 27 \text{ A}$$

Also power $P = V^2/R$

$$R = \frac{V^2}{P} = \frac{220 \times 220}{500} \text{ ohms}$$

$$= 96.8 \text{ ohms}$$

Energy consumed in 4 hours

$$= 500 \times 4 \text{ Wh}$$

$$\frac{2000}{1000} \text{ kWh}$$

$$= 2 \text{ kWh}$$

Since the cost of 1 unit is 15 paise, 2 units will cost 30 paise.

Problem

An automobile battery produced a potential difference of 12 volts between its terminals A head light bulb is to be connected directly across the terminals of the battery and dissipates 40 watts of power What current will it draw and what must be the resistance ?

ASSIGNMENT

1. The type of wall socket commonly found in houses are capable of delivering a current of 5 A. If this current flows through a copper wire with a diameter of 0.1 cm, what is the drift velocity V_d of the electrons ? Given, number of electrons per unit volume is 8.4×10^{20} per m^3.

Ans 5×10^4 m/s

2. A 110 volt, 60 watt electric light bulb is connected between the terminals of a bank of storage batteries producing 110 volt. How many electrons enter the bulb per second ?

Ans. 3.4×10^{18}

3. A 450 watt electric press is made to operate at 225 volts. Find (i) its resistance (ii) rate of heat produced in joule per second (iii) power of the press, if voltage drops to 200 volts.

Ans. 112.5 W, 450 W, 355.6 W

4. A house is fitted with 8 lamps, 5 of 40 watt and 3 of 100 watt each, 3

fans each taking 0 25 A and an electric iron having a heating element of 44 ohms. If all the appliances work on 220 V supply for 5 hours a day, find the bill for the month of April at 20 paise per unit.

(Ans Rs. 52.95)

5 A potential difference V is applied to a conductor of length L, and diameter D. How are the electric field E, the drift velocity V_d and the resistance R affected when (i) V is halved, (ii) L is tripled, (iii) D is quadrupled

6. (a) The temperature coefficient of resistance of a thermistor is negative and varies greatly with temperature How will the I-V curve look like in this case

   (b) Show that P, the power per unit volume transformed into joule heat in a resistor can be written as

$$P = J^2 \rho$$

where the symbols have their usual meaning.

SELF-ASSESSMENT

1. (a) Calculate the drift velocity of electrons in a metallic conductor of length 2 m, when the voltage applied across its ends is 150 V.

   Given that the average frequency of collisions of electrons is 10^{14}/s.

   (b) If a current of 1.6×10^{-19} A is noted through a wire, what is the net number of electrons passing through any cross section inside the wire in 10 seconds.

2. The copper windings of a motor have a resistance of 50 ohms at 20°C, when the motor is idle. After running for several hours the resistance rises to 58 ohms. What is the temperature of the windings ?

3. A 500 watt immersion heater is placed in a pot containing 2 kg water at 20°C (a) How long will it take to bring the water to boiling temperature, assuming that 80% of the available energy is absorbed by the water ? (b) How much longer will be taken to boil half the water away ?

4. Deduce Ohm's law using drift velocity equation.

5. Resistance R, of a conductor varies with temperature t according to the formula

$$R = R_0 (1 + a\,t + \beta\,t^2)$$

Given $a = 10^{-3}/K$, $\beta = 10^{-4}/K^2$, and $R_0 = 1$ ohm.

Plot a graph of t against dR/dt. Determine the temperature t at which dR/dt is zero. At that point how will resistance depend upon temperature ?

F-15787(12)

UNIT 18

Physics for Class XI

ELECTRICITY II

F-15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is eighteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr. K. J. George, Senior Project Fellow, helped in correcting the proofs.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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Reader
Department of Education in Science
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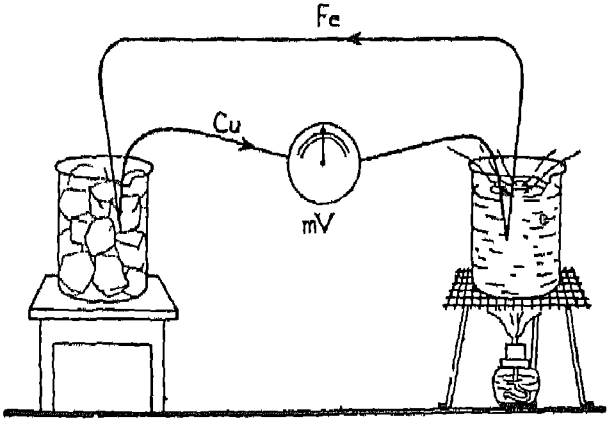
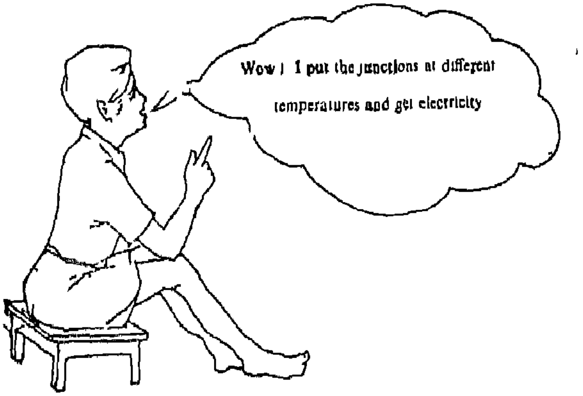
N C.E.R.T.

New Delhi
20 November 1982

INTRODUCTION

Take two wires of dissimilar metals (say, Cu and Fe) and join them in the end to end fashion Maintain the junctions at different temperatures inserting a milli-voltmeter in the circuit. You will notice that the voltmeter shows deflection. This is an effect of electric current (like the heating effect of current, you studied in the previous unit) called thermoelectric effect, which you will study in the present unit.




Wow ! I put the junctions at different temperatures and get electricity

Here we are also going to discuss about another effect of current where deposition of metal occurs when electric current passes through a solution. This is known as chemical effect of current. In the next unit you will study about other effects of electric current,

OBJECTIVES

To be able to

1. State and explain Seebeck effect; and state principle behind the thermoelectric series.
2. Describe the variation of thermo *emf* with temperature.
3. Explain how temperature is measured by thermocouples, and describe a *thermopile* giving its sketch and requisites.
4. State, and apply laws of electrolysis.

SUGGESTED READING MATERIAL

1. Thermoelectric effect (Seebeck effect)

Let us take two wires of dissimilar metals e.g. one of copper and other one of iron. These two wires are joined together at the ends A and B and a galvanometer of very low resistance is also included in the circuit as shown in Fig. 1. Junction A is kept at room temperature and when junction B is heated, a small electric current is observed as shown by arrows.

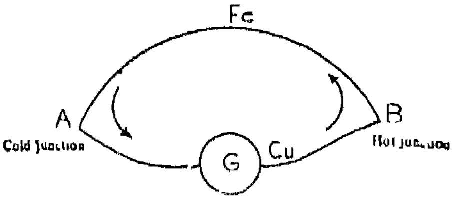


Fig. 1

Thus an *emf* could be produced by purely thermal means in a circuit composed of two different metals, whose junctions are maintained at different temperatures. This is known as Seebeck effect and *emf* in the circuit is called thermal *emf*. The origin of thermal *emf* can be explained in the following fashion. We know that all conductors contain a large number of free electrons Normally the concentration of electrons at the inter-face of two metals is not the same as it depends upon the nature of metals and temperature. Hence, when two dissimilar metals are brought in contact and connected to form a junction, electrons diffuse from one into other i.e. from metal of higher electronic concentration to that of lower electronic concentration, giving rise to a contact difference of potential. When the temperatures of the two junctions are same, this contact difference of potential is also identical across the two junctions and so there is no resultant *emf* in the circuit. However, when there is a temperature difference across the two junctions, the contact potential across the hot junction becomes more than the potential at the cold junction and the difference acts as the resultant *emf* which drives the current through the circuit.

In fact Seebeck investigated the effect for a large number of metals and arranged them in a series called *thermoelectric series*, such that when any two of them form a circuit called thermocouple, current flows at the hot junction from the metal occurring earlier in the series to the one occurring later. Given below are some metals belonging to the thermoelectric series in proper order.

Bi, Ni, Pt, Au, Pb, Ir, Fe, Sb.

Thermo *emf* is rather small, hence to measure it with reasonable accuracy, potentiometers are most suitable devices. Keeping cold junction at 0°C, the experimental graph of thermo *emf* E and temperature difference between hot and cold junction t is a parabola shown in Fig. 2. The magnitude of thermo *emf* is maximum at t_N called the *neutral* temperature At t_i which is called the *inversion* temperature, thermo *emf* reverses its direc-

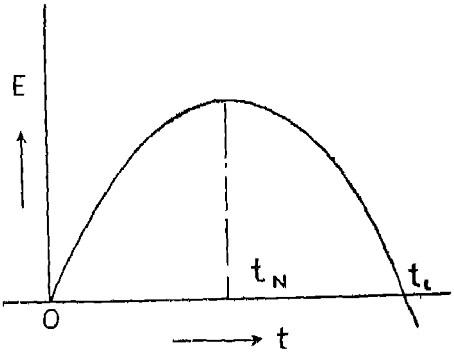


Fig. 2

tion. For a given thermocouple, the value of t_N is unique (for Cu-Fe thermocouple it is 275°C), but the value of t_1 is not unique; it depends upon the temperature of the cold junction. If the cold junction is at 0°C, T_1 for Cu-Fe thermocouple is 550°C, but it decreases with the rise in temperature of the cold junction

Temperature measurement by thermocouple

When the temperature T_1 of the cold junction is kept constant, Seebeck *emf* E is found to be a function of the temperature T_2 of the hot junction. This fact enables the thermocouples to be used as thermometers. Since the junction is small and has a small mass consequently small thermal capacity, it comes quite rapidly to thermal equilibrium with the system whose temperature is to be measured It, therefore, can follow the temperature changes rapidly.

To connect a thermocouple to a measuring instrument, it is necessary to break the thermocouple circuit at some point and introduce a third metal thereby creating two new junctions. But if both the new junctions are maintained at the same temperature, no disturbing effects are produced.

The temperature T_2 at the site of test junction can be measured noting down the *emf*, if the couple is previously calibrated. For considerable range of temperature, the thermo *emf* is almost a linear function of temperature, as it is clear from Fig. 2; therefore calibration is not difficult. Frequently galvanometer in the circuit is calibrated to read the temperature directly. Copper-constantan couple is used to measure temperatures ranging from 190°C to 300°C Thermocouples of platinum and platinum-rhodium alloy are used for measuring temperatures up to 1600°C.

As temperature measuring device, the thermocouple has following merits.

1. Because of the smallness of the test junction, it is particularly suitable for measuring temperatures in very small regions and cavities.

2. It is accurate and the *emf* can be measured with an accuracy of about 10^{-6} volt.

3. Since the test junction attains thermal equilibrium with the surroundings quickly, it can be used for measuring variations in temperature in various parts of the bodies of animals, insects etc.

Question

Fig. 2 depicts the variation of E against t. The curve is a parabola and we can write

$$E = a + bt + ct^2$$

where a and b are constants. Suggest a method to determine the temperature by a thermocouple by using the above formula. Will it be more convenient to plot dE/dt against t ? Explain why.

Example 1

The thermo *emf* of copper-constantan thermocouple at room temperature is 50 μV per degree. A galvanometer of resistance 50 Ω and capable of detecting currents as low as 2 μA is used. Calculate the smallest temperature difference that can be detected with this galvanometer.

Solution

Current $I = 2\mu$ A

$$= 2 \times 10^{-6} \text{ A}$$

Resistance of galvanometer $G = 50\ \Omega$

Corresponding voltage V, using Ohm's law is

$$V = I\,G$$

$$V = 2 \times 10^{-6} \text{ A} \times 50\ \Omega = 10^{-4} \text{ volts}$$

If θ is the smallest temperature difference which can be detected by this galvanometer then

$$10^{-4} \text{ volts} = 50 \times 10^{-6} \text{ volts/degree} \times \theta$$

$$\theta = 2 \text{ K}$$

Problem

Near room temperature, a copper-constantan couple and a galvanometer of 200 Ω resistance is capable of detecting current as low as 10 μA, are used. If the smallest temperature difference that can be detected with such couple is 1 K, what is the value of thermo *emf* of such a couple per degree ?

(Ans 2 mV per degree)

Thermopile

In a previous unit it has been shown that heat is a form of energy, and the propagation of heat by radiation consists merely of a transference of energy. But the radiant energy, in the process of transference, does not make itself evident unless it falls on matter. When it falls on matter and is absorbed, it is converted to heat and can then be detected by the use of thermocouple.

You have seen that the *emf* generated by a single thermocouple is only of the order of a few millivolts. But if a number of thermocouples are connected in series, the *emfs* are added up. Such a combination is called a thermopile and is used to measure the intensity of heat radiation. In its original form, Melloni's thermopile consisted of a number of bismuth and antimony rods, placed alternately with their ends joined, as indicated in Fig. 3. The left faces are blackened and form the hot junction, while

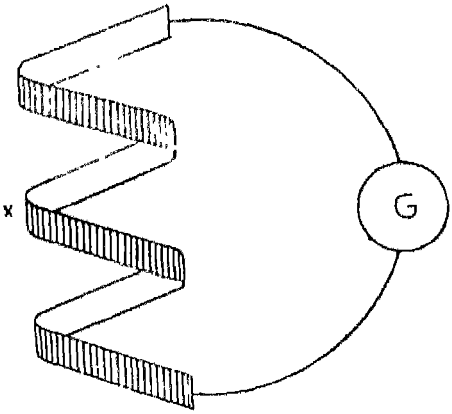


Fig. 3

the right ones are covered with insulation to form cold junction. Radiant energy falling on the hot junction, heats it and thereby producing an *emf* which sends a current through the circuit, and this is recorded by the galvanometer **G**.

Requisites of a sensitive thermopile

1. The sensitiveness of the thermopiles depends on the couple used.
2. The sensitiveness of the thermopile also depends upon the sensitiveness of the galvanometer. Thus the galvanometer must be of low resistance and voltage sensitive.
3. The sensitiveness also depends upon the number of thermo-junctions, but this cannot be multiplied indefinitely as the external resistance increases. The best procedure is to have the piles as light as possible and to choose a number so that the total resistance of the junction is equal to the galvanometer resistance.

Question

How will you investigate the emission or absorption of a substance, using a thermopile ? Explain.

Problem

It is desired to construct a Bi-Ag thermopile that will develop a thermal *emf* of 1 μV when temperature difference between hot and cold junction t is 0.001 K. How many separate thermocouples must be connected in series ?

Given $E_{\mathrm{Bi-Ag}} = a\,t + b\,t^2$

where $a = -46.20\ \mu\mathrm{V/K}$

$b = -0.482\ \mu\mathrm{V/K^2}$

What will the ve sign of E_{H-Ag} indicate ?

(Ans. Approx. 22)

2. Chemical effects of electric current

Electrolysis

If two small strips of platinum are connected to the terminals of a battery and dipped into a beaker of distilled water, it is difficult to pass current between them. But if a few drops of sulphuric acid are added, current begins to pass and gas bubbles appear at both plates. These bubbles are found to be that of hydrogen and oxygen, the two constituent elements of water Similarly if two plates of copper are connected to the terminals of a battery and then dipped into copper sulphate solution, current begins to flow.

Faraday made a thorough study of this process of chemical action by electricity. This chemical action is called *electrolysis*. The liquid conducting electricity is called an *electrolyte*. The two plates such as those of platinum, copper etc, are called *electrodes*, the one where the current enters, is called the *anode* and the one where it leaves, the *cathode*. The vessel in which the electrolyte is contained for electrolysis is called an *electrolytic cell* or a *voltameter*. The majority of electrolytes are liquids, and the most important ones are aqueous solutions of acids, bases and salts.

A *copper voltameter* consists of a glass vessel containing copper sulphate solution as electrolyte and two copper plates as electrodes (Fig. 4). When copper sulphate is dissolved in water it splits up into Cu^{++} and SO_4^{--} ions. The copper ions carry two units of positive charge and the sulphate ions, two units of negative charge.

$$CuSO_4 \rightarrow Cu^{++} + SO_4^{--}$$

When the electrodes are connected to a battery through an ammeter and a rheostat the Cu^{++} ions move towards the negative plate and after delivering their charge become neutral atoms. The SO_4^{--} ions go to the positive plate and combine with the copper atoms of the electrode to form $CuSO_4$. The $CuSO_4$ so formed goes into the solution and thus the concentration of the solution is maintained. Thus the cathode gains as much copper as the anode looses.

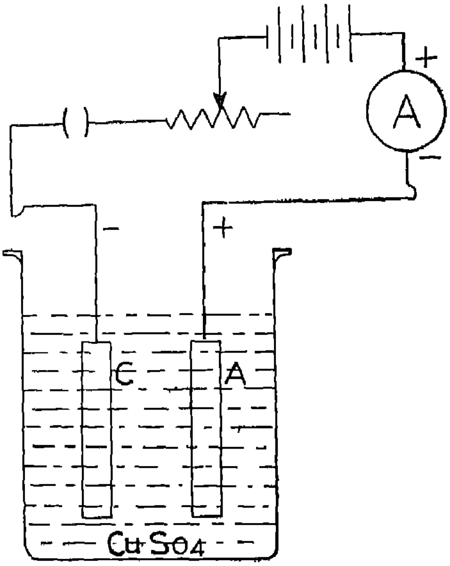


Fig. 4

Faraday's laws of electrolysis

Faraday stated the following two laws of electrolysis.

Law I

The masses of the substances deposited or collected at either electrode are proportional to the current and to the time during which current passes i.e. to the quantity of charge passed.

If m is the mass of ions deposited or collected at the electrode, then

$$m \propto I\,t \quad \text{or} \quad m \propto q$$

where I is the current in amperes flowing for t seconds, and q is the total charge in coulombs passing through the electrode.

$$m = Z\,I\,t = Zq \qquad \ldots(1)$$

where Z is the constant of proportionality and is called the *electro chemical equivalent* (E.C.E.) of the substance. Its value depends upon the nature of the substance. Thus we may define *E.C.E. as the mass in grams liberated by one coulomb of charge passing through an electrolyte*. Hence

$$\text{E.C.E.} = \frac{\text{Mass of the substance deposited}}{\text{Current} \times \text{Time}} \qquad \text{...(2)}$$

The unit of E.C.E. will thus be g/C.

Law II

The masses of different substances liberated by the same quantity of electricity, are proportional to their chemical equivalent $\left(\frac{\text{atomic mass}}{\text{valency}}\right)$ when voltameters containing different electrolytes are connected in series.

According to this law if a current is passed through a solution of $CuSO_4$, $AgNO_3$ and acidulated water for the same time, then for every one gram of hydrogen liberated, 108 grams of silver, and 31.5 grams of copper will be liberated. We may write it as

$$\frac{\textit{Mass of substance } \text{A} \textit{ deposited } (M_A)}{\textit{Mass of substance } \text{B} \textit{ deposited } (M_B)} = \frac{\textit{Chemical equivalent of } \text{A}}{\textit{Chemical equivalent of } \text{B}}$$

From Eq. (2) we get

$$\frac{M_A}{M_B} = \frac{Z_A}{Z_B} \qquad \text{...(3)}$$

Thus

$$\frac{\textit{Mass of } \text{A} \textit{ deposited}}{\textit{Mass of } \text{B} \textit{ deposited}} = \frac{\textit{Chemical equivalent of } \text{A}}{\textit{Chemical equivalent of } \text{B}}$$

$$= \frac{\textit{E.C.E. of } \text{A}}{\textit{E.C.E. of } \text{B}} \qquad \text{...(3a)}$$

Problem

For what time must a current of 2.5 amperes pass through a solution of zinc sulphate to deposit 1 g of zinc ? (E.C.E. of zinc=0.0003387 gram per coulomb.)

Faraday's laws of electrolysis can be put in a slightly different form. You know that a mass equal to the atomic mass divided by the valency is called an equivalent mass. Let us define one *faraday* (*F*) *of charge as the amount of electricity required* to liberate one gram equivalent mass of electrolyte. From Eq. (3) one can find out the value of one Faraday (How ?) and it is found that one F=96,519 coulombs Faraday's law of electrolysis is therefore, *that the number of gram equivalent masses of a substance deposited, liberated, dissolved, or reacted at an electrode is equal to the number of Faradays of electricity transferred through the electrolyte*

Let us put this statement mathematically. Let a mass *m* of a substance of atomic mass M and of valency J be deposited, due to the passage of I amperes of current for time t seconds. If Z be the E.C.E. of the substance, from the definition of Faraday

$$(M/J)=Z\,F \qquad \ldots(4)$$

and from first law

$$m=Z\,I\,t \qquad \ldots(5)$$

Substituting from Eq. (4) in Eq. (5), we get

$$m=\left(\frac{M}{J\,F}\right)I\,t$$

or,

$$\frac{m\,J}{M}=\frac{I\,t}{F} \qquad \ldots(6)$$

Eq. (6) is the mathematical form of above statement. Let N is the Avogadro's number and we consider the electrolysis of $AgNO_3$, resulting in the deposition of Ag on the cathode (say). If a mass of silver equal to the atomic mass is deposited, silver being monoatomic, it follows that N Ag atoms [illegible] transferred is then Ne, wh[illegible] novalent, the total

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charge is one faraday; then

$$Ne = F \qquad ...(7)$$

Eq. (7) enables us to determine Avogadro's number N

3. Application of electrolysis

1. Electroplating

Electroplating is a common application of electrolysis. The object of electroplating is to give articles made of one metal coating of another metal, either to improve their appearance or to protect them against rusting and tarnishing

2. Electrotyping

Electrotyping is commonly used in printing books. A wax impression of the type is made and the surface of wax is covered with a layer of graphite or other material to make it conducting. This conducting wax impression is then made the cathode in a copper voltameter and a layer of copper of sufficient thickness is deposited on the wax. The copper replica of this type is then heavily backened with an alloy of lead, tin and antimony and is used for printing. Metals other than copper may be used in the same manner.

3. Purification of metals

Electrolysis has been found very useful in purifying metals such as copper. Large anodes of impure copper are placed in large vessels containing a solution of $CuSO_4$ with 3.2% copper by mass and 16% free sulphuric acid The copper anodes contain many impurities, such as lead, arsenic, iron, and possibly selenium and some gold. By using suitable current and keeping the potential difference between anode and cathode little above 0.34 volt, all the materials but gold go into the solution, the gold falling to the bottom If the potential difference is less than 0.34 volt, only copper deposits on the cathode, the other metals remain in the solution. Zinc, cadmium and other metals may be separated in a similar manner.

4. *Decomposition of salts*

In 1806 Sir Humphry Davy first obtained small quantities of metallic sodium and potassium by electrolysis. Today most of the supply of these metals is produced by electrolysis of sodium or potassium hydroxide with iron electrodes.

5. *Electropolishing*

Electropolishing is the reverse of electroplating and is of great interest to the metallurgists. If an object is to be electropolished, it is placed in an electrolytic bath and made the anode; whereas copper or lead could be taken as the cathode. Then with a suitable electrolyte, the elevated portions of the anode are preferentially dissolved (i.e. removed from the anode). The result on the anode is similar to that of polishing.

ASSIGNMENT

1. Define thermo *emf*. How does the thermo *emf* vary with temperature difference between hot and cold junction ? What are neutral and inversion temperatures ?

2. The international ampere was defined as that unvarying current which in one second will deposit 0.001118 g of silver from a solution of $AgNO_3$. Determine from this definition the value of one Faraday.

   Ans. 96,600 C

3. Two electrolytic tanks are arranged in series, one containing a solution of $CuSO_4$, the other $AgNO_3$. How much silver will be deposited during the time in which 4.8 g of copper are deposited ? (Atomic mass of Ag=108; Atomic mass of Cu=63.5).

   Ans. 16.3 g

4. A potential difference of 3 volts is established across a copper electroplating cell. How much does it cost to deposit 500 g of copper if the electricity costs 50 paise per unit ?

Ans. 63 paise

5. (a) Explain the origin of thermo *emf*.

(b) List the requisites of a sensitive thermopile.

6. If the E_{ML} is given by the formula

$$E_{ML} = a\,t + b\,t^2$$

(M = Any metal; L = Lead)

(t = Difference of temperature between hot and cold junction)

compute for a copper-nickel thermocouple the *emf* when t = 100°C.

Given $a_{Cu\text{-}Pb} = 2.76$ μV/deg; $a_{Ni\text{-}Pb} = 19.1$ μV/deg

$b_{Cu\text{-}Pb} = -0.012$ V/deg^2, $b_{Ni\text{-}Pb} = 0.030$ μV/deg^2

and $E_{MM} = E_{ML} - E_{ML}$

Ans. 1.42 mV

SELF-ASSESSMENT

1. (a) Define E.C.E. of an element. What are its units ? How is it related to chemical equivalent mass ?

(b) How do you determine Avogadro's number starting from Faraday's laws ? Explain.

2. (a) What is thermoelectric series ? Explain.

(b) How do you make temperature measurement with the help of a thermocouple ? Explain with the help of a circuit diagram.

3. (a) How many Faradays of electricity are required to deposit 159 g of copper from a solution of $CuSO_4$?

(b) How many Ag atoms are deposited by a current of 4 amperes in 3 hours from a solution of $AgNO_3$?

4. A thin rectangular plate 4 cm × 3 cm is to be plated on both sides with gold to a thickness of 0.001 cm. How long must a current of 1.5 amperes be maintained through the plating solution ?

(Given specific gravity of gold = 20; Atomic mass of Au = 197; Valency of Au in plating solution = 3; E.C.E. of hydrogen = 0.00001 g/C.)

5. Calculate the volumes of the gaseous products at 0°C and 1 atmospheric pressure liberated during the electrolysis of water for one hour, with a current of 5 amperes.

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UNIT 19

Physics for Class XI

ELECTROMAGNETISM

F 15/87

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is nineteenth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help, from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit, I was assisted by some teachers. Dr K. J. George, Senior Project Fellow, helped in correcting the proofs.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

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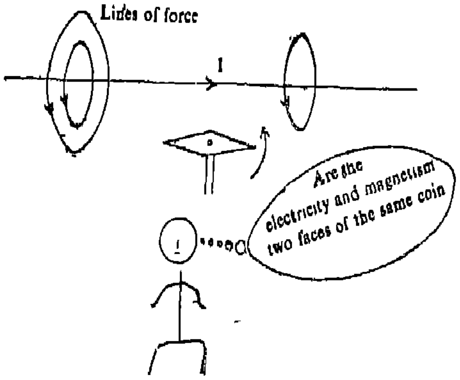
Department of Education in Science
and Mathematics
NCERT

New Delhi
22 November 1982

INTRODUCTION

By now you have learnt a little about electricity from two preceding units and we suppose you have some idea about what is magnetism. In the present unit we are going to make a synthesis of the two It is surprising that the linkage between electricity and magnetism eluded great many physicists; it was Oersted who first experimentally observed magnetism as a consequence of electricity. His experiment was very simple—pass electric current through a wire and put a magnetic needle near it; notice a deflection in the needle

The contents of the present unit will be devoted to the effect of electricity on magnet and vice versa; the subsequent unit will also be pertaining to practical applications of these concepts.


Lines of force
I
Are the electricity and magnetism two faces of the same coin

OBJECTIVES

To be able to

1. Define magnetic field intensity at a point in a magnetic field.
2. Apply Biot-Savart law to determine the magnetic field due to current in simple cases such as, (a) circular coil. (b) long conductor, and (c) solenoid etc.

3. Determine the force experienced in a magnetic field by (a) moving charge, and (b) a current carrying conductor.

4. Derive force between two parallel conductors carrying current.

5. Derive the torque acting on a coil placed in a magnetic field

SUGGESTED READING MATERIAL

1. Magnetic field intensity

Perhaps, you are familiar with basic ideas of magnetism. We straight away here introduce the Coulomb's law in case of interaction between magnetic poles.

Let there be two magnetic poles of pole strength M and m separated by a distance d (Fig. 1), according to Coulomb's law the force of interaction F is given by

Fig. 1

$$F = \frac{\mu_0}{4\pi} \frac{M m}{d^2} \qquad \text{...(1)}$$

where μ_0 is called the permeability (power of conduction of magnetic lines of force) of the vacuum, its value being $4\pi \times 10^{-7}$ webers/ampere metre.

Now the magnetic field is defined as the region of space where a unit N-pole experiences a force and *magnetic field intensity at a point in a magnetic field is the force experienced by a unit north pole placed at that point.* You may say that isolated polarities do not exist, so the definition above is bad from the operational point of view. You are absolutely right. That is why in the next section, we are going to introduce the practical type of definition for the magnetic field intensity. However, from Coulomb's law we get an expression for the magnetic intensity.

Consider a pole of pole strength M placed at a point x and consider another point y distant d from it. If you want to determine the magnetic intensity at y, put a test pole of strength m over there and determine the force F experienced by it, which will be given by Eq (1) The magnetic field strength B, sometimes also called the field of induction, at point y, is defined as

$$B = \frac{F}{m} = \frac{\mu_0}{4\pi} \frac{M}{d^2} \quad \text{.. (2)}$$

The direction of B is same as the direction of force F experienced by a north pole placed at point y.

Similar to the electric lines of force, we define magnetic lines of force (also called lines of induction) to visualize the magnetic field. B is related to these lines in two ways ·

1. Tangent to the lines of force at any point gives the direction of B at that point.
2. The number of lines of force per unit area is proportional to magnitude of B. Closer lines of force represent stronger field.

Moving charge in a uniform magnetic field

Consider a charge q moving with velocity v in a magnetic field B as shown in Fig. 2. This charge will experience a sideways force called the Lorentz force F given by

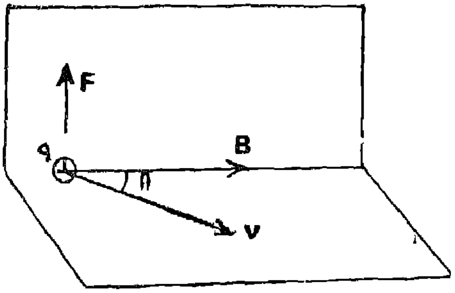


Fig. 2

$$F = q\ (v \times B) \quad \text{...(3)}$$

The definition of cross product such as $v \times B$ is given in unit I which you may review. As shown in Fig. 2, F is perpendicular to the plane of vectors **v** and **B** and has magnitude

$$F = B q v \sin\theta \qquad \text{...(4)}$$

where θ is the angle between **v** and **B**. We define the magnitude of magnetic field B from Eq. (4) as

$$B = \frac{F}{q v \sin\theta} \qquad \text{.. (5)}$$

When F is expressed in newtons, q in coulombs, and v in m/s, units of B are weber/m² (or Wb/m²) or tesla. *Thus one tesla (Wb/m²) is the magnetic field in which one coulomb of charge, moving with a component of velocity perpendicular to the field ($v_\perp = v \sin\theta$) equal to one metre per second, is acted on by a force of one newton.*

$$\text{Hence} \quad 1 \text{ tesla} = 1 \text{ Wb/m}^2$$
$$= 1 \text{ N/(C m/s)}$$
$$= 1 \text{ N/Am}$$

Since $F \perp v$, the net effect of the force F will be to change the direction of motion of the charge, since it does not produce change in speed. (How ?) *However by changing the direction of motion of the charge a direction will be found in which the magnetic field will exert no force. Such a direction will be defined as the direction of magnetic field.* (In this case since **v** and **B** are in the same direction $v \times B = 0$, i.e. no Lorentz force, consequently charge will pass through the field undeviated.)

2. The Biot-Savart law and its applications

The Biot-Savart law gives the magnetic field produced by a current distribution. Consider a small length segment dl (Fig. 3) in a conductor carrying a current I as shown. Its direction is along the tangent to the conductor (dotted lines). It is experimentally determined that the magnitude of magnetic intensity dB at a point O distant r from dl will be

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2} \qquad \text{...(6)}$$

where θ is the angle between the direction of dl and r You may notice

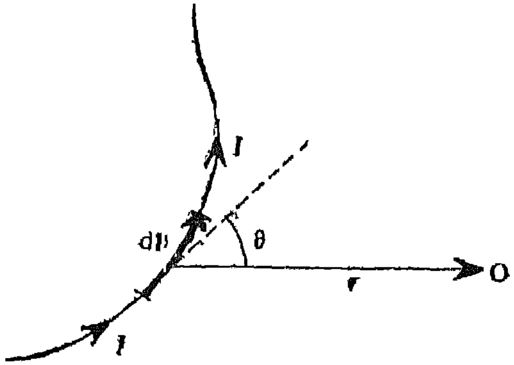


Fig 3

that dB obeys inverse square law of distance Eq (6) can be written in the form of vector product as

$$dB = \frac{\mu_0}{4\pi} \, I \left(\frac{dl \times r}{r^3} \right) \qquad (7)$$

In this case (Fig. 3) since dl and r, are in the plane of paper, dB will be going into the paper.

The resultant field at O can be found by integrating Eq. (6) or

$$B = \int dB$$

Questions

1. How will you determine the direction of dB by Eq. (7) ?
2. Why didn't we consider current I to be a vector, instead of dl in Eq. (7) ? Explain.
3. How have we synthesised electricity and magnetism in Biot-Savart law ?

(*i*) *Field at the centre of a circular coil*

Consider a circular coil of radius r carrying counter clockwise current

I as shown in Fig. 4. The field dB due to a small length segment dl at the centre O of the circular coil will be in the direction of $dl \times r$ which is

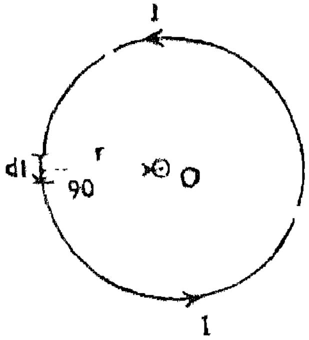


Fig. 4

coming out of the paper towards the reader as represented by The magnitude of the field is given by Eq. (6)

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

Let us divide the entire ring into many such length segments and fields at O due to all of them will be in the same direction (How ?). Hence the resultant field at O will be given by

$$B = \int dB = \int \frac{\mu_0}{4\pi} I\,dl/r^2$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} 2\pi r$$

Thus

$$B = \frac{\mu_0 I}{2r} \qquad \text{.. (8)}$$

1. Given two circular coils carrying same current with radius of one being four times of the other. If you want the fields at the respective centres to be in the ratio 1 : 3, what should be the turn ratio between the former and the latter ?

2. Assuming that the earth's magnetic field is due to a large circular loop of current in the interior of the earth, what is the plane of the loop and what is the direction of current around it ?

(ii) Field on the axis of a circular coil

We don't intend to deduce the expression for the field *B* at a point A distant *x* from the centre O of the circular coil of radius *a*, but we give some hints.

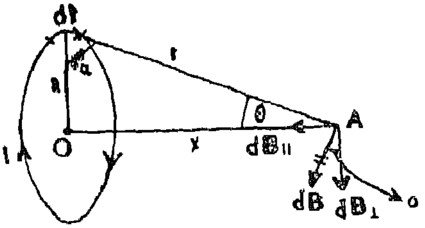


Fig 5

Hints : Field at A due to *dl* is (Fig. 5)

$$dB = \frac{\mu_0}{4\pi}\,(I\,dl\,\sin 90^\circ)/r^2$$

where $r^2 = a^2 + x^2$; $\cos a = \dfrac{a}{(a^2+x^2)^{1/2}}$

Resultant field *B* at A due to entire coil is

$$B = \int dB = \int dB \cos a = \frac{\mu_0 I a^2}{2\,(a^2+x^2)^{3/2}}$$

Problem

A convenient rule to determine the direction of B in this case is as follows :

Curl the fingers of your right hand in the direction of the current (knuckles to nails), the stretched thumb then points in the direction of the field.

Problem

Magnetic field due to a circular coil of 250 turns, radius 15 cm, carrying a current of 3.2 ampere at a point on the axis of a coil is 8×10^{-2} Wb m^{2} Determine the distance of the point from the centre of the coil.

$$(\mu_0 = 4\pi \times 10^{-7} \text{ WbAm}^{-1})$$

(*iii*) *Field due to a straight current*

Consider a straight infinite conductor carrying a current I. We want to determine the field at O distant R from the conductor as shown in Fig. 6.

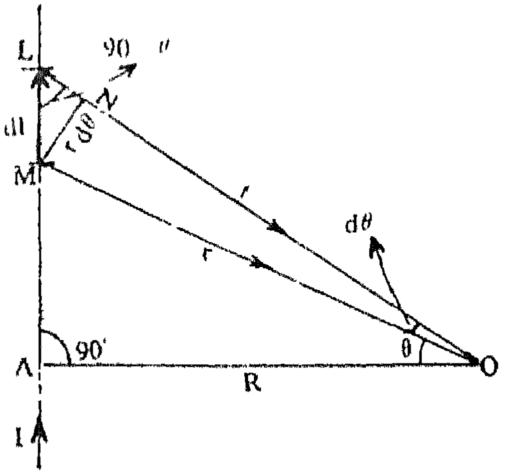


Fig. 6

In $\triangle$ MNL $\qquad \frac{r\,d\theta}{dl} = \cos\theta$

In $\triangle$ M AO $\qquad R/r = \cos\theta$

Hence

$$\frac{r\,d\theta}{dl} = \frac{R}{r}$$

$$\frac{dl}{r^2} = \frac{d\theta}{R} \qquad \text{...(9)}$$

The field at O due to *dl* can be obtained by using Eq. (7).

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin[180-(90-\theta)]}{r^2}$$

Using Eq. (9) we get

$$dB = \frac{\mu_0}{4\pi} \frac{I}{R} \cos\theta\, d\theta \qquad \text{...(10)}$$

The field due to whole length of the conductor (assuming it to be of infinite length).

$$B = \int_{-\pi/2}^{\pi/2} \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{R}\right) \cos\theta\, d\theta \qquad \text{...(11)}$$

(Try to explain why have we taken the lower and upper limits as $(-\pi/2)$ and $(\pi/2)$ respectively). After integration we get

$$B = (\mu_0 I)/2\pi R \qquad \text{...(12)}$$

The directions of $\boldsymbol{B}$ is the direction of $(\boldsymbol{dl} \times \boldsymbol{r})$ which is going into the paper at O. A convenient rule is given below :

Grasp the wire in the right hand with the thumb pointing in the direction of current, the field lines then follow the direction of curled fingers from [illegible] circles around a current (Fig. 7). The tangent at any point on the field lines will give us the direction of the field at that point.

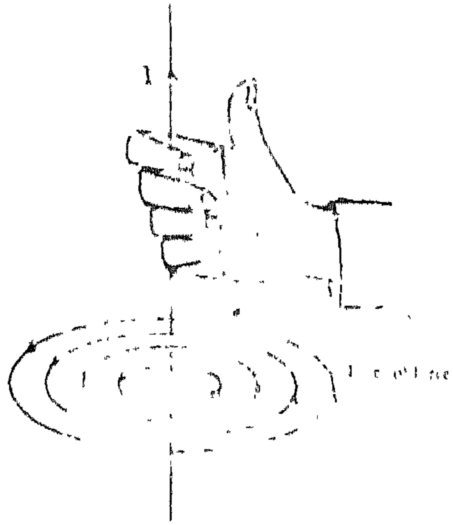


Fig. 7

In Fig. 8, we have shown the magnetic field lines due to a cylindrical coil carrying a current. Such a coil is normally known as a *solenoid.* The field is uniform in the interior of the coil and is given by

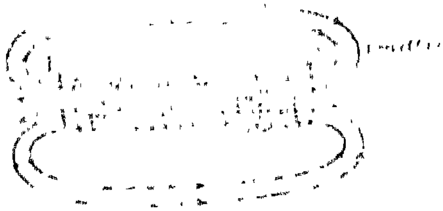


Fig. 8

$$B = \mu_0 I n \qquad ...(13)$$

where I is the current and n the number of turns per unit length of the solenoid.

Problems

1. Calculate the current through a straight conductor if the magnetic field due to it at a point distant 0.5 m is 6.28×10^{-9} Wb/m². (Given $\mu_0 = 4\pi \times 10^{-7}$ WbAm)

2. Calculate the magnetic field at a point on the axis of a solenoid at a distance 10 cm from one end of the solenoid.

   (Given, current $I = 1.5$ A, no of turns $N = 200$ and length of the solenoid $l = 30$ cm)

Magnetic force on a current carrying conductor

A charge moving in a magnetic field experiences a force. Since current in a conductor is an assembly of moving charges, it will experience a force when placed in a magnetic field. Let us determine the force experienced by a length element dl of a conductor carrying current I when placed in a magnetic field $\boldsymbol{B}$. The force experienced by charge dq can be obtained from Eq. (3)

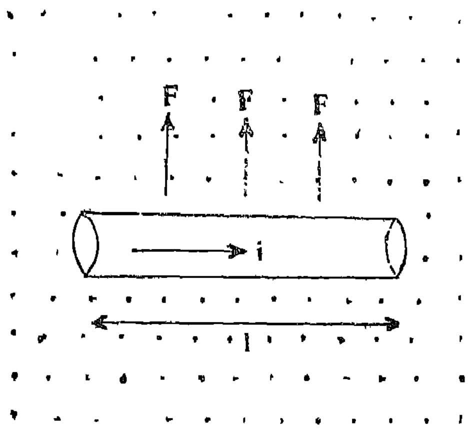


Fig 9

dF |

$$dq\left(\frac{dl}{dt}\times B\right)$$

$$\frac{dq}{dt}\left(dl\times B\right)$$

Since $I=\frac{dq}{dt}$, we get

$$dF = I\,(dl\times B) \qquad \text{...(14)}$$

which gives the force experienced by length dl of the conductor. If the conductor is kept at right angles to the field (Fig. 9), the magnitude of the force is

$$F = I\,l\,B \qquad \text{...(15)}$$

Interaction between current carrying conductors

Consider two infinitely long parallel conductors carrying currents I_1 and I_2 and separated by a distance r, currents being in the same direction (Fig. 10). Assuming conductor II to be lying in the field B of I where

$$B = \frac{\mu_0 I_1}{2\pi r} \qquad \text{...(16)}$$

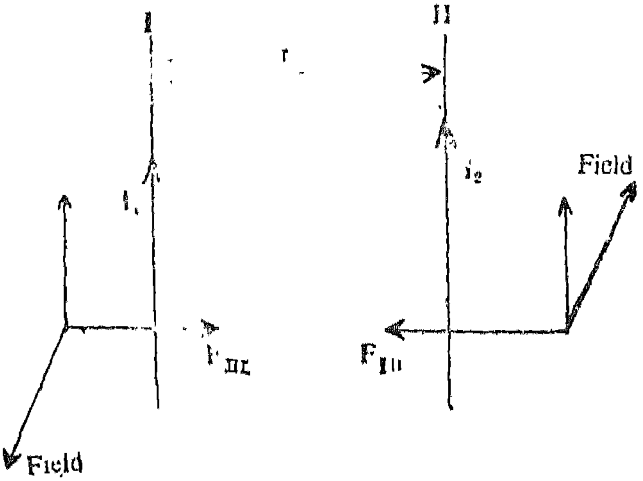


Fig. 10

Field B is directed towards the page and is at right angles to conductor (II). Hence, from Eq. (15) force per unit length at (II) will be given by

$$F_{\rm I\,II}=BI_2=\frac{\mu_0 I_1 I_2}{2\pi r} \qquad ...(17)$$

The direction of $F_{\rm I\,II}$ will be as shown in Fig. 10. Try to ascertain yourself that indeed $F_{\rm I\,II}$ is in the direction as shown.

Similarly force experienced by conductor (I) placed in the magnetic field of (II) ($F_{\rm II\,I}$) can be obtained, of course, the magnitude of $F_{\rm II\,I}$ will be same as given by Eq. (17), but direction will be as shown in Fig. 10.

What do you conclude from the above discussion ? *When currents through two parallel conductors are in the same direction, they will attract each other.*

Questions

1. Why do two parallel wires carrying currents in the opposite direction repel each other ? Explain.
2. Consider oblique currents as shown in Fig 11, Case-I and Case-II. In which case do you get attraction and in which repulsion ?

   (Hint : In both the cases take the component of second current in the direction of the first)

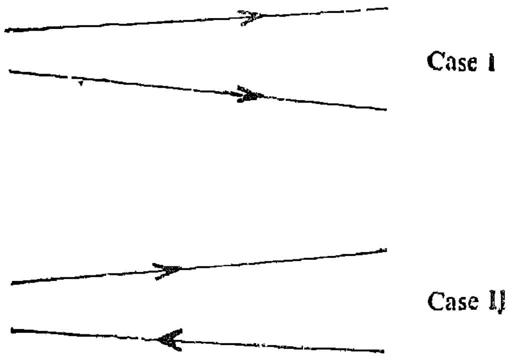


Fig. 11

Example 1

A wire making an angle of 30° with the horizontal (carrying current of 5A) is supported by a magnetic field of 0.65 Wb/m² (perpendicular to the plane of Fig. 12). Determine the mass per unit length m of the wire

(Take g = 10 m/s²)

Solution

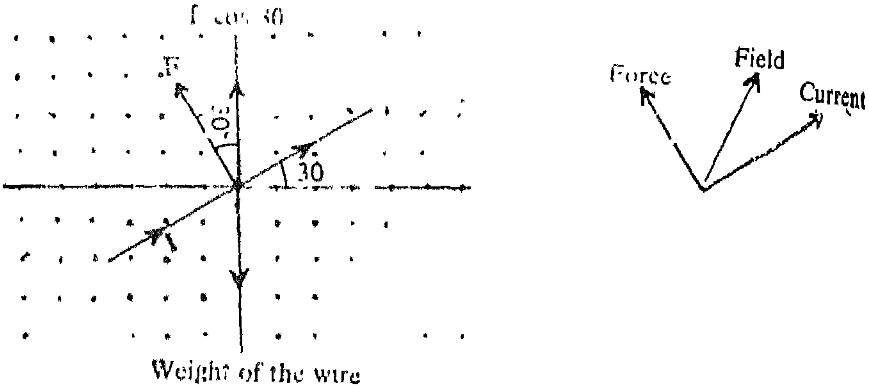


Fig. 12

Lorentz force on the wire

$$F = B\,I\,l\,\sin 90^\circ$$

$$= B\,I\,l$$

Component of the Lorentz force in the vertically upward direction

$$= F\cos 30^\circ$$

$$= B\,I\,l \times (\sqrt{3}/2)$$

Weight of the wire acting in the vertically downward direction

$$= m\,l\,g$$

Hence $B\,I\,l \times (\sqrt{3}/2) = m\,l\,g$

$$m = \frac{B\,I \times (\sqrt{3}/2)}{g}$$

$$m = \frac{0.65 \times 5 \times \sqrt{3}/2}{10}\ \text{kg/m}$$

$$m = 0.28\ \text{kg/m}$$

Example 2

A long straight wire carries a current of 2A. An electron travels with a velocity of 4.0×10^4 m/s parallel to the wire 0.1 m from it, and in a direction opposite to the current as shown in Fig. 13 What force does the magnetic field of current exert on the moving electron ?

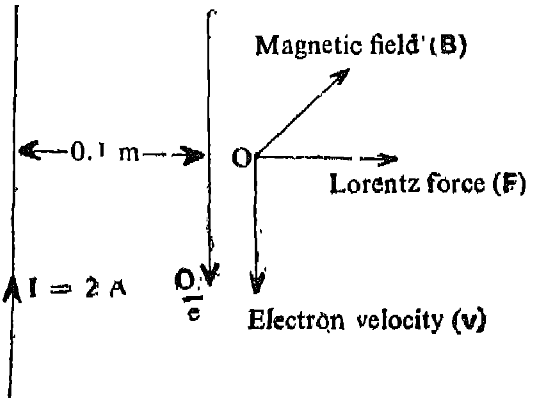


Fig. 13

Current $I = 2$A

Distance $r = 0.1$ m

Velocity of electron $v = 4.0 \times 10^4$ m/s

Magnetic field due to wire

$$B = \mu_0\, I/2\pi\, r$$

$$= \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 0.1} \text{ Wb m}^{-2}$$

$$= 4 \times 10^{-6} \text{ Wb m}^{-2}$$

Lorentz force $F = I(l \times B)$

or

$$F = e(v \times B)$$

Since v and B are at right angles, magnitude of F will be

$$F = Bev = 4 \times 10^{-6} \times 1.6 \times 10^{-19} \times 4 \times 10^{4} \text{ N}$$

$$= 2.56 \times 10^{-20} \text{ N}$$

Example 3

Consider two electrons moving on parallel paths in free space side by side, with same speed v as shown in Fig. 14. The distance between them, at right angles to their velocity, is r. Determine the force acting on one of them, owing to the presence of the other, as observed in the frame of reference moving with the electrons.

Solution

Lorentz force is velocity dependent. Since electrons are moving in the same direction with the same velocity their relative velocity is zero or in a frame of reference moving with the electrons, they are at rest.

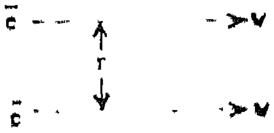


Fig. 14

Thus there is no Lorentz force between them; the only force F acting on them is Coulomb's repulsion given by

$$F = \frac{1}{4\pi\epsilon_0}(e^2/r^2) \qquad ..(18)$$

This is why although two parallel wires carrying currents in the same direction attract each other, two beams of electrons travelling in the same direction with same speed repel each other.

Magnetic force and torque on a complete circuit

Consider a rectangular coil carrying current I placed in a magnetic field B provided by concave pole pieces which give a radial field as shown in Fig 15 Let $AB=CD=b$ and $AD=BC=l$. The horizontal sides AB and CD of the rectangular coil are parallel to the direction of the field, hence Lorentz force does not act upon them (Since Lorentz force $F=B\,I\,l\sin\theta$ and $\theta=0$ in this case), whereas Lorentz force acting over the vertical sides AD and BC are $B\,I\,l\sin 90°$ ($\theta=90°$) in the directions as shown in Fig. 15.

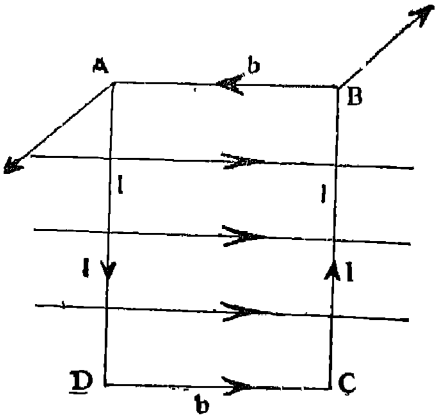


Fig. 15

The forces acting upon AD & BC are unlike parallel forces of equal magnitudes, hence they will constitute a couple, which will try to deflect the coil.

The moment of the deflecting couple

$$\Gamma = (n\,B\,I\,l) \times b \qquad \ldots(19)$$

(where n is the number of turns in the coil.)

Area of the coil $A = l \times b$

Hence Eq. (19) can be written as

$$\Gamma = n\,B\,I\,A \qquad \ldots(20)$$

Instead of concave pole pieces if we use flat pole pieces, then we get rectangular field, for which (Box 2)

$$\Gamma = n\,B\,I\,A\cos\theta \qquad \ldots(21)$$

where θ is the angle between the plane of the coil and the magnetic field.

ASSIGNMENT

1. A uniform resistance wire is given in the form of a circular loop Any two points (A and B) are connected to the terminals of a cell as shown in Fig. 16. What should be the magnetic field at the centre of this loop ?

(Ans. Zero)

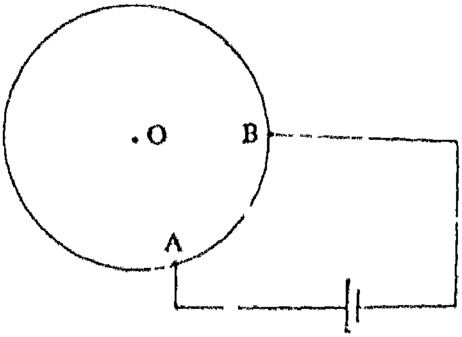


Fig. 16

2. Fig. 17 shows a rectangular coil suspended from one arm of an analytical balance. It hangs between the poles of an electromagnet with

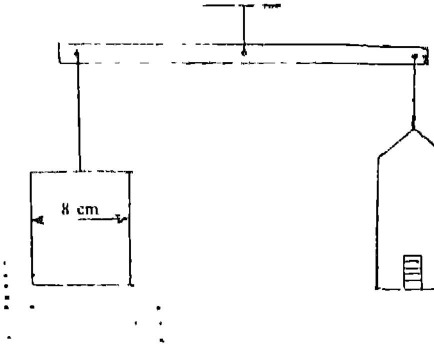


Fig. 17

the plane of coil parallel to the pole faces. The magnetic field is uniform in the shaded region and negligibly small in the neighbourhood of the top wire. The coil has 15 turns and a bottom edge of length 8 cm. The system is balanced and then current of 0.5 amp is passed through the coil. If an extra weight of 60.5 g must be added to the right pan to rebalance the system, what is the strength of the magnetic field B in tesla ? (Take $g = 10$ m/s^2).

(Ans. $B \sim 1$ tesla)

3. A long straight wire is bent into a hair pin like shape as shown in Fig 18. What is the magnetic field at the centre O of the half circle of radius r.

(Ans. $\mu_0 I/4r$)

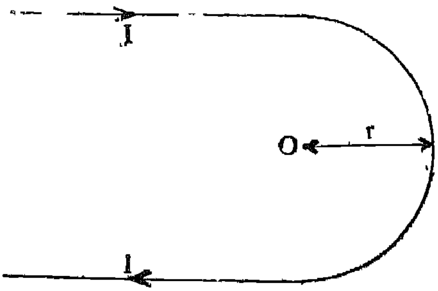


Fig. 18

4. A straight wire carrying current I_2 runs along the axis of a circular current I_1, as shown in Fig. 19. What is the force of interaction per unit length of the wire between the currents ?

(Ans. Zero)

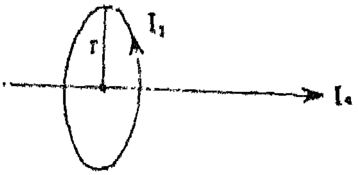


Fig. 19

SELF-ASSESSMENT

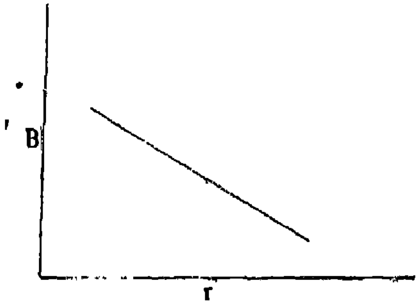
1. An electric current exists in a long wire which at a distance of 2 cm produces a magnetic field of 2×10^{-8} Wb/m^2. What will be the field at 4 cm from the wire ?

2. What will be the force experienced by a wire placed parallel to the lines of force in a magnetic field B when a current I flows through the wire ?

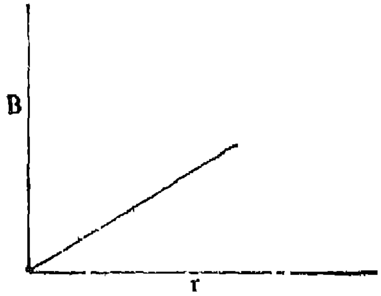
3. Two long straight wires are set parallel to each other. Each carries a current I in the same direction and the separation between them is $2r$. Determine the intensity of the magnetic field midway between them.

4. Two long parallel wires 4 cm apart, carry currents of 2 and 6 amperes respectively, in the same direction. What is the force between the wires per metre of wire length ?

5. Which of the following graphs (Fig. 20) represents the variation of magnetic flux density B with distance r for a straight long wire carrying an electric current ? Explain.



(a)



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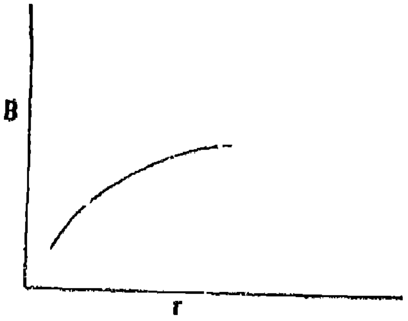

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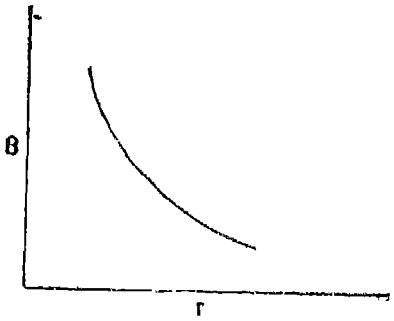

(d)

UNIT 20

Physics for Class XI

APPLIED ELECTROMAGNETISM

F. 15187

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

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PREFACE

This unit is twentieth in a series of units developed for try-out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding and next units. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self-study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful

In preparing this unit, I was assisted by some teachers. Dr. K. J. George, Senior Project Fellow, helped in correcting the proofs.

At this stage this unit is an experimental edition I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
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New Delhi
10 January 1983

INTRODUCTION

You know that atoms are made up of fundamental particles, one of which being electron. But how are you going to detect the electron ? Its detection by looking at is out of question. There are a set of quantities which have different values for different particles. One of them is specific charge (charge to mass ratio) and in the present unit you will learn how to determine it for electron with the help of the theory you learnt in the previous unit. You will also apply your knowledge of the last unit to understand the principles of galvanometer, ammeter, and voltmeter.

The next question is how are we going to know what is happening inside the atomic nucleus. Obviously, a clearer picture will only emerge out if it is smashed by bombardment with high energy beam of particles such as α-particles, protons, deuterons and so on. However, bombardment will only be effective if particles are highly accelerated.

In the present unit you are also going to study about a particle accelerator called cyclotron. Thus in this unit we will confine ourselves to the world of microscopic entities. In the next unit we want to go to the other extreme, i.e., to peep inside the macroscopic world.

OBJECTIVES

To be able to

1. Describe the principle, working with the help of a labelled diagram, and factors affecting sensitivity of a moving coil galvanometer.
2. Convert a galvanometer into an ammeter, and voltmeter.
3. Determine the trajectory of a charged particle in a uniform magnetic field.
4 Derive an expression for e/m of electrons and describe, with the help of a labelled diagram, the Thomson's method for determination of e/m

5. Describe with the help of a labelled diagram the construction, principle, working, and limitations of a cyclotron.

1 Moving coil galvanometers

A galvanometer, as you may be knowing, is a device to detect and measure the electric current. Let us now discuss the theory of a moving coil galvanometer. Consider a rectangular coil k carrying current I placed in a radial magnetic field B provided by concave pole pieces as shown in Fig. 1 (a). At the core of the coil we put a soft iron cylinder Z because

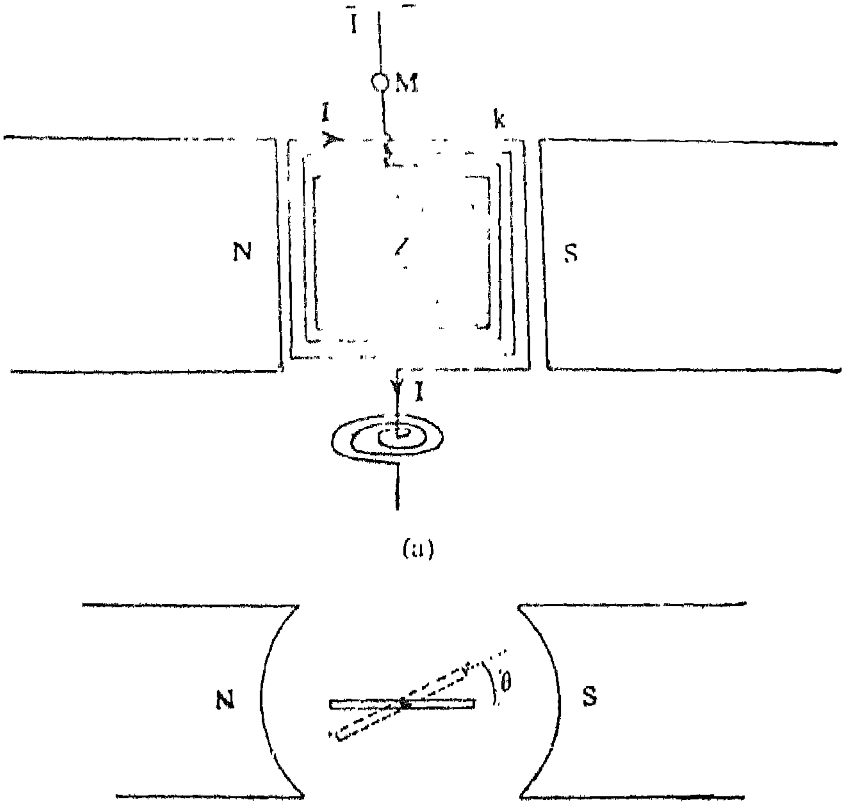

Fig. 1. Moving coil galvanometer
a) Front view
b) Viewed from top. The thin rectangle at the centre represents the coil. The dark one represents its initial position and the dotted one its deflected position.

the permeability of iron is maximum which makes the lines of force more crowded (hence making the field stronger inside the coil). Radial field is always preferred as it gives a linear scale. The field provided by flat pole pieces will not give linear scale.

As shown in the last unit a coil carrying current, when placed in a magnetic field B, experiences a torque given by

$$\Gamma = n\,B\,I\,A \qquad \text{.. (1)}$$

where A is the area of the coil and n is the number of turns

Due to this deflecting torque the coil will rotate. This rotational effect will be counter balanced by a restoring couple in the suspension fibre F. Let the coil deflect through an angle θ [see Fig. 1 (b)] before coming to equilibrium. If C is the torque per unit twist of the fibre, the restoring torque will be $C\theta$. Thus for equilibrium deflecting torque will equal the restoring torque.

$$n\,B\,I\,A = C\theta \qquad \text{. (2)}$$

$$\text{Therefore} \quad I = \left(\frac{C}{n\,B\,A} \right) \theta$$

$$\text{or} \quad I = R\,\theta \qquad \text{...(3)}$$

$$\text{where} \quad R = \frac{C}{n\,B\,A}$$

$$\text{Hence} \quad I \propto \theta$$

Thus in this case we get a linear relationship between I and θ.

Instead of concave pole pieces if we use flat pole pieces, then we get rectangular field, which gives

$$\Gamma = n\,B\,I\,A \cos\theta \qquad \text{(how ?)} \qquad \text{. .(4)}$$

In which case

$$I = (C/n\,B\,A)\,(\theta/\cos\theta)$$

$$\text{or} \quad I \propto (\theta/\cos\theta) \qquad \text{...(5)}$$

Eq. (5) shows that [illegible] flat pole pieces we will not be able to get a linear scale.

The instrument described above is the moving coil galvanometer. The deflection of the coil can be measured by noting the reflection of a narrow beam of light from a small mirror M attached to the suspension fibre. This instrument is very sensitive and can measure currents of the order of microamperes, but it is very delicate and requires careful handling. A more rugged type is the moving needle galvanometer shown in Fig. 2. The coil is pivoted between two jewel bearings and the restoring torque is provided by two hairsprings which also serve as current leads. The current is directly measured by means of a needle and scale.

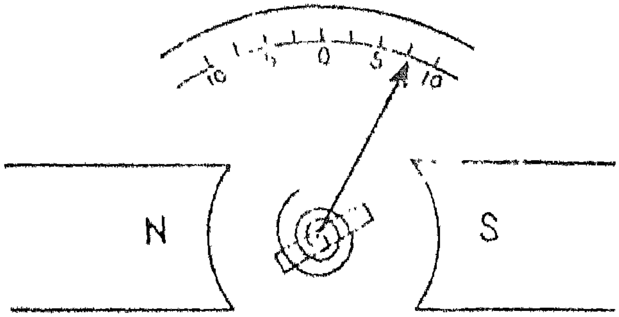

Fig. 2

Let a galvanometer of coil resistance $G = 20\ \Omega$ show full scale deflection for a current $i_g = 10$ mA. The potential difference across the galvanometer terminals for 10 mA is

$$V = i_g G = 0.010\ \text{A} \times 20\ \Omega = 0.20 \text{ volts}$$

Thus this galvanometer can measure currents up to 10 mA or voltages up to 0.20 volts.

On sensitivity of a moving coil galvanometer

The constant R appearing in Eq. (3) is called the *reduction factor* of the galvanometer. In order to get a very large deflection for a rather small current (so that the galvanometer is very sensitive), R must be very small.

Now, since $R = C/n\,B\,A$, for R to be small

(i) C has to be small,

(ii) B has to be large.

To make C small, suspension fibre of phosphor bronze is used, for which C is minimum.

A soft iron cylinder is put at the core of the coil to increase B since permeability of soft iron is high. Of course, here we cannot make n and A very large which will increase the inertia of the coil.

Due to these arrangements, sensitivity of the moving coil galvanometer is very high This is why it is chiefly used for the identification of very small currents.

Problem

Given a moving coil galvanometer (0.02 m × 0.08 m) of 200 turns. If the magnetic field provided by concave pole pieces is 2 Tesla, determine the sensitivity (reduction factor) of the galvanometer.

(Given torque per unit twist of the suspension fibre

$C = 6 \times 10^{-5}$ Nm/radian)

Ans. 9.4×10^{-5} A/radian

2. Ammeters

An ammeter is a moving needle type galvanometer with a shunt S (a conductor of low resistance) connected in parallel to the galvanometer coil G as shown in Fig. 3.

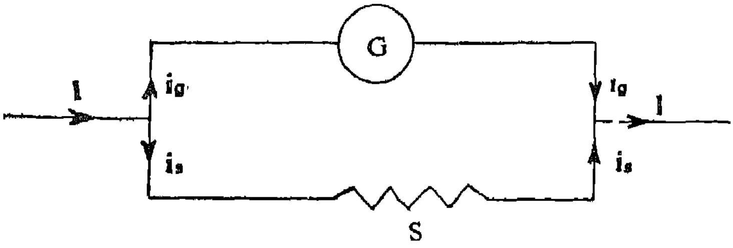

Fig. 3

The reason for connecting a shunt is that the galvanometer, as such, has got finite resistance and if introduced in a circuit without shunt, it will increase the total resistance of the circuit i.e., current will get decreased. Thus current measured by it will be less than the actual value.

The introduction of low resistance shunt also protects the galvanometer as most of the current passes through the shunt. Let I be the total current in the ammeter out of which i_g passes through the galvanometer coil and i_s through the shunt :

$$i_s S = i_g G, \qquad I = i_g + i_s \qquad \text{..(6)}$$

or
$$S = \frac{i_g}{i_s} G = \frac{i_g}{I - i_g} G$$

The equivalent resistance of the ammeter R will be given by

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S}$$

where G = Resistance of the galvanometer coil only

S = Resistance of the shunt.

But since $S \rightarrow 0$, $\quad 1/S \rightarrow \infty$

Hence $\quad \frac{1}{R} \simeq \frac{1}{S}$; or $R \simeq S$

Thus the introduction of the shunt decreases the equivalent resistance of the ammeter considerably.

3. Voltmeters

A galvanometer is capable of measuring low voltages. However, if higher potential difference has to be measured, a high resistance r is connected in series with the galvanometer. The instrument so prepared is called a voltmeter as shown in Fig. 4 (a). The voltage measured when current in the galvanometer coil is i_g is

$$V = i_g\ (r+G) \qquad \text{...(7)}$$

The voltmeter is a very high resistance meter which is connected in parallel across the load resistance. The high resistance of the voltmeter compared to that of the load ensures that the voltmeter draws very small current and the voltage to be measured is not much altered. (How ?)

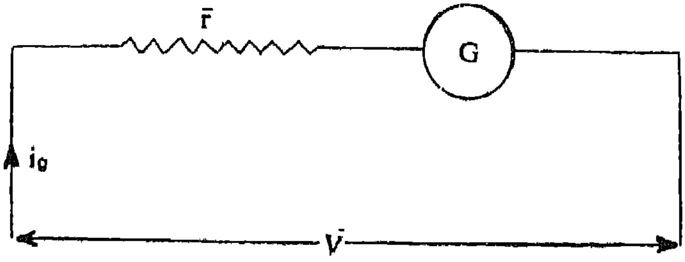

Fig. 4 (a)

Let (see Fig. 4 (b))

$G' = G + r$ = Resistance of the galvanometer (Coil resistance + High resistance)

R = Load resistance

I = Current through the resistance

R' = The equivalent resistance of G' and R

Then

$$1/R' = 1/R + 1/G'$$

But G' being very high, $1/G' \to 0$

$$1/R' \simeq 1/R$$

$$R' \simeq R$$

Had G been not very high

$$R' \neq R$$

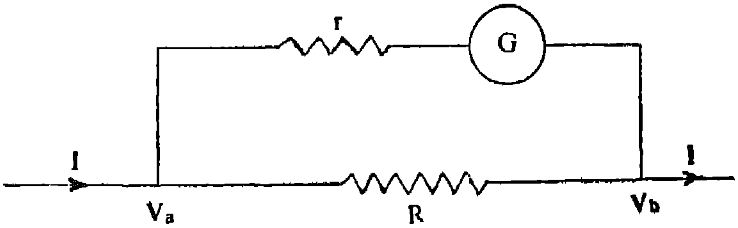

Fig 4 (b)

The voltage to be measured is $V_A - V_B = IR$

So if G' is not high, instead of measuring IR we will be measuring IR'; whereas if G' is very high

$$IR \simeq IR'$$

This discussion, perhaps, makes clear why one should use high resistance in series with the coil in a voltmeter

Example 1

The coil of a galvanometer is 0.02 m × 0.08 m. It consists of 200 turns of fine wire and is in a magnetic field of 0.20 Tesla. The restoring torque constant of the suspension fibre is 10^{-6} Nm/deg. Assuming the magnetic field to be radial

(i) What is the maximum current that can be measured by this galvanometer if the scale can accommodate 45° deflection ?

(ii) What is the smallest current that can be detected if the minimum observable deflection is 0.10 degree ?

Solution

Area of the coil $A = 16 \times 10^{-4}$ m²

No. of turns $n = 200$

Magnetic field $B = 0.20$ Tesla

Torque/unit twist of the suspension fibre $(C) = 10^{-6}$ Nm/deg.

(i) Maximum current I_{max} is given by

$$I_{max} = \frac{C}{n\,B\,A}\,\theta_{max}$$

$$I_{max} = \frac{10^{-6} \times 45}{200 \times 0.2 \times 16 \times 10^{-4}}\ \text{A}$$

$$= 7 \times 10^{-4}\ \text{A}$$

(ii) Minimum deflection $\theta_{min} = 0.1$ deg. Therefore minimum current is

$$I_{min} = \frac{C}{n\,B\,A} \times \theta_{min}$$

$$I_{min} = \frac{10^{-6} \times 0.1}{200 \times 0.2 \times 16 \times 10^{-4}} \text{ A}$$

$$= 1.6 \times 10^{-6} \text{ A}$$

Example 2

A galvanometer has an internal resistance G of 1.0 ohm. It gives maximum deflection (full scale) for a current of 50 mA. Show how this instrument can be converted into (a) a voltmeter with a maximum reading of 2.5 volt; (b) an ammeter with a maximum reading of 2.5 A.

Solution

(a) Internal resistance of the galvanometer

$$G = 1\ \Omega$$

For voltmeter, a high resistance r must be connected in series to the galvanometer coil. Then combined resistance is

$$= G + r$$

Maximum current $I = 50$ mA $= 5 \times 10^{-2}$ A.

Maximum voltage $V = 2.5$ volts

$$V = I\,(G + r)$$

$$2.5 = 5 \times 10^{-2}\,(r + 1)$$

$$r = 49\ \Omega$$

(b) For an ammeter a shunt resistance S must be connected in parallel.

Then combined resistance R will be given by

$$\frac{1}{R} = \frac{1}{S} + \frac{1}{G}$$

or $$R = \frac{S}{S + 1}$$

Maximum voltage (without shunt) will be

$$V = 5 \times 10^{-2}\ \text{A} \times 1\ \Omega$$

$$= 5 \times 10^{-2}\ \text{volt}$$

When the shunt is connected, maximum current I, we want, is 2.5 A.

Hence $V = I\,R$

$$5 \times 10^{-2} = 2.5 \times \left(\frac{S}{S+1}\right)$$

$$\text{or} \quad \frac{1}{50} = \frac{S}{S+1}$$

$$\text{or} \quad S = 1/49\ \Omega$$

4 Orbits of charged particles in magnetic fields

Let a positive charge q of velocity v enters a magnetic field B as shown in Fig. 5 The field B is perpendicular to the plane of the paper

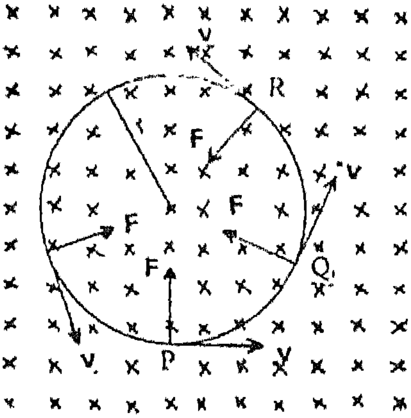

Fig. 5. Moving charge in a uniform magnetic field. The crosses represent the magnetic field perpendicular to the plane of the paper and directed into the plane of the paper.

going away from the reader, and v is in the plane of the paper. The vector v is perpendicular to vector B. Let the charge be at point P initially. You know from Unit 19 that the force experienced by this charge is

$$F = q\,(v \times B)$$

which is perpendicular to v, and in the plane of the paper as shown. Since F is perpendicular to v, it does not change its speed v (why ?) but merely changes the direction of velocity. At points Q and R the directions of velocity and force are as shown. Since the charge moves under the influence of a force of constant magnitude which is always perpendicular to velocity, its orbit is a circle and F provides the necessary centripetal force. Thus

$$q\,v\,B = \frac{m\,v^2}{r} \qquad \ldots(8)$$

which gives radius of the circle as

$$r = \frac{m\,v}{q\,B} \qquad \ldots(9)$$

If the initial velocity is not perpendicular to the field, the particle moves in helix as shown in Fig. 6. In this figure the field B is along the axis of the helix.

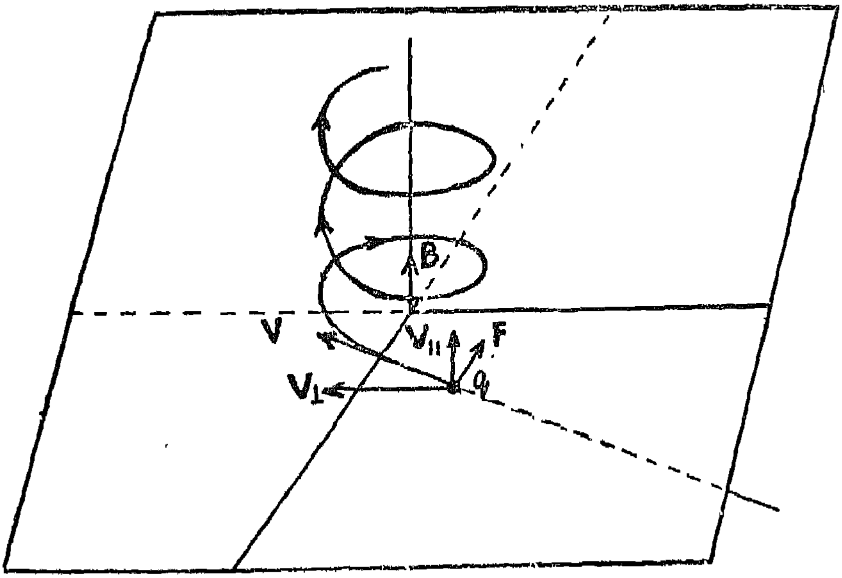

Fig. 6. Path of a ch [illegible] field when the velocity of the particle [illegible] the magnetic field.

The particle enters the magnetic field B with a velocity v, which makes an angle with the direction of the magnetic field. The velocity v is resolved into two components, $v_\perp$ in a direction perpendicular to B and $v_\parallel$ in a direction parallel to B. The particle will experience a Lorentz force $F = q\,(v_\perp \times B)$. Due to this force the particle will execute a circular path of radius $r = \frac{m}{q}\,\frac{v_\perp}{B}$. Simultaneously it will execute a linear motion in the direction of B due to the velocity component $v_\parallel$. The resultant path is a helix.

Problem

A particle has a mass of 0.5 g, charge 2.5×10^{-8} C, and an initial horizontal velocity of 6×10^{-4} m/s. What are the magnitudes and directions of magnetic fields that will leave the particle undeviated.

Ans. Any field in the
horizontal direction.

5. The cyclotron

The only way to understand the physics of nucleus, is to make the nucleus disintegrate and for that highly accelerated particles are needed. The electric field E can serve as a particle accelerator. Let us explain this by considering a particle of charge q moving through an electric field E. Since $q\,E$ is the electrical force, from Newton's second law

$$qE = m\,a \qquad \text{...(10)}$$

where m and a are mass and acceleration of the particle respectively.

Eq. (10) shows that higher the electric field higher will be the acceleration. However, with such an arrangement particles can be accelerated to energies of the order of a few million electron volt (MeV) (where 1 eV = 1.6×10^{-19} joules) depending upon the voltage available. If however the charge can be made to go through the same field repeatedly, it can then be given much higher energy. This is done in the charged particle accelerator called the cyclotron.

The main parts of the cyclotron consists of two hollow half cylinders D_1 and D_2 (Fig. 7). They are referred to as 'dees' since they are in the shape of D. They are mounted inside a vacuum chamber between the poles of a powerful electromagnet. The 'dees' are connected to the two terminals of a high frequency oscillator which changes the electrical polarities of the 'dees' very rapidly. The electric field exists only between the dees and as the charged particle passes from one dee to the other it is accelerated provided the polarities are correct The entire device is in high vacuum so that molecules of air will not interfere with the acceleration of the particles. The ionic source is placed near the mid-point of the gap between the dees. The magnetic field B inside the dees is perpendicular to the velocity of the ionic beam, so the particle will describe circular path The frequency of alternating voltage applied across D_1 and D_2 is so adjusted that the particle, describing half-circular path in D_1 when on the verge of going over to D_2, experiences repulsion by D_1 and attraction by D_2. Thus in D_2 it describes a path of greater radius of curvature And again when it is about to leave D_2, polarities of D_2 to D_1 change with the result that the particle speeds up in going from D_2 to D_1 and then describes a circular path of greater radius. This process is maintained until the particle acquires sufficiently high acceleration; then it is taken out of the orbit with the help of a magnetic field and directed on to a target made up of the material to be investigated.

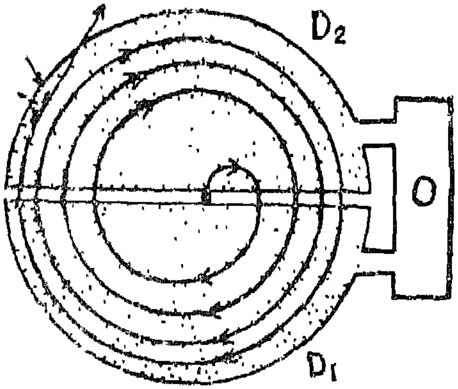

Fig 7. Cyclotron : O-Oscillator, P-Deflector plate, D_1 and D_2-Dees. The dots represent the uniform magnetic field perpendicular to the plane of the paper and directed towards the reader. The charged particle describes semicircular paths in dees as shown

The angular velocity ω of the particle is

$$\omega = \frac{v}{r} = B\left(\frac{q}{m}\right) \qquad ...(11)$$

which depends only on (q/m) and is independent of the velocity and radius of the particle.

Thus if t is the time for half a rotation for particles in dees, the frequency of alternating voltage applied across the dees is adjusted to $1/2t$, then and only then particles will always be accelerated when they cross the gap.

If R represents the outside radius of the dees, then the maximum velocity which a particle can attain will be (from Eq. (8))

$$v_{max} = (q/m)\ BR \qquad \text{...(12)}$$

and maximum kinetic energy will be $\frac{1}{2}mv^2_{max} = \frac{1}{2}m\,(q/m)^2\ B^2 R^2$.

According to the special theory of relativity the mass of a particle m changes with its velocity v according to the formula

$$m = m_0/\sqrt{1-v^2/c^2} \qquad \text{...(13)}$$

where m_0 is the rest mass of the particle, and c is the velocity of light.

Until $v/c << 1$, we can regard mass to be a constant, but if $v/c \sim 1$, the change in the mass of the particle will no more be negligible, hence in Eq. (11), ω will depend on velocity v through the mass m. For very high velocities particles will be thrown off the track. This is one of the serious limitations of cyclotron. The other one being the demand that magnetic field B should be uniform throughout the dees. Now if you want particles of very high energy, correspondingly larger dees are to be used. Then maintaining a uniform magnetic field throughout the entire region will be a very difficult task.

To overcome these drawbacks many ingenious variations have been made in the construction of accelerators. We have as a result, the synchrotron, the betatron and other machines. These machines employ changing oscillation frequencies or changing magnetic field strengths or both, and can accelerate particles up to a few BeV.

Questions

1. For what purpose is the magnetic field used in a cyclotron ?
2. How is the constancy of angular velocity of a particle is made use of in the cyclotron ?

Example 3

Deuterons are to be accelerated in a cyclotron The value of the uniform magnetic field applied is 2.475 Wb/m². What is the frequency of the oscillating potential that must be applied to the dees of the cyclotron ?

(Mass of deuteron = 3.3×10^{-27} kg, and charge 1.6×10^{-19} C).

Solution

The frequency of the deuteron is

$$f = \frac{\omega}{2\pi}$$

$$= \frac{B}{2\pi} \frac{q}{m}$$

$$= \frac{2\,475 \text{ Wb/m}^2 \times 1.6 \times 10^{-16} \text{ C}}{2 \times 3.14 \times 3.3 \times 10^{-27} \text{ kg}}$$

$$\simeq 2 \times 10^7 \text{ Hz}$$

This is also the frequency of the oscillator

Problem

What is the period of revolution of a deuteron in a uniform magnetic field of 1 Wb/m² ?

Ans. 1.3×10^{-7} s

6. Measurement of *e/m* by Thomson's method

A cathode ray discharge tube is shown in the Fig. 8. The stream of electrons at right angles to the cathode C is collimated by slits S_1 and S_2. The slits act like directional filter because only the electrons with velocity vector perpendicular to the plane of the slits will be able to pass through. Then they are subjected to an upward electrical field (provided by plates A and B across which a potential difference is applied) and magnetic field coming out of the paper (provided by a pair of Helmholtz coils in the region shown by dots, Helmholtz pair of coils gives fairly uniform magnetic field) Finally electrons impinge on the screen S as shown.

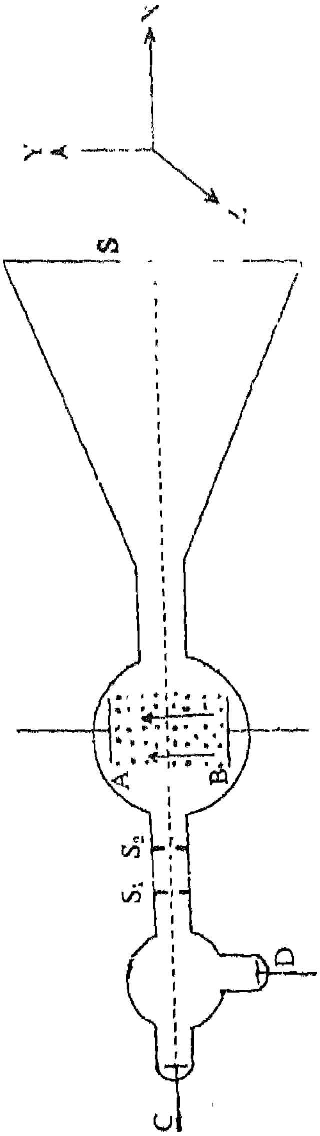

Fig. 8

Under the conditions described above you can verify that the electric force eE is acting downwards whereas magnetic force evB whose direction is given by the direction of $e\ (v \times B)$, is acting upwards. By proper adjustment of combined electric and magnetic fields, the deflection of the electrons can be made zero. Under these conditions the fields are said to be *crossed*, and we get

$$E e = B e v$$

or

$$v = \frac{E}{B} \qquad \ldots(14)$$

Only the electrons of velocity E/B go through the plates undeviated. Other electrons impinge on the plates and are lost. Thus the devise described above selects electrons of a particular velocity $v = E/B$, and is called a *velocity filter*.

If the P D. applied across C and D is V, then the kinetic energy acquired by the electrons will be

$$\tfrac{1}{2} m v^2 = e V$$

or

$$e/m = \frac{v^2}{2V} \qquad \ldots(15)$$

Substituting the value of v from Eq. (14) in Eq. (15), we get

$$e/m = \frac{E^2}{2B^2V} \qquad \ldots(16)$$

In the Eq. (16), all quantities are known, hence e/m can be determined. The value of e/m for the electron was found to 1.76×10^{11} C/kg.

Questions

1. How are electrons produced inside the discharge tube ?
2. How do you determine the direction of magnetic field, referring to Fig. 8, so that $F_E = - F_B$?
3. How do $(e/m)^s$ of proton and electron compare with that of α-particle ?

Example 4

What is the magnitude and direction of the magnetic field needed to nullify the deflection of the electron due to electric field (given that the voltage applied across two plates to get electric field is 400 V and distance of separation between plates is 25 cm), if the voltage applied across the anode and the cathode of discharge tube is 15,000 V.

(Specific charge of the electron $= 1.76 \times 10^{11}$ C/kg)

Solution

The velocity acquired by the electron

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 15000 \times 1.76 \times 10^{11}} \text{ m/s}$$

$$= 7.26 \times 10^{7} \text{ m/s}$$

If the deflection of the electron due to the electric field is nullified by the deflection due to magnetic field, then

$$v = E/B$$

$$\text{Where electric field } E = \frac{400 \text{ V}}{0.25 \text{ m}} = 1600 \text{ V/m}$$

$$\text{Hence } B = \frac{E}{v} = \frac{1600 \text{ V/m}}{7.26 \times 10^{7} \text{ m/s}} = 2.20 \times 10^{-5} \text{ Wb/m}^2$$

If the electron motion is taking place along +X axis (see Fig. 9) and electrical force F_E is acting along −Z axis, then in order that F_B should act along +Z axis, magnetic field B must act along +Y axis:

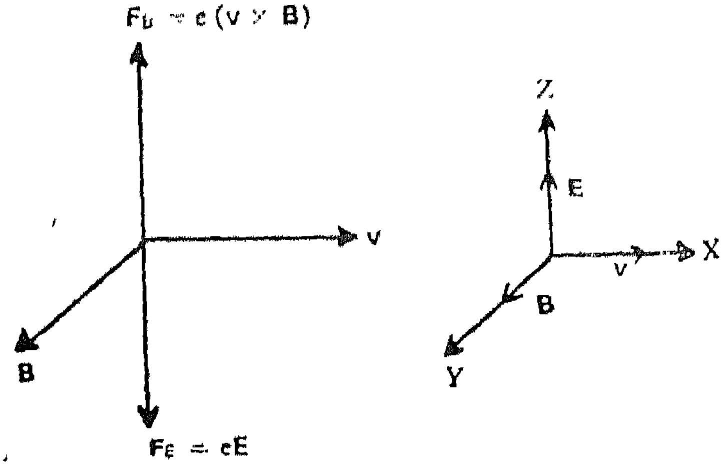

Fig. 9. Electric and magnetic forces acting on an electron moving along +X axis, in an electric field E acting along +Z axis and magnetic field B acting along +Y axis.

Problem

A Na$^+$ ion travelling at an angle of 45° to a magnetic field of 0.5 Wb/m^2 experiences a force of 3.84×10^{-13} N. What is the speed of Na$^+$ion ? Ans. 15.1×10^{6} m/s

ASSIGNMENT

1. An electron moving at right angles to a uniform magnetic field completes a circular orbit in 10^{-9} second. What is the magnitude of the magnetic field ?

   (Ans. 3.6×10^{-2} Tesla)

2 An electron moving with kinetic energy 8×10^{-6} joules, enters a magnetic field 4×10^{-3} Wb/m^2 at right angles to its motion What is the radius of its path ?

   (Ans. 0.015 m)

3. A cathode ray beam is bent in a circle of radius 4 cm by a magnetic field 4×10^{-3} Wb/m^2. Determine the velocity of the electron.

   (Ans. 2.82×10^{7} m/s)

4. A cyclotron has an oscillator frequency of 12×10^{6} Hz and dee radius of 2 m. (a) What value of magnetic induction B is needed to accelerate deuterons ? (b) What maximum deuteron energy results ?

   (Ans. 1.6 Wb/m^2, 17 MeV)

5. (a) Imagine that you are sitting in a room with your back to one wall and that an electron beam, travelling horizontally from the back wall towards the front wall, is deflected to your right What is the direction of the magnetic field existing in the room ?

   (b) If an electron is not deflected in passing through a certain region of space, can we be sure, that there is no magnetic field in that region ?

6. (i) What are the primary functions of (a) the electric field, and (b) the magnetic field in the cyclotron ?

   (ii) For Thomson's e/m experiment to work properly, is it essential that the electrons have a fairly constant speed. Explain why ?

(iii) If the magnetic field is 2.5 Wb/m^2, determine the frequency at which a proton in cyclotron is operating.

(Ans. Approx. 4×10^7 Hz)

7. (a) If one-tenth of the main current is allowed to pass through a galvanometer of resistance R_g shunted with a shunt (resistance R_S). What fraction of R_g is R_S ?

(Ans. $R_s = \frac{1}{9} R_g$)

(b) Find the value of resistance that must be connected in series with a galvanometer (resistance 120 Ω), showing a full-scale deflection with a current of 0.0005 A, so that it can measure a maximum potential difference of 6 volts.

(Ans. 1.19×10^4 Ω).

8. A proton, a deuteron and an α-particle, accelerated through the same potential difference, enter a region of uniform magnetic field at right angle to B. (a) Compare their kinetic energies. (b) If the radius of the protons circular path is 10 cm, what are the radii of the deuteron and α-particle paths ?

(Ans. $K_p = K_d$ $\frac{1}{2}K_\alpha$; R_d 14 cm, $R_\alpha = 14$ cm)

SELF-ASSESSMENT

1. (a) If a moving electron is deflected sideways in passing through a certain region of space, can we be sure that a magnetic field exists in that region ?

(b) A beam of protons is deflected sideways. Could this deflection be caused by

(a) An electric field ?

(b) A magnetic field ?

2. (a) In a nuclear experiment 1.0 MeV proton moves in a uniform magnetic field in a circular path. What energy must (a) an alpha particle, and (b) a deuteron have if they are to circulate in the same orbit ?

(Ans 1.0 MeV; 0.50 MeV)

(b) A 10 eV electron is circulating in a plane at right angles to a uniform field of magnetic induction 10^{-4} Wb/m^2. (a) What is its orbit radius ? (b) What is the cyclotron frequency ? (c) What is the period of revolution ? (d) What is the direction of circulation as viewed by an observer sighting along the field ?

(Ans. 11 cm, 2.8×10^6 Hz, $3\ 6\times10^{-7}$ s)

3. The electrons in the beam of a television tube have an energy of 12 keV. The tube is oriented so that electrons move horizontally from south to north The vertical component of the earth's magnetic field points down and has $B = 5.5\times10^{-5}$ Wb/m^2. (a) In what direction will the beam deflect ? (b) What is the acceleration of a given electron ? (c) How far will the beam deflect in moving 20 cm through the television tube ?

(Ans. East, 6.3×10^{14} m/s^2, 3 0 mm)

4. An electron beam consisting of electrons with velocity ranging from 3×10^8 m/s to 5×10^8m/s enter into crossed electromagnetic field where $B = 0.5$ Wb/m^2 and electrical field is provided by two plates having P.D. of 400 V separated by a distance 25 m. Determine the velocity for which electrons will pass through undeviated. Suggest a practical use of this sort of arrangement.

(Ans. 3.2×10^8 m/s)

5. Calculate the frequency of alternating voltage applied across the dees of the cyclotron used to accelerate protons, if the magnetic field applied at right angles is 30 Wb/m^2.

(Ans. 4.6×10^8 Hz)

6. (a) What is the cyclotron frequency of an electron with an energy of 100 eV in the earth's magnetic field of 10^{-4} Wb/m^2 ? (b) What is the radius of curvature of the path of this electron if its velocity is perpendicular to the magnetic field ?

(Ans. 2.8×10^6 Hz, 0.34 m)

7. A resistor of 250 Ω, a voltmeter and a 2 volt accumulator of negligible resistance are all connected in series. If the voltmeter reads 0.8 volt, calculate its resistance.

8. A moving coil galvanometer has resistance 10Ω and its scale is graduated from 0 to 10 mA. Explain how you will convert it into an ammeter reading 0–5 A and into a voltmeter reading 0–100 V.

UNIT 21

Physics for Class XI

ASTRONOMY

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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PREFACE

This unit is last in a series of units developed for try out of the Individually Guided System of Instruction (IGSI) in class XI Physics course of the Central Board of Secondary Education. A booklet wherein IGSI is described is available under a separate cover. This unit contains an introduction for motivation, arousing interest, and to link the present unit to preceding unit. The objectives given in this unit are the expected learning outcomes, stated discretely so that the pupil will know what to master. Several questions and problems are included to arouse curiosity so that the pupil is encouraged to think creatively. These units are suited for self study with occasional help from a tutor. However, pupils following the traditional system may also find these units useful.

In preparing this unit I was assisted by some teachers and Dr. K. J. George, who is working as a Project Fellow on this project.

At this stage this unit is an experimental edition. I will welcome any criticism and suggestion for the improvement of this unit.

The project for which this unit is prepared, is supported financially by the National Council of Educational Research and Training. However, the responsibility for the facts stated herein, the opinion and views expressed and the conclusions drawn are entirely mine and not of NCERT.

RAJESHWAR N. MATHUR
Reader
Department of Education in Science
and Mathematics
N.C.E.R.T.

New Delhi
20 January 1983

INTRODUCTION

In the last unit you were exposed to the micro world of subatomic particles. The present unit takes you out to the other extreme of endless universe. When you look at the sky at night you may see almost 10,000 crore stars collected in our Milky Way galaxy. Our sun is only one of these stars The dimensions of Milky Way are such that light travelling at the speed of 30 crore metres per second will take over 100,000 years to travel from one edge of the galaxy to the other. Some of the bright objects, that to a naked eye, appear like stars, are in fact collection of stars. One such object is the collection of stars called the Great Galaxy in Andromeda which, when viewed with a telescope, appears to have a spiral like structure much like Milky Way galaxy. But it is so far away that even its light takes 20 lakh years to reach us. We see the Andromeda as it appeared 20 lakh years ago. This is just another galaxy. The universe is so vast that astronomers have identified 10,000 crore galaxies. Have they seen the edge of the universe ? Most probably not, for more and more remote objects are detected by powerful radio telescopes, and man keeps stretching his imagination to understand the mysteries of his universe. In this unit we will initiate a brief study of the universe. Although early physics is an offspring of astronomy, we will apply laws of physics to understand some facts about the universe.

OBJECTIVES

To be able to answer the following questions

1. Describe the functions of two types of optical telescopes drawing neat diagrams. Which one is more advantageous, reflector type or refractor type ?
2. Describe a radio telescope. In what ways do they differ from optical telescopes ?
3. How do you measure the distances of the heavenly bodies in

solar system by applying parallax method and Kepler's third law ?

4. How do you estimate the masses of the celestial bodies forming binary system, by Kepler's third law ? Justify your answer giving mathematical derivations.

5. How does the angular size of a planet help us to determine its linear size ? Justify your answer.

6. How do you estimate the surface temperature of planets or stars by applying Stefan's law ?

7. Describe the composition of atmospheres of various planets in the solar system.

8. How do the space probes e g , Voyagers, Mariners etc. enrich our knowledge about the universe we live in ?

9. Which planet of the solar system is most likely to have life in some form and why ?

10. Supposing that there is a possibility of intelligent beings like us in the solar system, point out the difficulties encountered with regarding the communication with the species living in some peopled planets in some remote corner of our universe.

11. What are sun spots and solar flares ? What are the consequences of solar flares ?

12. Describe the solar spectrum and, conditions at the centre of the sun. Explain how does the sun (already 4.5 billion years old) seem to be a never-exhausting source of energy.

13. How are the distance and brightness of a star measured ?

14. How are the masses of binary stars measured ? How do you determine the radius of a star approximately ?

15. Describe the following physical properties of stars

    1. Stellar spectra

    2. Brightness (Magnitude)

16. Explain in brief the mechanism behind the formation of a star. When does a star become stable in size and temperature ?

17. Describe very carefully different stages of stellar evolution. What do you understand by the following

    (a) Neutron star

    (b) Black hole

    (c) Pulsars

18. Describe in fifteen lines the essential features of our home galaxy—the Milky Way.

19. Discuss in brief about the classification of galaxies.

20. What do you understand by the statement :

    "Quasars are the glimpses of cosmological past."

    Differentiate clearly between radio galaxies and quasars.

21. State and explain Hubble's red shift law.

22. Give brief ideas about the three cosmological models regarding the origin, and evolution of the universe.

23. What must happen in the universe to satisfy the steady state theory ? Explain.

    What recent finding tends to substantiate the big-bang theory ?

24. List some distinguishing features of each planet.

SUGGESTED READING MATERIAL

. Astronomical instruments

In the last two decades man has been able to send space probes only o his nearest neighbours such as the moon and the planets. The knowedge about other heavenly bodies is derived from observations made on the electromagnetic radiations received from these bodies. For this purpose photographic plates, optical telescopes, and radio telescopes described below, are used.

The main purpose of a telescope is to collect electromagnetic radiation over a large surface and to concentrate that energy at a point thus producing a bright image. You are familiar with telescopes of refractor type which use a convex lens for objective, such as astronomical telescopes. A simple telescope of this type is shown in Fig. 1 (a) where an objective convex lens of large focal length F_0 gathers radiation from a heavenly body and brings it to focus on the focal plane of an eye piece E of shorter focal length F_E. The image formed by refracting telescope suffer from (i) *chromatic* aberration caused by different extent of bending of different colours as they pass

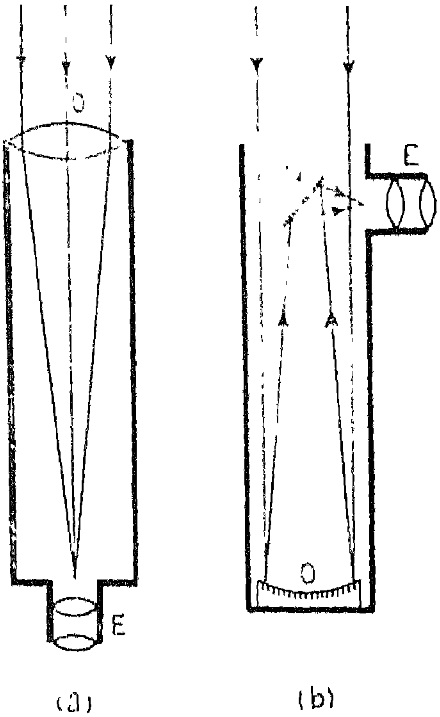

Fig. 1 Optical telescopes (a) Refraction type : O is the objective lens and E is the eyepiece (b) Reflection type : O is the objective parabolic mirror, N is a plane mirror, and E is the eye piece

through the objective lens as shown in Fig. 2 (a) which results in blurred coloured image (ii) *spherical aberration* caused due to the fact that in spherical shaped lens the rays near axis pass through greater thickness than the rays near edges, resulting in different foci [Fig. 2 (b)] and blurring of the image (iii) *coma* caused by oblique entrance of light due to off axis sources and results in an image of a point object in the form of coma (,) Fig. 2 (c) The first two defects can be eliminated by using a reflecting telescope [Fig 1 (b)] where the objective is a parabolic reflector mirror Fig. 2 (d) (How ?). The radio telescope uses large metallic mesh of parabolic shape to gather waves and to focus them on a receiver which can be tuned to desired wavelength. The optical telescopes are generally coupled

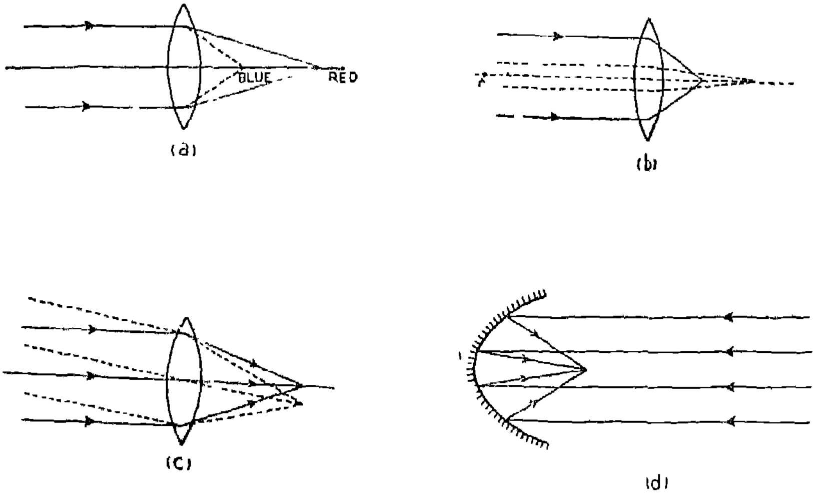

Fig. 2 Defects of refraction type telescope

a Chromatic aberration, b. Spherical aberration. c. Coma.
d. Parabolic reflector . rays falling on the boundaries and near the centre are brought to same focus.

with camera or photographic plate for taking permanent and longtime photographs. Sometimes these telescopes are placed in space laboratories to minimize the effects of earth's atmosphere.

Question

List some advantages of observing heavenly bodies outside the earth's atmosphere. Why do stars twinkle ?

The size of the image of an object of 1′ diameter, formed by the objective of a telescope of focal length F_0 is given by

$$S = 0.00687\ F_0$$

where F_0 is in centimetres Thus for bigger image the telescope should have longer focal length objective

Question

Compute the size of the image (in centimeter) of the full moon (diameter 0.5°) as observed by a telescope of $F_0 = 150$ cm.

But the *brightness* of the image formed is given by $B =$ constant $\left(\frac{A}{F_0}\right)^2$ which is small for large focal length objective But higher aperture A (diameter) obviously results in greater brightness. Also greater aperture results in better resolution. The *resolving power* which is the smallest angle between two objects that can be distinguished as separate is given by

$$a = 2.1 \times 10^5\ \frac{\lambda}{A}\ \text{seconds of arc}$$

This is why most powerful telescopes of the world are reflectors of very large aperture (200 inch at Mount Palomar, U. S A , 234 inch in USSR).

Question

Determine the resolving power of 48 inch reflector of Osmania University for light of wavelength 4000 A.

The radio waves have longer wavelength, therefore, to see same detail radio telescopes require much larger diameter. The Arecibo Antenna in Puerto Rico has a diameter of 1000 feet.

Question

Determine the aperture of a radio telescope tuned at 21 cm wavelength which has same resolving power as a 48″ optical telescope receiving wavelength of 5000 A.

2. Astronomical measurements

The astronomical instruments described above are used to study physical properties of heavenly bodies. The methods used for these studies are discussed below.

(*a*) *Distance*

The method of parallax described in Unit 2 is used to determine distances to planets and near stars. For far away stars the reliability of results by this method becomes poor.

Question

One *parsec* is defined as the distance to a star whose parallax is one second of arc. Express one parsec in astronomical units. One Astronomical Unit (A U) is the average distance between the sun and the earth.

$$1 \text{ A U.} = 1.49 \times 10^8 \text{ km}$$

Ans. 1 parsec$=2.06 \times 10^5$ A.U.

Question

Using a base line equal to the radius of the earth (6.38×10^6m) the parallax of the moon is 57′, compute the earth-moon distance.

Ans. 3.84×10^8m.

If the distance to one planet is known, the distance to other planets can be determined knowing their periods of revolution.

Problem

The period of revolution of Mars is 1.88 years. Use Kepler's laws (Review Unit 8) to determine the distance to Mars in astronomical units

Ans. 1 52 A.U.

The distances to other planets and stars are given in Table 1 and Table 2 respectively.

Knowing the distance r (See Unit 1) and the angular size θ in radians the size D of an object can be computed from the equation

$$D = r\theta$$

Question

The angular diameter of the sun is about 0.5^0 and its distance from the earth is 1.5×10^{11} m, determine the solar diameter

Ans. 1.4×10^9m

For distant stars the angle of parallax cannot be measured with desired accuracy. However we can calculate the radius of a star if its surface temperature is known. Using Stefan's law the total energy L radiated out per second from a star of radius R is

$$L = (4\pi R^2)\sigma T^4$$

L can be determined if the distance to the star is known.

(*b*) *Mass*

Nearly half of the stars that we see with naked eye are in reality multiple star systems containing two, three or more stars. The masses of binary stars can be easily determined. In Fig. 3 a binary system of stars of

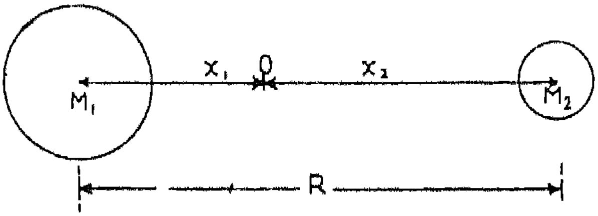

Fig. 3 Binary system

masses M_1 and M_2 is shown. The stars revolve around a point O called the *barycentre*. The distances x_1 and x_2 of the stars from the barycentre are determined by the proportion

$$\frac{M_2}{M_1} = \frac{x_1}{x_2} \qquad (1)$$

and the separation of the stars

$$R = x_1 + x_2 \tag{2}$$

From Eq. (1) and Eq (2) we get

$$x_2 = \left(\frac{M_1}{M_1+M_2} \right) R \tag{3}$$

The centripetal force on the stars is provided by the force of gravitation. Thus for a star of mass M_2

$$\frac{M_2 V_2^2}{x_2} = G \frac{M_1 M_2}{R_2} \tag{4}$$

The orbital velocity V_2 of the star M_2 is related to its time period T.

$$V_2 = \frac{2\pi x_2}{T} \tag{5}$$

Substituting for V_2 from Eq (5) into Eq (4) we get

$$\frac{1}{x_2}\left(\frac{2\pi x_2}{T}\right)^2 = \frac{G M_1}{R^2}$$

Using Eq. (3) we get

$$M_1+M_2 = \frac{4\pi^2}{G} \frac{R^3}{T^2} \tag{6}$$

Using Eq. (1) and Eq. (6) masses of individual stars can be determined. Eq. (6) can be applied to any binary system.

Problem

Two components of the star Sirius, Sirius *A* and Sirius *B*, are observed to be separated by 3.7 × 10^{-5} radians and are at a distance of 20.3 A.U. Further it is observed that the two components revolve around their barycentre in 49.0 years such that $x_2/x_1 = 2.35$. Determine the masses of Sirius *A* and Sirius *B*.

Ans. 2.35 M (Sun), 1.00 **M** (Sun)

Masses of large number of stars determined by this method are found to vary from $\frac{1}{10}$ to 40 times solar mass.

Eq (6) can be used to determine the mass of the sun in the sun-earth system, and masses of planets that have moons (See Table 1)

Question

Assuming M (sun) [illegible] M (earth), radius of the earth's orbit $R = 1.5 \times 10^{11}$ m, and period of the earth $T = 365$ days. Determine the mass of the sun. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2).

Ans. 2×10^{30}kg

(c) *Surface temperature*

All planets shine by reflecting light from the sun Only a small fraction of solar radiation is reflected, rest is absorbed. Solar radiations are first absorbed by the planets and then re-radiated as infrared and radio waves On measuring this self radiation the temperature can be computed using Stefan-Boltzmann law (See Unit 14).

The temperature of the sun can likewise be calculated. The temperature of stars can be estimated from their colour (see Unit 14) and spectra analysis.

Problem

Assuming solar constant to be 1.39×10^3 W/m^2 compute :

(a) The total radiation emitted in watts by the sun. This is called the luminosity of the sun.

Ans. 3.9×10^{26} W.

(b) W/m^2 from the surface of the sun. Solar radius is 6.96×10^8 m.

Ans. 6.4×10^7 W/m^2

(c) Surface temperature of the sun using Stefan's constant of 5.7×10^{-8} W/m^2 K^4.

Ans. 5500 K

(*d*) *Atmosphere*

The composition of atmosphere of planets and stars is determined from the spectra of these bodies. You are already familiar with Fraunhofer lines which indicate the presence of certain elements in solar atmosphere. Space probes are also used to study the atmosphere of planets.

3. The planets

All the planets move around the sun in elliptical (low eccentricity or nearly circular) orbits nearly in the same plane. One glaring exception is Pluto which has highest eccentricity of 0.248 and highest orbital inclination of 17.2°. Most of the planets revolve around the sun in direct motion, which is considered counter clockwise as viewed from the 'north side' of the solar system, the north side being determined by the north pole of the earth. All the planets, with the exception of Venus and Uranus, rotate in direct (west to east) motion. The approximate distance d (in A U.) of the planets from the sun is given by Bode—Titius relation

$$d=\frac{3\ 2^{n-1}+4}{10}$$

where $n=1,2,3,4,5,6,7,8,$ and 9 for Venus, earth, Mars, Asteriods, Jupiter, Saturn, Uranus, Neptune and Pluto respectively. For Mercury $d=4/10$ A.U. Except for Pluto which is actually at 40 A. U., this relation gives nearly the actual distances. Some observers wonder whether this relation holds a key to the origin of the solar system, or is a mere coincidence. Some of the characteristics of planets are described below. Space probes used by the U.S.A and USSR have given significant information about the planets.

Mercury

Looking from the earth, Mercury never appears more than 28° (*maximum elongation*) from the sun. Since the earth rotates 15° in one hour, Mercury can rise no more than 2 hours before the sun when it is a morning object and can set not more than 2 hours after the sun when it is an evening object. It is the smallest planet, having extreme variation of temperature (see Table 1). The *albedo* of a planet is the percentage of light that it reflects, and depends on the thickness of its atmosphere. The albedo of Mercury is only 7%. No life can exist on Mercury due to lack of atmosphere. The escape velocity for Mercury is low 2 5 miles/second which is responsible for escape of molecules of gases. The surface of Mercury is pockmarked by craters of different sizes.

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Table 1

DATA OF PLANETS

Name of the Planet	Mean distance from sun (in 10^6 km)	Radius (in 10^3 km)	Mass ($\times 10^{24}$ kg)	Period of rotation	Period of revolution	Albedo	Escape velocity km/s
Mercury	58	2.4	0.30	58 d	88 d	0.06	4.3
Venus	107	6.1	4.82	225 d	225 d	0.76	10.3
Earth	149	6.4	[illegible]	23 h 56m	365 d	0.40	[illegible]
Mars	227	3.3	[illegible]	24 h 37m	687 d	0.15	5.1
Jupiter	773	70.4	1900.00	9 h 50m	12 y	0.45	57.5
Saturn	1,386	60.0	[illegible]	10 h 14m	29 y	0.61	35.4
Uranus	2,853	24.8	[illegible]	10 h 49m	84 y	0.35	21.9
Neptune	4,470	26.4	102.00	15 h 40m	165 y	0.35	24.4
Pluto	5,866	2.9	0.66	6 d 9 h	248 y	0.14	7

Venus

The maximum elongation of Venus is 47°. It can, therefore, be best seen about 3 hours after sunset when it is a night object and about 3 hours before sunrise when it is a morning object Venus has a retrograde (clockwise as viewed from northerly position) rotational period of 250 days The diameter of Venus, as determined by reflecting radiowaves from its surface, is 12,101 km. Hence the planet is almost the twin of the earth in size. It has the highest albedo of 8% and is covered by a thick atmosphere containing 95% CO_2 and some hydrogen and water vapour but no nitrogen. This atmosphere also traps lot of heat (greenhouse effect) which results in a high surface temperature of about 48°C in daytime. Due to high temperature and negligible amount of oxygen, there is no possibility of existence of life on Venus. Its axis of rotation has a tilt of 23°.

Question

Venus shows phases similar to our moon. It is half full at maximum elongation. Draw a diagram showing phases of Venus.

Earth

You have much knowledge about the earth from your previous classes (See also Table 1). We will only pose a question for you.

Question

Assuming that an outer space astronomer sends a space probe to the earth. What clue would he be looking for to determine the existence of life on the earth ?

Mars

A glance on Table 1 will reveal the similarities between the Mars and the earth. Mars' rotational period is 24 hours and 37 minutes, and its tilt of axis is approximately 25° to the normal This results in seasons on Mars which last for almost 6 earth months each Due to the higher eccentricity of Mars' orbit these seasonal changes are more extreme. Just like the earth, Mars has polar caps composed of frozen carbondioxide. Mars has atmosphere composed mostly of CO_2 with traces of water vapour. Average atmospheric pressure near the surface of Mars is equivalent to earth's atmospheric pressure at a height of about 32 km Photographs reveal dry river beds which indicate that sometimes in the past Mars was warmer and water flowed over its surface. At present Mars appears to be going through a cold glacial period and there seems to be no possibility of life there. Mars is sometimes called the *red planet* because of its red soil like colour. Its surface has crater like our moon. Mars has two moons of less than 16 km diameter each.

Jupiter

Jupiter is the largest and most massive planet but its density is only 1.33g/cm^3. Despite its tremendous size, approximately 11 times the diameter of the earth, Jupiter rotates once in 9 hours and 0 minutes. Due to this a particle on Jupiter's equator rotates almost at 48,000 km/hr. This results in large equatorial bulge of the planet. It has a thick atmosphere which consists mostly of hydrogen and helium with small amounts of methane and ammonia. This planet has a banded appearance of atmosphere along with a striking permanent marking called the *Great Red Spot*. The Great Red Spot represents rising column of gases caused by a depression of or protrusion on the planet's surface. The size of this spot is such that three earths, side by side, could be dropped into it. Being far away from the sun, Jupiter has low average temperature of 140°C which results in highest escape velocity of 57.5 km/s. Owing to this and its heavy mass, Jupiter is able to retain such thick atmosphere. It also has highest number of moons which are 14 in number. One of its moons is larger than the planet Mercury.

Saturn

It is second in mass and size but perhaps the most beautiful planet. Its distance from the sun is 9.5 A. U. and average surface temperature is 175°C. Saturn has an equitorial diameter of 121,000 km. This size is such that 9 earths, side by side, would fit into Saturn. Its period of rotation is 10 hours and 40 minutes. The density of Saturn is 0.62 g/cm^3 which is less than the density of water. If you could find a large enough lake of water Saturn would float in it. Saturn has the highest albedo of 68 per cent and its atmosphere is predominantly composed of hydrogen and methane with small quantities of helium and ammonia. Due to its large mass Saturn retains 17 moons and at least 7 beautiful rings surround it. The rings are divided in a natural way due to gravitational effects of the planet's moons, into 3 main divisions. The outer ring is called the *grey ring* and the inner *crape ring*. It is likely that the rings were formed from particles of a satellite that was torn apart by the tremendous gravitational force of the planet. The particles, perhaps surrounded by frozen gas, are distributed in a band little more than 60 km thick.

Uranus

The period of rotation of Uranus is 10 hour and 49 minutes It has the highest tilt of axis which is 98°. This results in retrograde rotation of the planet Its atmosphere contains mostly hydrogen, helium and methane. It has 5 moons.

Neptune

The discovery of Neptune was in July 1856. Before that time scientists had predicted the existence of an object which was disturbing the orbit of Uranus This object was later found to be Neptune. It has several characteristics that are common with the planet Uranus (see Table 1) It has 2 moons.

Pluto

Pluto is the most unusual planet with highest eccentricity of 0.248, and highest inclination of orbit, which is 70° Its average distance from the sun is 48 A. U., but it travels within less than 30 A U. at perihelion and out to almost 50 A. U. at aphelion. The diameter of Pluto is about half the diameter of the earth. Consequenty even with the largest telescope Pluto appears only as a tiny speck. The exact information about the atmosphere of Pluto is unknown but its surface temperature is expected to be about—240°C. Some people feel that Pluto was originally not a member of the planetary system but was captured by the sun at a later time. Recent observations reveal that it has one moon.

4. The stars

(*a*) *Position of stars*

You probably have seen a model of the globe. It has the circles of latitude and longitude which are used to locate a city on the surface of the earth.

Question

Exactly how are latitudes and the longitudes defined ? Where are they measured from? What are the latitude and longitude of the city you live in ?

Similarly to locate the position of a star, let us imagine a large sphere, concentric with the earth, on the surface of which all the stars are studded. Such a sphere is called the *celestial sphere*. The projection of earth's equator into the celestial sphere marks the *celestial equator* and the intersection of extended earth's axis with the celestial sphere determines the *celestial poles* as shown in Fig. 4. The apparent yearly path of the sun as seen from the earth, after elimination of *diurnal* (daily) motion, is the

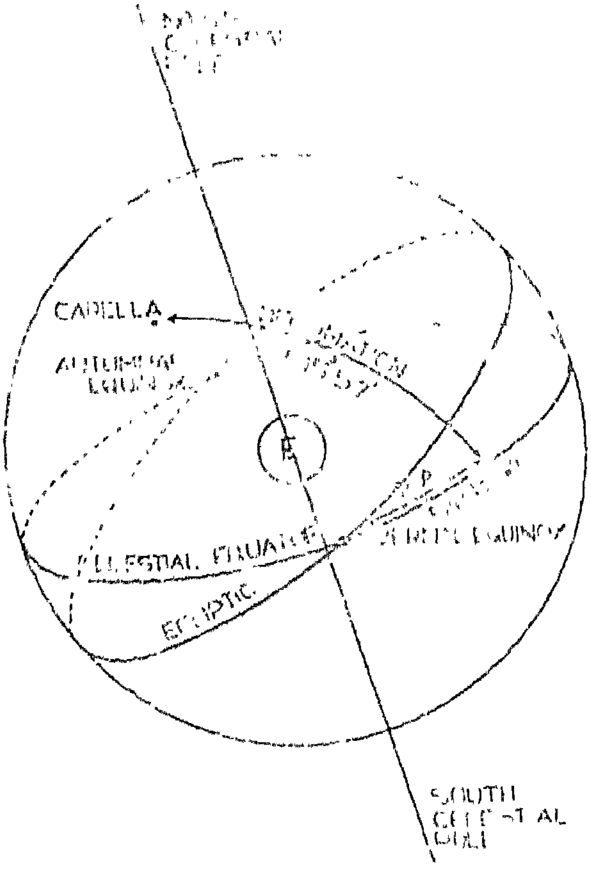


Fig. 4. Celestial sphere

ecliptic. It is called ecliptic because the eclipse of the moon and the sun takes place when the moon is crossing the ecliptic. The sun crosses the celestial equator on March 21 at a point called *vernal equinox*, and again on September 21 at a point called *autumnal equinox*. The projection of the meridians and the parallels of the latitude on to the celestial sphere complete the coordinate system in the sky. The projected celestial meridians are called *hour circles*. The hour circle passing through vernal equinox is called the zero *hour circle*, and successive circles measured to the east are called first, second, third hour circles, and so on to the twenty third hour circle, the twentyfourth hour circle is same as zero hour circle. This angular measure of a star is called *right ascension* and is measured in hours and minutes. The second coordinate required to locate a star is called *declination* which is the angle a star makes with the celestial equator. It is specified as positive (+) if north of equator and negative (−) if south of equator. The position of a star is completely specified by giving its right ascension and declination. The star Copella, for example, has a right ascension (R.A.) $=5^h\ 13^m$ and a declination (Dec.) $=+45°57'$

(b) Stellar distribution

Astronomers have divided the entire star system into 88 constellations. A constellation is a definite region of the sky around group of stars that by its shape suggests an object, person, or animal

A band on the celestial sphere that is centred on the ecliptic and is 16° wide is named by astrologers as the Zodiac belt. It contains 12 constellations and includes all apparent positions of sun, moon, and planets. These constellations and their signs are Aries or Ram, Taurus or Bull, Gemini or Twins, Cancer or Crab, Leo or Lion, Virgo or Virgin, Libra or Balance, Scorpio or Scorpion, Sagittarius or Archer, Capricorn (us) or Goat, Aquarius or Water carrier, Pisces or Fish (es). Within a constellation we can find small and large groupings of stars. A small group is called a galactic cluster and a big one is called a globular cluster.

A galactic cluster contains some twenty to several thousand stars. A familiar example of a galactic cluster is Plough The well known star Sirius belongs to this cluster. The Big Dipper or Ursa Major is another galactic cluster which has a size of about 100 light years from one end to the other. Taurus group consists of 80 stars. These 80 stars are lying within a radius of about 16 light years. The centre of this group is 130 light years away from us. The well known star Aldebaran belongs to this group. Pleiades, popularly known as seven sisters consists of six naked eye stars. With the help of a telescope 20 to 30 individual stars of this group can be seen.

Unlike a galactic cluster, a globular cluster is extremely stable. As its name suggests, it has a spherical shape. They contain different types of stars as compared to a galactic cluster. The number of stars that makes a globular cluster varies from several thousands to a few million; on an average around half a million The diameters of globular clusters are normally between 70 and 600 light years, the average being approximately 200 light years.

The brightest globular cluster is Omega Centuri. It is at a distance of 22000 light years from us. The globular cluster M13 in the constellation of Hercules is 3400 light years away from us. Around 100 globular clusters are identified in the Milky Way More than 30 per cent of them belong to the constellation of Sagittarius. Globular clusters have also been detected in external galaxies

(c) *Magnitude of stars*

From the earliest time, observers of heavens tried to group the stars into some kind of a pattern. The earliest classification was based on the differences in brightness. More than 2000 years ago Hipparchus divided the naked eye stars into six classes depending upon the magnitude of their brightness. The brightest star was placed in the first magnitude group, the next brightest star in the second magnitude group and so on upto the sixth magnitude group. The ratio of brightness between successive magnitude groups was found to be 2.512. A difference of five magnitudes (say between 1 and 6) represents a ratio in brightness 2.512x2.512x 2.512x2.512x2.512=$(2.512)^5$ =100. This means a star of magnitude one is 100 times brighter than a star of magnitude 6.

Use of telescope made it possible to observe stars fainter than 6th order magnitudes. The stars fainter than that of sixth order magnitudes were successively placed in 7th order, 8th order magnitudes etc. It has become possible today to observe stars of 28th order magnitude brightness which is 10^{22} times dimmer than sun.

Modern astronomers have been able to measure the brightness of stars very accurately. The star Aldebaran has a magnitude 1.1. Regulus slightly dimmer than Aldebaran is of magnitude 1.3. Polaris (North Star) has a magnitude of 2.1. Electra belonging to the galactic cluster Pleiades has a magnitude 3.8.

There are brighter stars than Aldebaran. Procyon, for example, is of magnitude 0.5. Vega is of magnitude 0.1. In this way we can have magnitude zero also.

Stars brighter than those of magnitude zero are assigned negative magnitudes. Canopus has a magnitude −0.7. Sirius is of magnitude −1.4.

The brightness we were discussing so far was the apparent brightness, i.e. the brightness as it appears to us. As stated previously, a star may appear bright due to its nearness to us or due to its intrinsically greater brightness. If the sun and Sirius were at equal distance, Sirius would have appeared 3.5 magnitude brighter than the sun, i.e. $(2.512)^{3.5} = 25$ times as bright as the sun. Therefore for comparison we define absolute magnitude of stars as their brightness when placed at a distance of 1 parsec = 32.6 light years (see Table 2 on page 20).

(*d*) *Colour of stars*

Stars have different colours, for example the star Antares is of red colour, Capella is of yellow colour, Sirius is white and Vega bluish-white The colour of a star is actually related with the temperature of the star, according to Wien's law $\lambda_m T$=constant (see Unit 14). A cool star (temperature around 3000 K) appears red and a hot star (temperature around 20,000 K) appears blue. The hottest known star has a temperature of 100,000 K. Sun with a surface temperature of 6000 K may be considered as a cool star.

(*e*) *Stellar spectra*

Spectroscopic analysis of the stellar radiation provide new insight into the nature and composition of stars. The stellar spectrum varies from star to star The spectral characteristics of stars are intimately connected with its temperature. The spectrum of a hot star is quite different from that of a cool star. Each element in stars has its own characteristic spectrum

By observing the lines in the stellar spectrum we can identify the elements present in the star. We can also have an idea about the state of ionization of various elements and hence an idea about its temperature.

Stellar spectra is mainly classified into ten groups. Letters are used

Table 2

DATA ON SOME STARS

Star (Spectral type)	Right Ascension h	 m	Declination (°)	 (′)	Distance (Parsec)	Colour	Temperature in K (approximate)	Absolute Magnitude
Rigel (B)	5	12.1	—8	15	250	Blue	20,000	—6.8
Sirius (A)	6	42.9	—16	39	2.7	White	9,500	1.4
Canopus (F)	6	22.8	—52	40	90	Green	7,000	—3.1
a-Centuri (G)	14	36.2	—60	38	1.3	Yellow	5,800	4.4
Arcturus (K)	14	13.4	+19	27	11	Orange	4,500	—0.3
Betelgeuse (M)	5	52.5	—7	24	200	Red	3,500	—5.5

to designate this spectral sequence. They are O, B, A, F, G, K, M, R, N, S. The sun belongs to spectral class G.

The R, N, and S type spectra from a separate sequence. Stars belonging to this spectral group have chemical composition slightly different from other stars and they are relatively cool.

O-type stars have a temperature so high that it is difficult to measure it. A rough estimate is that it is about 50000K or more. The dominant lines in O-type spectrum are that of ionized helium. Lines of ionized silicon and nitrogen are also found.

B-type stars have a temperature around 15000K. The prominent lines are that of neutral helium, ionized oxygen and carbon. Since at this temperature hydrogen atoms are ionized, hydrogen lines are not prominent in this spectrum.

A-type stars have a temperature about 9500K The prominent lines are that of hydrogen. Lines of ionized magnesium, silicon, iron, and titanium are present. Lines of neutral atoms are absent because at this temperature almost all metals will be singly on doubly ionized.

F-type stars have a temperature about 6600K. The hydrogen lines are very intense. Weak lines of neutral metals and comparatively stronger lines of ionized metals like calcium, iron, etc. are present.

G-type stars have temperature of 5500K. Hydrogen lines are more intense. Lines of singly ionized iron are also present.

K-type stars have a temperature around 5000K. Weak hydrogen lines are present; Lines of neutral atoms and molecular bands are stronger.

M-type stars have temperature around 1500K. This temperature is too low to excite electrons to higher energy states in several of the metals. So lines of most easily excitable atoms alone are present in M-type spectrum.

From the above description we have seen that the sequence O, B, A, F, G, K, M is arranged in the order of decreasing temperature.

(*f*) *Interstellar space*

The interstellar space contains hydrogen, helium, some metals and small dust particles of non uniform density A cloud of gas or dust in space is called a *nebula*. Clouds of dust particles bigger than the atom reflect light from stars and reveal themselves as *reflection nebula*. Clouds of much smaller particles re-emit light absorbed from stars and appear as *emission nebula*, the examples of which are Orion nebula and Lagoon nebula. Nebulas of higher density block radiations from the stars and appear as *dark nebula*.

(g) *Stellar evolution*

Like anything else in the universe stars are also born, live for sometimes and finally die. Starts are formed when a portion of nebula, for reasons unknown, starts contracting under the influence of gravity This contraction results in compression of the gas whose potential energy changes to heat. More contraction results in higher temperature and at 500,000K energy producing nuclear reactions (hydrogen→helium) start giving birth to a star.

Question

A stellar nuclear reaction can be summarised as

$$6\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 2\,{}_1e^0$$

Mass of hydrogen (${}_1H^1$) → 1.0076 amu, mass of helium (He^4) 4,0027 amu. Compute the mass difference between the R. H. S. and L. H. S. This loss in mass is converted into energy by the equation $E = mc^2$. The sun converts almost 4.5 million tons of mass into energy every second. Compute the solar power.

Ans. 4x10^{26}W

As the star glows it emits radiation. At some stage an equilibrium between gravitational forces pulling the matter inwards and the force due to radiation pressure pushing its matter outwards, is reached which determines the diameter of the then full fledged star. More massive stars have greater gravitational force which requires stronger radiation pressure and consequently higher temperature or luminosity for equilibrium. Thus at this stage high temperature stars have greater mass, and low temperature stars have smaller mass as shown in the main sequence of the H-R diagram of Fig. 5. In this main sequence stars burn for billions of years consuming the energy producing fuel (hydrogen). Bigger stars consume fuel faster than the smaller stars. The sun has consumed about 1/4th of its fuel.

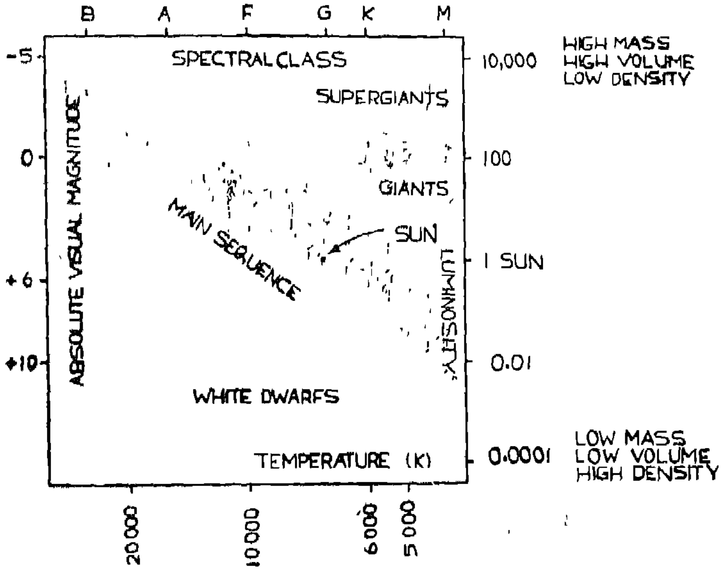


Fig 5 H-R diagram. Temperature of the star is taken along the horizontal axis. On the vertical axis, absolute visual magnitude or luminosity is plotted. Every point in the H—R diagram represents a star. There is a higher concentration of main sequence stars. Sun is a star in this sequence High mass, high volume, low density stars are at the right hand top part and low mass, low volume, high density stars at right hand bottom part of the H—R diagram. White dwarfs are at the left hand bottom part.

When the hydrogen fuel in the core of the star is exhausted, the core begins to contract, releasing more energy, due to nuclear reactions converting helium into heavier elements, that forces the outer region to expand and hence cool. The radius of the star increases and temperature decreases such that luminosity increases and the star becomes a *giant*. Much bigger stars at this stage of evolution are called *supergiants*. The time spent by a star in giant stage is much smaller than the time spent in main sequence. It consumes helium at a very fast rate. Ultimately all energy producing reactions run out of fuel. The star shrinks although not probably without a few brief flare-ups, which we see from the earth as a *nova* (or new star) shining brilliantly for a week or so and then subsiding into insignificance. Shrinking ultimately reduces the star into a collapsed state in which it is called a *dwarf star*.

A star becomes a giant in its middle life as indicated on the right side of the main sequence of the H-R diagram (Fig. 5) and it becomes dwarf in its old age, possessing high density, as shown on the left side of the main sequence. Heavy stars at the upper end of the main sequence having masses greater than 1.5 solar masses, have a different story. They consume hydrogen very quickly and helium reactions are so very strong that the star explodes and appears as *supernova*. It is about 100,000 times brighter than a novae. During explosion its mass reduces and it becomes a dwarf. Due to shrinking its diameter ranges from 6 to 12 kilometres and density becomes several billion tons per cubic centimetre. So high internal pressure is generated that all electrons and protons cancel charges and what remains is a *neutron star*. Some of them also retain strong magnetism and send radio frequency pulses. These are called *pulsars* The crab nebula which is remains of a supernova seen in 1054 A. D still emits x-rays. The remains of supernova explosion of some stars of mass greater than 5 solar masses, has very high gravity in collapsed state. Nothing can escape its gravitational pull. Any particle approaching it is swallowed up Such remains are called *black holes*.

Our sun is a star in the main sequence. It will shine for billions of years without much change. Then its temperature will become very high and life on the earth as we know today, will become impossible.

5. Sun—the super star

The sun is a ball of gas having a diameter of about 1400000 km. Its mass is 333000 times that of the earth and volume 1300000 times. Its average density is 1.4 time that of water on earth. The gravitational force on the surface of the sun is 28 times greater than that on earth's surface.

The outermost surface of the sun is called photosphere. The photosphere consists of gases (mostly hydrogen). Photosphere has a temperature of about 6000K. Photosphere acts like a black body emitting a continuous spectrum.

Sun's atmosphere

Unlike the earths' surface and its atmosphere there is no abrupt change from the sun's surface and its atmosphere. Above the photosphere there are three layers that constitute sun's atmosphere. Immediately above the photosphere and extending to a height of 800 km is a layer of gas called the *reversing layer*. Above the reversing layer and extending to a height of 1000 to 12000 km above the photosphere is *chromosphere.* Above the chromosphere and extending to millions of km is a layer called *Corona.*

Sun's atmosphere consists of gases, mainly hydrogen Traces of other elements are also present.

Solar spectrum

Photosphere of the sun is responsible for the solar spectrum. Solar spectrum is continuous.

In Fig 6 we have shown a plot of observed values of emission for different wavelengths. The curve is for the integrated emission from the disc. Taking wavelength range for the visible light between 0.35μ and 0.75μ, we have shown that the energy emission corresponding to this range is about 44.5%. The emission corresponding to infra-red rays is about 53.5% and that of U. V rays is only about 2%.

From Fig 6, it can be seen that from the wavelength of maximum emission, the fall is too steep on the violet side, and the curve can, under no circumstances, be made to agree with that for any black body at any temperature. This rapid fall on the violet side is certainly to be ascribed to greater diminution of ultraviolet light by the scattering in the atmosphere In fact, this total absorption has been traced to a layer of ozone which is formed in the upper layers of our atmosphere.

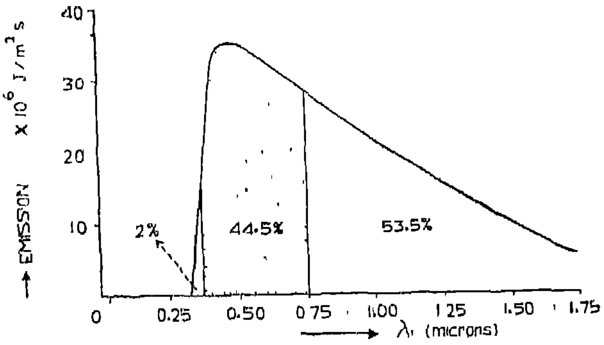


Fig. 6 Solar spectrum

The continuous spectrum is superimposed by dark absorption lines. When the continuous radiations emitted by the photosphere pass through sun's atmosphere, the atoms of various elements present in the atmosphere absorb these radiations. Each element absorbs its own characteristic radiations. The absorbed radiations give rise to dark lines in the spectrum. Therefore by identifying the dark lines in the spectrum we can identify the various element present in the atmosphere. By measuring the intensity of the dark lines (Intensity of an emission line is the amount of light in that line; intensity of a dark line is the amount of light subtracted from the continuous background spectrum) we can determine the concentration of the element responsible for that line.

Interior of the sun

The interior of the sun is of utmost importance The heat and light of the sun is derived from the energy generated in the interior of the sun due to thermonuclear reaction. Sun's interior has a temperature of about 15000000K. The pressure there is about $2 x 10^{11}$ atmospheres and density is 110 times that of water. Under these conditions atoms are forced together so fast that they combine to form a new atom and these thermonuclear reactions, release enormous energy.

Sunspots

Dark spots observed on the surface of the sun are called sunspots. Sunspots appear from time to time, some of the spots are small and do not last very long. Bigger spots may appear for several months. Some sunspots are single and travel along the surface of the sun in the direction of rotation of the sun. Sunspots are characterised by a lower temperature (compared to the 6000K of the photosphere) of the order of 2000K to 4000K and an intense magnetic field of the order of 2000 guass to 3000 guass. The magnetic field has a direction perpendicular to the surface of the sun.

Sunspots appear and disappear from time to time. The number of sunspots appearing varies from year to year. However, it is observed that the number exhibits a periodicity of 11 years i.e. if N sunspots are observed in a particular year γ, the same number of sunspots will be observed in the year $\gamma+11$, $\gamma+22$. This periodicity is known as sunspot cycle.

Many solar activities are directly related with the sunspot cycle. Solar flare up (sudden increase in the intensity of the red H-a light emitted from certain regions on the solar surface), global magnetic storm (the intense emission of streams of electrons, protons and α-particles by the sun during a solar flare up) growth of plants and trees on earth etc. are affected by sunspot cycle.

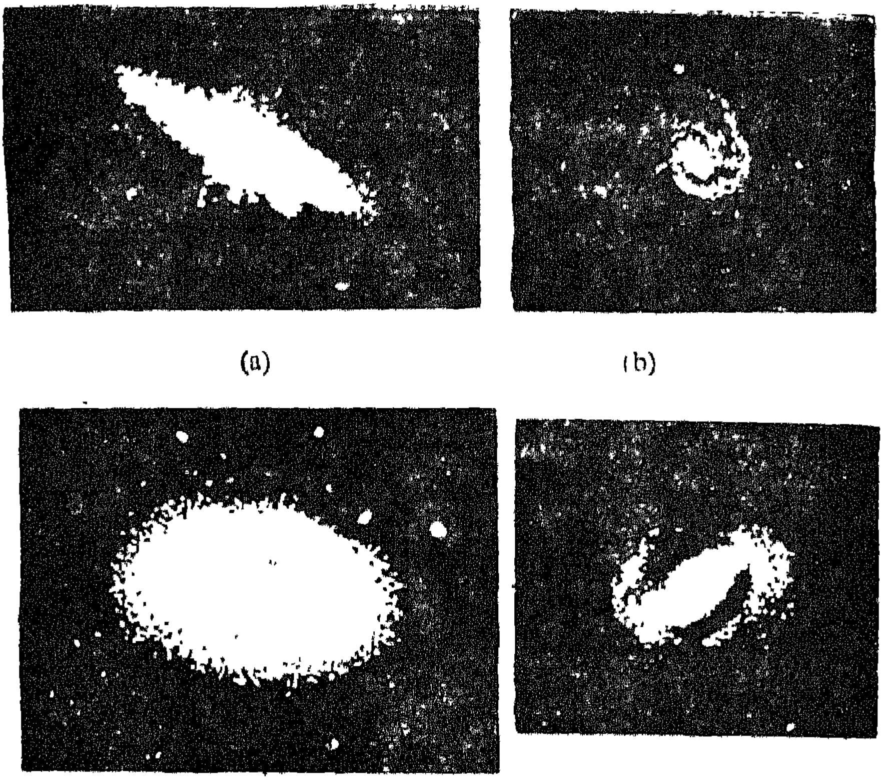
6. Galaxies

Galaxies are the basic building blocks of the universe. They are made up of stars and interstellar matter. The exact number of galaxies in the universe is not known. However, astronomers have been able to identify 100 billion of them.

Galaxies are seen in every portion of the sky. Even though they are enormously distant from us (the most distant galaxies that can be observed through our telescope is estimated to be at a distance of $6x10^6$ light years from us) they appear in our telescopes. They appear as patches of lights of different shapes.

Most of the galaxies have a diameter in the range 7000 to 150000 light years. About 75 per cent of the known galaxies have a disc shaped structure. The thickness of the disc is maximum at the centre and minimum at the boundaries. On an average each galaxy contains about one billion (10^9) to hundred billion individual stars.

Our sun and the solar system belong to the galaxy called Milky Way which is of spiral shape [Fig. 7 (b)]. The nearest galaxies to the Milky Way are the irregular shaped Large Magellanic cloud and the Small Magellanic cloud which are at approximate distances of 160000 and 100000 light years respectively. Their diameters are 3000 light years and 13000 light years respectively. Among the galaxies that are close to our Milky Way galaxy, the best known one is the Andromeda galaxy. This is at a distance of about $15x10^0$ light years from us and it has a diameter of about 100000 light years.



(a) (b)

(c) (d)

Fig. 7 Types of galaxies

a Irregular b. Spiral c. Elliptical d Barred spiral

Rotation of galaxies

Galaxies are rotating about an axis passing through their centre. This is evidenced by the flat shape of the galaxies. Since galaxies are made up of gases and dust its rotation will cause flattening.

The entire galaxy does not rotate like a solid wheel because relative shift of stars in a galaxy has been observed. Our sun revolves around the centre of the Milky Way with a speed of 250km/s The period of revolution around the galactic nucleus is about 240 million years

The Milky Way

As mentioned before, the Milky Way is our home galaxy. It can be seen in a clear night as a white band of light around the celestial sphere. Our sun and the solar system belong to the Milky Way galaxy.

The Milky Way has disc shaped form. The thickness of the disc is greater at the centre and it thins down towards the boundary. The thickness at the centre is estimated to be 10,000 light years and that at the boundary is about 5,000 light years. The diameter of the disc is 100,000 light years. This disc is symmetrical about a central plane called galactic plane. The sun is located towards the edge of this disc, relatively closer to the galactic plane (at about 40 light years from the galactic plane). The distance of the sun from the centre of the galaxy is about 27,000 light years. The model of the Milky Way when viewed edge-wise will have a shape like that shown in Fig. 8

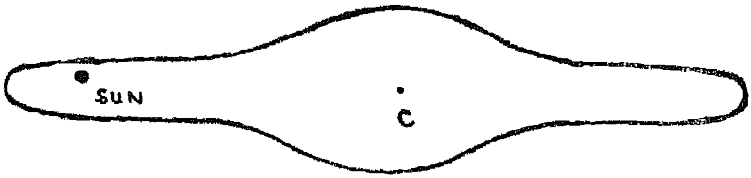


Fig. 8.

There are about 100 billion stars in the Milky Way. These stars are not uniformly distributed. There is a greater concentration of stars towards the centre of the galaxy which is the location of the constellation of Sagittarius.

Milky Way is a spiral type of galaxy, Spiral galaxies are characterized by very rapid rotation and the disc shape of the Milky Way is an indication of its rapid rotation

The arms of the spiral galaxies contain blue giant type of stars. It has been possible to locate blue giant stars in the Milky Way as specific distances and positions corresponding to the location of spiral arms. Our sun is located in one of the arms. The name of this arm is Orion arm. The spiral arms of the Milky Way are about 2,500 light years wide. Adjacent arms are separated by a distance of about 1,500 light years. The shape of the Milky Way when viewed from above or below will have a shape as shown in Fig. 7 (b).

An ordinary galaxy like our Milky Way emits about 10^{28} kilowatts of power in the form of radiowaves. But astronomers have identified sources of radiowaves called *radio galaxies* which radiate million times more energy. For example Cygnus A radiates 10^{34} kilowatts in radio region and a weak galaxy NGC 1098 emits 10^{30} kilowatts.

In addition, there are unusually small and unusually compact radio sources in the sky which are called *quasars* They emit a large quantity of microwaves as well as visible light.

Measurements have shown that quasars are at enormously large distance from us. These distances are of the order of billions (10^9) of light years. Since they appear bright even from such an enormous distance they should be extraordinarily luminous. They have to be thirty to hundred times as luminous as an entire galaxy. If this is to be the case, such a galaxy should contain up to hundred times as many stars as there are in an ordinary galaxy and its dimensions should be five to six times greater than that of an ordinary galaxy. Then, even from this enormous distance they should have shown a definite shape in the most powerful telescope. But the fact is that even in this telescope they appear as tiny stars. This forces us to conclude that in spite of the enormous luminosity quasars should be small in size. Another observation which is in favour of the smallness in size of quasars is the variation in the energy they emit which is characteristic of small bodies. Quasars are estimated to be about one fifth in size of a galaxy. The masses of quasars have been estimated to be of the order of 10^8 solar masses They are considerably less massive than ordinary galaxy.

7. Theories of the origin of universe

While the human knowledge about the universe is insufficient, theories regarding the origin of universe have been formulated based on the collected data. A study of spectra of galaxies, which form the building blocks of the universe, shows shift of spectral lines towards the red region implying galactic recession. Based on observations of near galaxies the Hubble's red shift law, stated below, was formulated.

The larger the red shift (or greater the recessional velocity) in spectral lines of a galaxy, the farther the galaxy is from the observer. Or mathematically,

$$V=HR$$

Where V is the recessional velocity of a galaxy, R is the distance from the earth to the galaxy; and $H=9.4\times10^{-18}\ \mathrm{s}^{-1}$ or 16,000 m/s for every million light years, is the Hubble's constant. This suggests that the universe of galaxies is ever expanding.

Question

Assuming that no galaxy can move faster than light, estimate the limit of the observable sky.

Ans. 10^{25} km.

The extent of red shift gives the velocity of a galaxy. Knowing the velocity we can find the distance and hence the density of galaxies Although more data is required, it appears that galaxies are distributed uniformly throughout the universe.

Question

If galaxies are uniformly distributed throughout the universe, are we at the centre ? Does it matter ?

The proponents of *steady state theory* assert that the universe always appeared as it does today. Although the universe is expanding more galaxies are created, by conversion of energy to mass, so that the density of galaxies remains constant. This theory suggests that creation of matter has been going on indefinitely in the ast and will continue indefinitely in future.

The big bang theory, on the other hand, postulates that the entire mass of the universe was initially concentrated in a primal nucleus. A big bang (explosion) took place in the primal nucleus throwing out masses in to space. This results in an ever expanding universe where density of galaxies is decreasing.

Question

Assuming that the galaxies are moving with constant velocity ever-since the big bang, estimate the age of the universe.

Ans. $\frac{1}{H}=3.5\times10^{9}$ years.

Astronomers have observed radiations of a few millimetre wavelength coming from all parts of the sky. This background radiation has a temperature of 3K and is believed to be the remnant of the original very hot radiation of 10^{10}K which existed at the time of big bang. The theory of *Oscillating universe* is similar to the big bang theory but suggests that the expansion of universe will stop one day and then universe will start shrinking. Ultimately the entire mass of universe will be concentrated in a small volume. Due to very high temperature there will again be a big explosion. The universe will keep *oscillating* in this manner. The cycle of one oscillation is estimated to take 80 billion years. But upto now only 10 billion years have elapsed. The evidence to support the idea of oscillating universe comes from distant galaxies and quasars which actually show that the rate of expansion of the universe is slowing down

Astronomers have succeeded in answering several questions but many more questions, have arisen. Man must continue the seemingly everlasting search for the unknown.

SELF-ASSESSMENT

1. (a) A man weighs 680 newtons on the earth. Determine his weight on the Mars.
(Given, radius of the Mars– 0.53 radius of the earth, and mass of Mars $\approx$ 0.11 mass of the earth)

(b) Calculate the values of solar constant at Venus and Mars—0.793 A.U. and 1.524 A.U. away from the sun respectively.
(Given, solar constant on earth—$1.388 \times 10^3 W/m^2$)

2. (a) If the two components of a binary system of stars have the same spectra while one component is about 1,000 times luminous than the other. Calculate the ratio of their radii, establishing the formula to be used.

(b) How far is the Venus from the earth on an occasion when the round-trip for a radar signal is 5 4 minutes ?

3. (a) How is it possible to deduce the mass of Jupiter with the help of observation on one of the satellites of Jupiter ? What quantities must be known before the mass can be found ?

(b) How many earths could fit into the volume occupied by Jupiter ?
(Given, R (*earth*)/R (*Jupiter*) = 1/11.23)

4. (a) The surface temperature of Procyon is about 7000 K, what is the approximate wavelength of the maximum emission ?

(b) At what rate energy is emitted by a square centimetre at the surface of Procyon ?

(Given, Stefan's constant $\sigma = 5.669 \times 10^{-8}$ MKS Units)

5. (a) What are some of the reasons why a reflector prforms better than a refractor of the same aperture ?

(b) Calculate the effective area and the resolving power of a radio telescope with diameter 70 metres and operating on a wavelength of 3 metres,

6. (a) Describe a likely birth place of stars and the process that brings them into being.

(b) Differentiate the following :

(i) Neutron star and Pulsars.

(ii) Radio galaxies and Quasars

(iii) Black hole and white dwarf.

(iv) Giant star and Nova.

7. (a) Recognizing that the facts are not all in, review our current but tentative ideas about the general character of the universe.

(b) Discuss some of the criticisms that have been made of the time varying models of the universe.

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